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SMART ROTOR UNDER UNCERTAIN PARAMETERS

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Abstract: This contribution aims at analyzing the effect of uncertain parameters on the dynamic response of flexible rotor with active vibration control. The uncertain parameters are considered as small variations with respect to their nominal values that are modeled by means of random variables and random fields. The random fields are discretized according to the spectral method by using Karhunen-Loève expansions. The dynamics of the smart rotor includes the flexible rotor and active vibration control. The structure response subjected to uncertain parameters is analyzed based on the stochastic modeling. Numerical simulations illustrate the dynamic response of the rotors showing the improvement of the performance with robustified controllers.

Keywords: Rotor Dynamics, Stochastic Modeling, Robust Control, Uncertain Parameters

INTRODUCTION

Smart rotor systems include a control system in order to attenuate the vibration of the flexible shaft. These mechanical systems are unavoidable subject to uncertainties, nevertheless they demand optimal performance (Koroishi *et al.*, 2015). The main sources of uncertainties include the variation of mechanical properties, such as damping, stiffness, and geometrical imperfections (Lalanne and Ferraris, 1998). Consequently, the quantification of parametric uncertainties is necessary to predict how the dynamic response varies, by means of numerical models of industrial applications and research testbeds (Koroishi *et al.*, 2012).

Several numerical methods have been applied in order to quantify the effects of uncertain parameters in the dynamic response of flexible rotors systems. Didier et al. (2012) quantified the uncertainty effects in the response of flexible rotors based on the Polynomial Chaos Theory. Koroishi *et al.* (2012) represented the uncertainties in the rotor parameters by using Gaussian homo-geneous stochastic fields discretized by Karhunen-Loève expansion; the dynamic response of the system with uncertainties was characterized through Latin Hypercube Sampling and Monte Carlo Simulation. Lara-Molina *et al.* (2015) used *Fuzzy Finite Element Method* and *Fuzzy Stochastic Finite Element Method* to quantify the uncertain response produced by the uncertain parameters which were modeled as fuzzy fields and fuzzy variables.

The present contribution is dedicated to analyze the effect of the uncertain parameters smart rotor in order to analyze how the uncertainties affects the overall performance of the system. In a flexible rotor system controlled by an Electromagnetic Actuator by using a modal active vibration control scheme. The Linear quadratic regulator (LQR) is incorporated to the control strategy. In the present case the modal displacements and modal velocities are used by the controllers to determine the control force. Additionally, the LQR was solved by Linear Matrix Inequalities (LMIs).

The main objective of this work is to analyze the effectiveness of the controller to attenuate vibration by using robust controllers for active modal vibration control in the presence of uncertain parameters, namely the linear quadratic regulator as solved by LMIs. In a previous contribution (Koroishi *et al.*, 2015) control techniques dedicated to rotor dynamics were investigated by using controllers in which uncertainties where considered in the estimator of the control approach which implies an recurrent identification of the model. However, in the present work robust controllers were tested numerically by evaluated the uncertain parameters aiming at study the performance of robust controllers in this case.

The remain of this paper is organized in four parts, as follows: the deterministic rotor model is defined by using the Finite Element Method (FEM) and the stochastic rotor model is formulated with the aid of stochastic finite element method; the control approach and the architecture of the control system are discussed; simulation results are performed for the FRFs and the orbits of the shaft; finally, conclusions are summarized.

FLEXIBLE ROTOR MODEL

In this section is presented the deterministic model and the stochastic modeling of the flexible rotor system. The stochastic modeling was presented in previous contributions, (Koroishi *et al.*, 2012), where additional details can be found.

Theoretical background on the deterministic finite element modeling of a rotor system

The dynamic response of the considered mechanical system can be represented by using principles of variational mechanics, namely the Hamilton‘s principle. For this aim, the strain energy of the shaft and the kinetic energies of the shaft and discs are calculated. An extension of Hamilton‘s principle makes possible to include the effect of energy dissipation. The parameters of the bearings are included in the model by using the principle of the virtual work. For computational purposes, the finite element method is used to discretize the structure so that the energies calculated are concentrated at the nodal points. Shape functions are used to connect the nodal points ((Morais, 2010)). In this model, 4 degrees of freedom per node are taken into account, namely two displacements (u and w) and two rotations (θ and ψ). The Fig. 1 shows the finite element used to model the rotor.

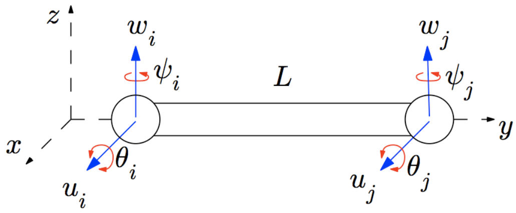


Figure 1: Illustration of the shaft finite element (Simões *et al.*, 2007).

The following assumptions are considered in the formulation of the model of the flexible rotor: i) the beam element is modeled according to Euler-Bernoulli’s theory, ii) the material presents linear behavior and it is homogeneous and isotropic; and iii) the effects of inertial rotations are taken into account.

According to (Lalanne and Ferraris, 1998) the model depicted in Fig. 1 can be represented mathematically by the following set of differential equations:

$$\mathbf{M}\ddot{\boldsymbol{\delta}}(t) + [\mathbf{C}{+}\Omega\mathbf{G}]\dot{\boldsymbol{\delta}}(t) + \mathbf{K}\boldsymbol{\delta}(t) = \mathbf{F}(t) \tag{1}$$

where $\mathbf{M} = (\mathbf{M}_S + \mathbf{M}_D) \in \mathbb{R}^{N\times N}$ and $\mathbf{K} = (\mathbf{K}_S + \mathbf{K}_B) \in \mathbb{R}^{N\times N}$ are, respectively, the mass and stiffness matrices, $\mathbf{C} = (\mathbf{C}_B + \mathbf{C}_P) \in \mathbb{R}^{N\times N}$ is the damping matrix formed by the contributions of the viscous damping matrix, $\mathbf{C}_B$ is the viscous damping matrix of the bearings, and the inherent proportional damping matrix, $\mathbf{C}_P = \alpha\mathbf{M} + \beta\mathbf{K}$, and $\mathbf{G} = (\mathbf{G}_D + \mathbf{G}_S \in R^{N\times N}$ designates the gyroscopic matrix formed by the gyroscopic contributions of the rigid discs and the shaft. $\boldsymbol{\delta}(t) \in R^N$ and $\mathbf{F}(t) \in R^N$ are, respectively, the vectors of the amplitudes of the harmonic generalized displacements and external loads, Ω is the angular speed of the shaft, and α and β represent, respectively, the proportional coefficients of mass and stiffness.

Parameterization of the deterministic FE model

At this point it is important to consider that, in order to study the system behavior when uncertainties are to be taken into account, the random responses have to be computed with respect to a set of uncertain geometrical and/or physical parameters associated with the flexible rotor. In general, such random variables intervene in a rather complicated manner in the finite element matrices. Hence, for evaluating the variability of the responses associated with these uncertainties, it becomes interesting to perform a parameterization of the FE model, which is understood as a means of making the design parameters factored-out of the elementary matrices. At the expense of lengthy algebraic manipulations, this procedure makes it possible to introduce not only the uncertainties into the flexible rotor model, but also to perform a sensitivity analysis in a straightforward way, leading to significant cost savings in iterative robust optimization and/or model updating processes. After manipulations, those parameters of interest can be factored-out of the elementary matrices as indicated below:

$$\begin{aligned}
\mathbf{M}_S^{(e)} &= \rho_S A_S \overline{\mathbf{M}}_S^{(e)} \\
\mathbf{K}_S^{(e)} &= E_S I_S \overline{\mathbf{K}}_S^{(e)} \\
\mathbf{G}_S^{(e)} &= \rho_S I_S \overline{\mathbf{G}}_S^{(e)} \\
\mathbf{K}_B^{(e)} &= k_{xx} \overline{\mathbf{K}}_B^{(e)} + k_{zz} \overline{\mathbf{K}}_B^{(e)} \\
\mathbf{C}_B^{(e)} &= d_{xx} \overline{\mathbf{C}}_B^{(e)} + d_{zz} \overline{\mathbf{C}}_B^{(e)}
\end{aligned} \tag{2}$$

where ρ_S, A_S, I_S and E_S represent, respectively, the mass density, the cross-section area, the inertia and the Young’s modulus of the shaft. d_{xx}, d_{zz} and k_{xx} and k_{zz} designate, respectively, the damping and stiffness coefficients of the bearings. It is worth mentioning that the matrices appearing in the right hand side of Eqs. (2) are those from which the design parameters of interest have been factored-out.

Stochastic Modeling

The design parameters which have been factored-out of the matrices appearing in Eq. (2) are considered to be random in order to model the system behavior when uncertainties are present in the shaft and bearing elements. The so-called Karhunen-Loève (KL) decomposition, which is a continuous representation for random fields expressed as the superposition of orthogonal random variables weighted by deterministic spatial functions, is used in this paper. A one-dimensional random field $H(y,\theta)$ is defined by its mean value, $E[H(y,\theta)]$, and covariance $C(y_1,y_2) = E\{[H(y_1,\theta) - E(y_1)][H(y_2,\theta) - E(y_2)]\}$, where y represents the spatial dependence of the random field, $E[.]$ represents the expectation operator, and θ represents a random process.

For a one-dimensional homogeneous Gaussian random field, it is possible to find a single projection of on an orthonormal truncated random basis as (Ghanem and Spanos, 2003):

$$H(y,\theta) = E(y) + \sum_{r=1}^{n} \sqrt{\lambda_r} f_r(y)\xi_r(\theta) \tag{3}$$

where the deterministic functions $f_r(y)$ and the scalar values λ_r are, respectively, the eigenfunctions and the eigenvalues of the covariance $C(y_1,y_2)$. Also, the functions $f_r(y)$ and the random variables $\xi_r(\theta)$ are orthonormal.

For the relatively simple geometry of the one-dimensional rotor model shown in Fig. 1 configurations, the analytical solution of the eigenproblem proposed by Ghanem and Spanos (1991) for the KL expansion into the domain, $\Omega_y = (y_1,y_2)$, is given by:

$$C(y_1,y_2) = exp\left(\frac{-|y_1-y_2|}{L_{cor,y}}\right) \tag{4}$$

Where $(y_1,y_2) \in [0,L]$ and $L_{cor,y}$ indicates the correlation length indicates the correlation length characterizing the decreasing behavior of the covariance with the distance between the observation points in the direction. Also, it can be noted that this continuous model of covariance function corresponds to a homogeneous (stationary) random field.

Taking into account the property of the covariance function (Eq. (4)), the eigenvalues and eigenfunctions are given as a function of the roots $\omega_r (r \geq 1)$ of two transcendental equations in a procedure summarized as follows:

- For r odd, with $r \geq 1$:

$$\lambda_r = \frac{2L_{cor,y}}{L_{cor,y}^2\omega_r^2 + 1}, \quad f_r(y) = \alpha_r \cos(\omega_r y) \tag{5}$$

  where $\alpha_r = 1/\sqrt{L/2 + \sin(\omega_r L)/2\omega_r}$ and the root ω_r is the solution of the following transcendental equation:

$$1 + L_{cor,y}\omega_r \tan(\omega_r L) = 0 \tag{6}$$

  defined into the domain $\left[(r-1)\frac{\pi}{a}, (r-\frac{1}{2})\frac{\pi}{a}\right]$.

- For r even, with $r \geq 1$:

$$\lambda_r = \frac{2L_{cor,y}}{L_{cor,y}^2\omega_r^2 + 1}, \quad f_r(y) = \alpha_r \sin(\omega_r y) \tag{7}$$

  where $\alpha_r = 1/\sqrt{L/2 - \sin(\omega_r L)/2\omega_r}$ and the root ω_r is the solution of the transcendental equation:

$$L_{cor,y}\omega_r + \tan(\omega_r L) = 0 \tag{8}$$

  defined into de domain $[(r-\frac{1}{2})\frac{\pi}{a}, r\frac{\pi}{a}]$.

The expansion detailed previously has been chosen in order to model the elementary random stiffness matrices of the shaft element, as follows:

$$\mathbf{K}_S^{(e)}(\theta) = \mathbf{K}_S^{(e)} + \sum_{r=1}^{n} \overline{\mathbf{K}}_{Sr}^{(e)}\xi_r(\theta) \tag{9}$$

where $\mathbf{K}_S^{(e)}$ is the elementary stiffness matrix (Simões *et al.*, 2007), and the random matrix is calculated as follows:

$$\overline{\mathbf{K}}_{Sr}^{(e)}(\theta) = \int_{y=0}^{L} \sqrt{\lambda_r} f_r(y)\mathbf{B}^T(y)\overline{\mathbf{E}}\mathbf{B}(y)dy \tag{10}$$

where $\overline{\mathbf{E}}$ is the elastic material property matrix in which the Youg's modulus, E_S, was previously factored out, and matrix $\mathbf{B}^T$ is formed by differential operators of the strain displacement relation.

Typically the stiffness and damping of the bearings are subjected to uncertainties. Additionally, these parameters should be identified for control applications, the identified parameters include small error that are introduced in the control algorithms. In accordance with the stochastic modeling, the uncertainties of the stiffness and damping of the bearings are included by means of random variables. The corresponding uncertainties are introduced by using the relation:

$$a_0(\theta) = a_0 + a_0\delta_a\xi(\theta) \tag{11}$$

where a_0 is the mean value of the parameter, δ_a is the dispersion level and $\xi(\theta)$ is the unite normal random variable with θ being a random process. The unite normal random variable is governed by a normal distribution, this distribution was selected in order to evaluate the uncertain parameters in this contribution.

The so-called Monte Carlo method combined with the Latin Hypercube sampling (Florian, 1992) is used to simulate the dynamic response of the robot with the considered uncertain random parameters. Additionally, with the aid of a convergence analysis helps determining the number of Monte Carlo samples n_s to obtain an accurate result in the simulations.

CONTROL APPROACH

Active modal control is used as control strategy for a rotor system in which an Electromagnetic Actuator (EMA) provides the control effort. The advantage of using active modal control is that this technique is very effective for flexible structures applications, requiring a reduced number of actuators and sensors. The estimator is responsible for determining the modal states required by the controllers. In a previous contributions (Mahfoud and Hagopian, 2011) used the modal estimation to calculate the air gap at the node where the EMA is located and a general control strategy was proposed. Additional details of the operation and model of the EMA can be found (Koroishi *et al.*, 2014). In the present contribution, this control strategy uses the Kalman Estimator performing two different roles: the first is to work as a filter of the dynamic response of the system, and the second (and most important function in this case) is to estimate the states and the displacement of the system at the actuator's position. The states are then used by the controllers to determine the control force. As the air gap in the EMA is very small (it is not feasible to insert sensors) the displacement at the actuator's position has to be estimated by the Kalman Estimator.

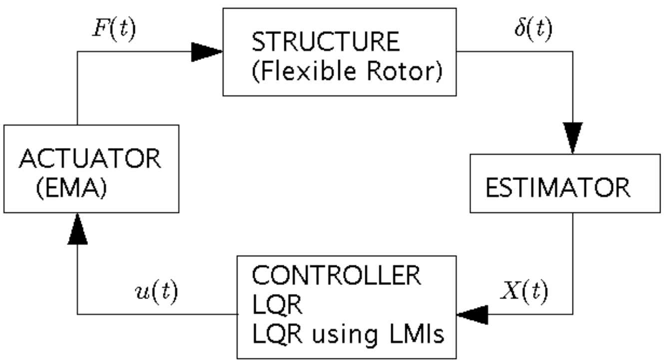


Figure 2: Control approach of the flexible rotor.

Structure Model

For control purposes, the use of a large number of degrees of freedom (dof) produces a high computational cost. For this reason, the size of the model should be reduced. In order to reduce the model size, the psudo-modal method is used in the present contribution. This method consists in changing the physical coordinates $\boldsymbol{\delta}(t)$ to modal coordinates $\mathbf{q}(t)$ as follows:

$$\boldsymbol{\delta}(t) = \mathbf{\Phi}\mathbf{q}(t) \tag{12}$$

Where $\mathbf{\Phi}$ is the modal basis that contains the m first modes of the non-gyroscopic conservative associated system. This pseudo-modal method was proposed by (Lalanne and Ferraris, 1998). The transformation of eq. (12) is applied to the model of the flexible rotor of eq. (1), additionally, the space-state representation is defined with the new set of equations

$$\begin{aligned}\dot{\mathbf{X}}(t) &= \mathbf{A}\mathbf{X}(t) + \mathbf{B}_w\mathbf{F}(t) + \mathbf{B}_u\mathbf{F}_C(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{X}(t)\end{aligned} \tag{13}$$

where

$\mathbf{X}(t)\begin{Bmatrix}\mathbf{q}\\ \dot{\mathbf{q}}\end{Bmatrix}$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbb{I} \\ -[\mathbf{\Phi}^T\mathbf{M}]^{-1}[\mathbf{\Phi}^T[\mathbf{K} + \phi\mathbf{K}_G]\mathbf{\Phi}] & -[\mathbf{\Phi}^T\mathbf{M}]^{-1}[\mathbf{\Phi}^T[\mathbf{C}_B + \dot{\phi}\mathbf{C}_G]\mathbf{\Phi}] \end{bmatrix}$$
$$\mathbf{B}_w = \begin{bmatrix} \mathbf{0} \\ [\mathbf{\Phi}^T\mathbf{M}]^{-1}\mathbf{\Phi}^T \end{bmatrix}$$
$$\mathbf{B}_u = \begin{bmatrix} \mathbf{0} \\ [\mathbf{\Phi}^T\mathbf{M}]^{-1}\mathbf{\Phi}^T \end{bmatrix}$$
$$\mathbf{C} = \mathbf{\Phi}$$

Controllers

In this contribution two different techniques are used to design the controllers.

The Linear Quadratic Regulator

The Linear Quadratic Regulator (LQR) has a very important role in the study of multivariable control of dynamic systems, not only as a powerful control technique, but also because it represents the source of many recently developed procedures for designing linear Multiple-Input-Multiple-Output (MIMO) systems. Besides providing a methodology to control the feedback gain, the LQR ensures good stability margins for the closed loop system.

The LQR is designed so that the minimization of the corresponding performance index leads to the optimization of pre-defined physical quantities (Ogata, 2010). Considering the feedback control given by

$$\mathbf{u} = -\mathbf{G}\mathbf{x}(t) \tag{14}$$

The gain $\mathbf{G}$ can be determined by minimizing the performance index of eq. 15

$$J = \int_0^{\infty} (\mathbf{x}(t)^T\mathbf{Q}_{LQR}\mathbf{x}(t) + \mathbf{u}(t)^T\mathbf{R}_{LQR}\mathbf{u}(t))dt \tag{15}$$

where $\mathbf{Q}_{LQR}$ is a positive defined hermitian matrix (positive definite or semi-definite) or real symmetric that weights each state, and $\mathbf{R}_{LQR}$ is a positive defined hermitian matrix or real symmetric that weights the energy cost of each controller (Simões *et al.*, 2007). By substituting eq. (14) into eq. (15), it is possible to obtain the controller gain $\mathbf{G}$. The controller gain $\mathbf{G}$ is the matrix that minimizes the performance index J.

Linear Quadratic Regulator using LMIs

Classic LQR control cannot cope with system uncertainty. In the present contribution the LQR is also solved by the LMI method with the purpose of obtaining a robust controller that is able to handle parametric uncertainties as previously presented by (Koroishi *et al.*, 2015).

A version of LQR solved by LMIs is illustrated in (Erkus and Lee, 2004), the authors of this contribution show that the problem LQR-LMI is described by:

$$\min_{\mathbf{X}_{lmi},\mathbf{Y}_{lmi},\mathbf{P}_{lmi}} tr(\mathbf{Q}_{lqr}\mathbf{P}_{lmi}) + tr(\mathbf{X}_{lmi}) + tr(\mathbf{Y}_{lmi}\mathbf{N}) + tr(\mathbf{N}^T\mathbf{Y}_{lmi}^T) \tag{16}$$

subjecto to:

$$\begin{gathered} \mathbf{A}\mathbf{P}_{lmi} - \mathbf{B}\mathbf{Y}_{lmi} + \mathbf{P}_{lmi}\mathbf{A}^T - \mathbf{Y}_{lmi}\mathbf{B}^T + \mathbf{B}_w\mathbf{B}_w^T < 0 \\ \begin{bmatrix} \mathbf{X}_{lmi} & \mathbf{R}_{lqr}^{1/2}\mathbf{Y}_{lmi} \\ \mathbf{Y}_{lmi}^T\mathbf{R}_{lqr}^{1/2} & \mathbf{P}_{lmi} \end{bmatrix} > 0 \end{gathered} \tag{17}$$

where $\mathbf{N}$ is a noise position vector, $\mathbf{P}_{lmi}$, $\mathbf{X}_{lmi}$ and $\mathbf{Y}_{lmi}$ are the LMIs solutions and $tr()$ denotes the matrix trace.

The objective of the present contribution is designing robust controllers. Equation (17) was used to obtain robust controllers for the LQR control strategy solved by LMI that allows the inclusion of uncertainty in the models.

NUMERICAL RESULTS

The rotor's model is represented by the set of differential equations as stated by (Lalanne and Ferraris, 1998) in eq. (1). The rotor's model was obtained by discretizing the shaft into 32 Euler-Bernoulli beam elements. The disks D_1 and D_2 are located at nodes $\#13$ and $\#22$, the bearings B_1 and B_2 are located at nodes $\#4$ and $\#31$ and the sensors to measure the two orthogonal displacement (along x and z axis) located at nodes $\#8$ and $\#25$.

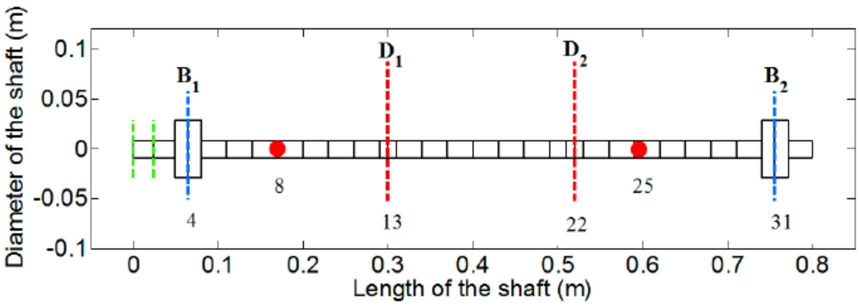


Figure 3: Finite Element Model of Rotor.

Table 1: Parameters of the rotor bearing system.

Rotor Parameter	Value	Bearings Parameter	Value
Mass of shaft (Kg)	4.1418	k_{x1} (Nm)	$7.73X10^5$
Mass of Disk D_1 (Kg)	2.6495	k_{z1} (Nm)	$1.13X10^5$
Mass of Disk D_2 (Kg)	2.6495	k_{x2} (Nm)	$5.51X10^8$
Thickness of Disk D_1 (m)	0.0100	k_{z2} (Nm)	$7.34X10^8$
Thickness of Disk D_2 (m)	0.0100	c_{x1} (N.s/m)	5.7876
Diameter of shaft (m)	0.0290	c_{z1} (N.s/m)	12.6001
Young Modulus (GN/m^2)	205	c_{x2} (N.s/m)	97.0231
Density (Kg/m^3)	7850	c_{z2} (N.s/m)	77.8510
Poisson Coefficient	0.3		

The parameters of the bearings were determined experimentally in previous works (Koroishi *et al.*, 2015). These parameters together with the rotor properties are presented in Tab. 1.

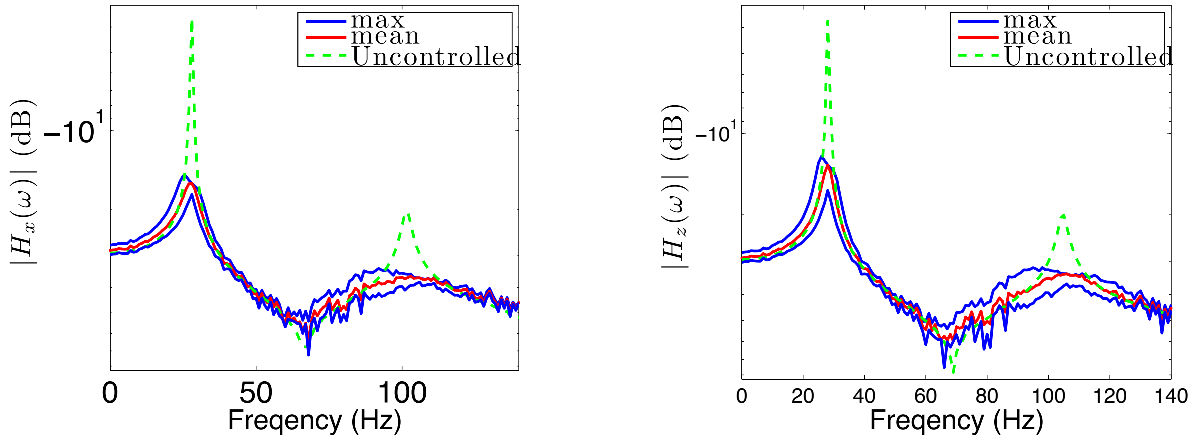
The dynamics of rotor system was numerically simulated in order to analyze the structure response in the presence of the parametric uncertainties. LQR and the LQR using LMIs controllers were used in the active modal control. Thus, the structure response was analyzed in terms of orbits and FRFs of the rotor system. The simulations were implemented in Matlab software platform. The number of vibration modes taken into account are defined according to the controllability and observability of the rotor system represented by eq. (13). In this contribution, four modes were taken into account (two for each direction), for this case the system is controllable and observable.

The dynamic response of the rotor system was computed considering $\%5$ of dispersion in the Young's modulus of the shaft, and the stiffness and damping of the bearings. Consequently, nine different parameters are considered in this contribution: E_S, k_{x1}, k_{z1}, k_{x2}, k_{z2}, c_{x1}, c_{z1}, c_{x2}, c_{z2}.The computations of the stochastic stiffness matrix of the shaft element is performed by assuming the correlation length $L_{cor,y}$ = 0.02725m, corresponding to the length of the beam elements, according to FE mesh.

Figures 4 and 5 show the FRFs amplitudes that correspond to the displacement at node $\#25$ of the shaft in the directions x and z for an impulsive excitation applied in x and z direction at the first disk, D_1. Figure 4 shows the FRFs when the LQR controller is applied, and similarly, Fig. 5 presents the FRFs for LQR using LMIs.

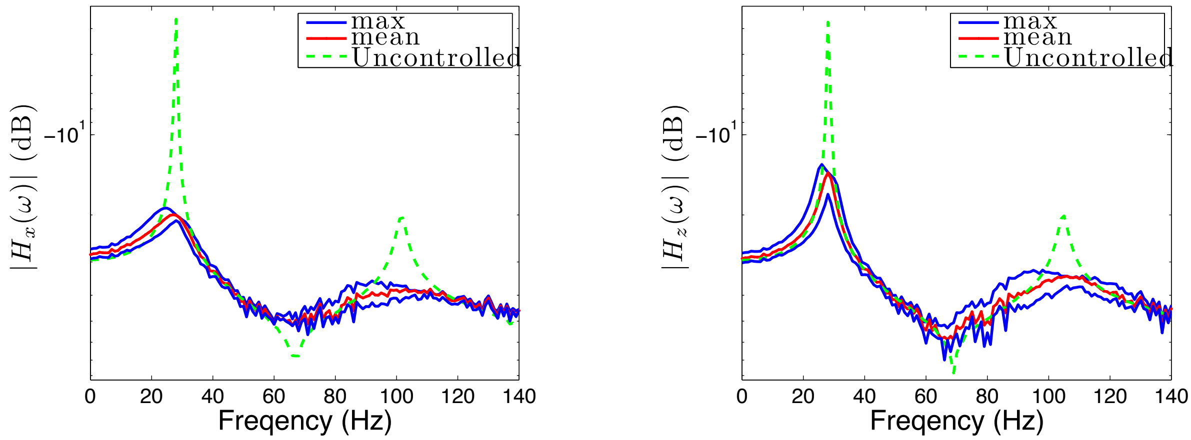
As expected, when the LQR using LMIs controller is applied, the uncertain FRFs exhibits a reduction in the dispersion because this controller was designed in order to take into account the uncertain parameters of the model (see Fig. 5). Specifically, comparing the FRFs in x directions, that were obtained with LQR controller (see Fig. 4(a)), and the LQR controller using LMIs (see Fig. 5(a)), it is observed an smaller variation of FRF amplitude at first vibration mode, additionally, a greater attenuation was also obtained. In a similar way this result is also observed in the FRFs in the z directions, however with less influence than in the previous case since the gravity force is acting along z axis (see Figs. 4(b) and 5(b)).

The time domain response of the rotor system with the aforementioned uncertain parameters was analyzed in agreement the FRF's analysis. The orbits of the shaft were computed by considering a rotation speed of 1600 RPM in which the rotor operates above its first two critical speeds. Figure 6 shows that the orbits have a larger variation, when the LQR controller is used, than the orbits obtained with LQR using LMIs. As indicated in the FRFs the variation in the x direction of the orbits is less expressive than the variation in z direction. Thus, it is observed that the LQR controller using LMIs reduces the effect of the uncertain parameters in the time domain response of rotor system.



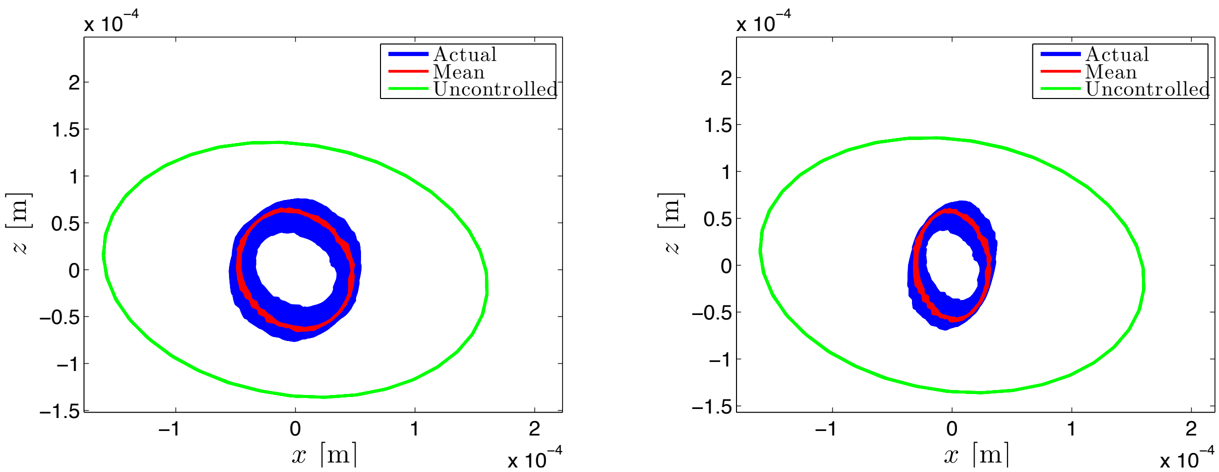
(a) (b)

Figure 4: FRF with LQR controller: a) in x and b) z direction.



(a) (b)

Figure 5: FRF with LQR using LMIs: a) in x and b) z direction.



(a) (b)

Figure 6: Orbits with: a) LQR and b) LQR using LMIs.

CONCLUSIONS

This contribution presented a study to analyze the effect of uncertain parameters of a smart rotor system. The deterministic modeling of the flexible rotor bearing system was presented. Additionally, the stochastic modeling based on the Stochastic Finite Element Method was used in order to take into account the uncertain parameters. A control approach based on modal method was used to implement an active vibration control technique. The dynamic response of the smart rotor system subject to uncertain parameters was evaluated in terms of the time and frequency domain.

The numerical results indicate that including robust techniques in the design controller minimize the variation of the dynamic response of the flexible rotor system subject to uncertain parameters. Future work will encompass several robust control techniques in order to improve the performance of the system.

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