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Quantification of Uncertainties in Experimental Modal Parameters Estimation: An Industrial Case Study

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Abstract: In an industrial metrology environment, surrounded by a large amount of variation sources, where time to experiment execution must be minimized without jeopardizing results quality, knowledge about the measurement processes is a key aspect in guaranteeing confidence in the outcome results. A Measurement System Evaluation (MSE) applied to a modal testing rig allows qualifying and/or quantifying the measurement process according to its reproducibility, precision, discrimination and stability, obtaining the contribution of each variation source in the experimental modal analysis to a determined confidence interval. The methodology presented in this paper differs from the usual MSE due the use of process mapping to organize the steps of the measurement process, tracing the sources of variation, applying the rational subgrouping concept in order organize these sources in an experimental tree. Once executed and analyzed with control charts, this method allows the evaluation of the variation in each step of the measurement process, the analysis of the discrimination threshold, measurement precision, measurement reproducibility and stability of the measurement process. The selected case study presents a MSE of a modal testing rig and shows that it is possible to evaluate the expected variation in the first natural frequency measured on a washing machine component, in the context of a workshop metrology quality control test.

Keywords: Measurement System Evaluation, Experimental Modal Analysis, Uncertainty Quantification.

INTRODUCTION

The definition of a process as a continuous activity, where inputs are changed in outputs, is widely used all over the academic and industrial environment as is possible see in (Herhard Pahl, 2005) and (Back, et al., 2008). This definition meets manufacturing processes, as plastic injection or measuring processes, for example, using a caliper rule. In this context, it is understandable that since all the processes' steps are subject to input variance, both known as unknown, as well will be the outputs.

In usual measuring process, as dimension or mass measurements, knowledge about variation can be used as a quality index, for sorting out good from bad parts of a certain manufacturing process. In a similar fashion, vibration, acoustic or even modal properties can be used as quality control parameters (Reynders et al., 2016, Carnero, et al. 2010). Knowledge on the variability in a modal analysis is important to assess the reliability of the collected data, whose scatter can be due a set of expected components of variation caused by inadequate device or measurement techniques, knowledge gap on the physical phenomenon of interest or even inherent process variation.

Variations measured in an input or output of a process are called variance, defined as the arithmetic average of the deviation between a sample and the sample average or population (Barbeta, et al., 2010). In a measurement process, the knowledge about the variance implies in quantifying some of its characteristics, which, according to the Join Committee for Guidance in Metrology (JCGM, 2008), are defined as:

- Discrimination Threshold: largest values variation of a quantity being measured that causes no detectable readings in the corresponding indicator;
- Measure Precision: Closeness of agreement between indications or measured quantity values obtained by replicate measurements on the same or similar objects under specified conditions;
- Reproducibility Condition of Measure: A set of conditions that includes different locations, operators and replicate measurements on the same or similar objects;
- Stability: The capability of an instrument to give statistically constant readings throughout time.

There is a wide range of statistical methods that could be used to perform a Measurement System Analysis (MSA) or Measure System Evaluation (MSE), however, just a few of them provides subsidies for taking decisions about the measurement process stability.

This work presents an MSE developed specifically to investigate the stability of an Experimental Modal Analysis (EMA) test jig in an industrial metrology environment. The statistical analysis performed, characterizes the full EMA process, providing its precision, reproducibility and stability. The full MSE accounts for different users, suspension stiffness and assembly, accelerometers and impact hammer positions, etc., eventually, providing the full EMA variation in terms of the structure resonance frequency.

The remainder of this paper is organized as such: Section 0 describes a review on statistical tools, addressing control charts and rational subgrouping concepts. Sections 0 and 0 describe the experimental plan to the MSE and the experiment execution, respectively. Section 0 presents the experimental results and statistical analyses and Section 0 presents the final conclusions.

MEASURING SYSTEM EVALUATION

The effective usage of any measuring system is about understand the variation sources in a certain measurement (Wheeler, et al., 1989). Once the entire variation sources are connected to the measurement, it is necessary to study the entire process to know the measurement process robustness.

Control charts are not the only method to perform a MSE, however, it has been used with large success in many companies. In applications focused on variation detection in a measurement system, the $\bar{X}$-Chart or Sample Mean Chart, and the R-Chart or Range Chart are widely used.

Wheeler and Chambers (Wheeler, et al., 2010) affirm that the effective usage of control charts is also connected to the correct process subgrouping, according to the data acquisition structure and experimental hierarchy. The division of a measurement process in stages is known as Rational Subgrouping.

In order to better understand the strategy and the rational subgrouping adopted in this experiment, an Experimental Tree is devised, which will be demonstrated in the following section

Rational Subgrouping

Rational Subgrouping may be defined as a piece region of space, time, product or any other variable of the process, which can be grouped as one, due its relative homogeneity (Wheeler, et al., 2010). By grouping different inputs of the process in subgroups, it is possible quantify if the major variation is inside or outside that subgroup.

The creation of the subgroups is profoundly linked to the experimental plan, therefore, linked to how the components are collected and evaluated. Figure 1 shows an experimental tree with three subgroups, Machine, Component and Measure, in which two parts from each machine have been collected and three measures performed in each part, resulting in a total of 12 measurements.

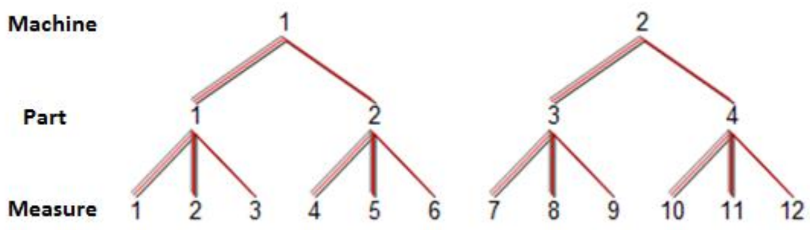

Figure 1 – Example of an Experimental Tree

$\bar{X}$-Chart and R-Chart

Control cards are commonly applied in Statistical Process Control (SPC) in manufacturing to perform a reliability and robustness analysis. There are different kinds of control charts, however, this paper will focus on $\bar{X}$-Charts and R-Charts. An R-Chart, or Range Chart, is a point graph that shows the range of measurements, in other words, the higher minus the lower measurement value, inside a specific subgroup, as showing in Figure 2.

The limits UCL (Upper Control Limit) and LCL (Lower Control Limit) from R-Chart are approximately calculated using Equations (1) and (2).

$$UCL_R = D_4\bar{R} \tag{1}$$

$$LCL_R = D_3\bar{R} \tag{2}$$

where $\bar{R}$ is the mean of the amplitude inside a subgroup and D_4 and D_3 are data from Table 1, which are function of the number of samples inside a subgroup (Ross, et al., 1995).

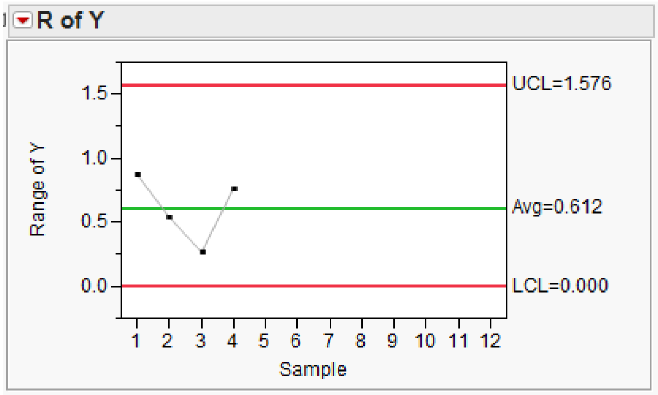

Figure 2 - R-Chart

The main utility of the R-Chart in a MSE context is analyzing the variation inside a specific subgroup. In case any value is outside the upper or lower control limit, there are evidences that the measurement system cannot be classified as stable, predictable and controllable (SPC), i.e., the Stability of the measurement system may be compromised. Therefore, it will not be possible to conclude that the system will keep its discrimination threshold, measurement precision and reproducibility constant at all times.

It is also in R-Chart that the Discrimination Threshold is analyzed by counting the quantity of values that each point can assume inside the chart limits. In other words, what is the minor variation that the system is able to capture and how many times the range of limits are larger than the smallest variation measured by the system. In general, the more steps between limits, the better. More details can be found in (Wheeler, et al., 1989).

$\bar{X}$-Chart or Mean Sample Chart is a point graph that shows the mean of measurements inside a certain subgroup, as shown in Figure 3. The limits UCL and LCL of the $\bar{X}$-Chart are calculated using the Equation (3).

$$CL_{\bar{X}} = \bar{\bar{X}} \pm A_2\bar{R} \tag{3}$$

where $\bar{\bar{X}}$ are the mean of the mean inside a determined subgroup, $\bar{R}$ is the mean of the ranges inside the subgroup and A_2 is given by the Table 1, function of the number of samples inside the subgroup (Ross, et al., 1995).

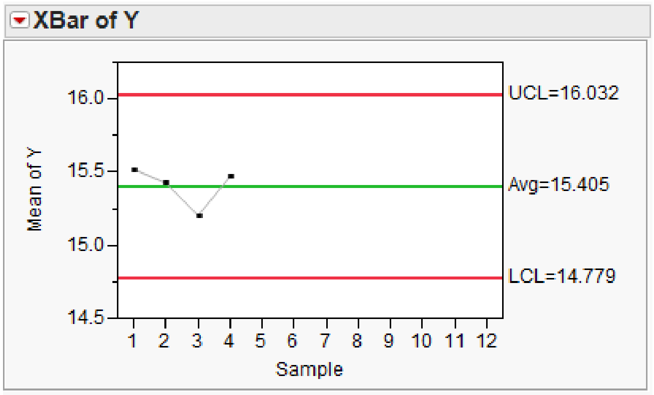

Figure 3 - $\bar{X}$-Chart

The $\bar{X}$-Chart shows where the large variations can be found, in case of all values being inside the chart limits, there is a large variation inside the subgroup, in case of a few values outside the limits, there are evidences to suggest the larger variation is in between the subgroups, and in case half or more points are outside the limits, the large variation is in between the subgroups.

The total variance squared inside a subgroup can be approximated by Eq. (4), in which the value of d_2 is given by Table 1.

$$\hat{\sigma}^2 = \left(\frac{\bar{R}}{d_2}\right)^2 \tag{4}$$

It is necessary to highlight that both $\bar{X}$- and R-Charts are used in all hierarchy levels of the experimental tree, beginning at the button and until the top. Good practice suggests defining the component where the higher variation is expected to the top of the hierarchy, allowing an easier analysis of the results. As far as MSE is concerned, it is common to place samples as near as possible to the top of the hierarchy.

Table 1 - Constants for Equations (1) to (4).

Sample size inside a subgroup	A_2	D_3	D_4	d_2
2	1.880	0	3.267	1.128
3	1.023	0	2.574	1.693
4	0.729	0	2.282	2.059
5	0.577	0	2.114	2.326
6	0.483	0	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.964	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078

EXPERIMENTAL PROCEDURE

The evaluation of the experimental modal analysis measurement system has been performed using 5 samples of a structural component, made of steel by a progressive stamping process, in which 8 coordinate points have been selected for each sample (Fig.4).

In order to create an Experimental Tree, the execution process was mapped and the variation sources in each step were identified. The resulting Experimental Tree is shown in Figure 5, where the variation sources Sample, Operator, Reply and Point are used to evaluate the Reproducibility Condition of the Measurement and the source Repetition is used to evaluate the Measurement Precision.

The analysis of the Experimental Tree allows a thorough investigation of some limitations and advantages of the performed experiment, such as:

- Running order is aligned with other variability components. Via the R-Chart, it should be possible to visualize the operators' learning effects during test execution
- Modal Identification Method can be mistaken by other sources of variation as Hammer Orthogonality and Hammer Position, therefore it will not be possible to estimate the error around the frequency response function approximation.
- Accelerometer Positioning can be confused with Experimental Reply, therefore, all the 8 frequency response functions in each reply will be performed with the same sensor attachment.
- The variability of the Sample is above that of the Operator, which makes the calculation of experiment total variation using control charts a simpler procedure.

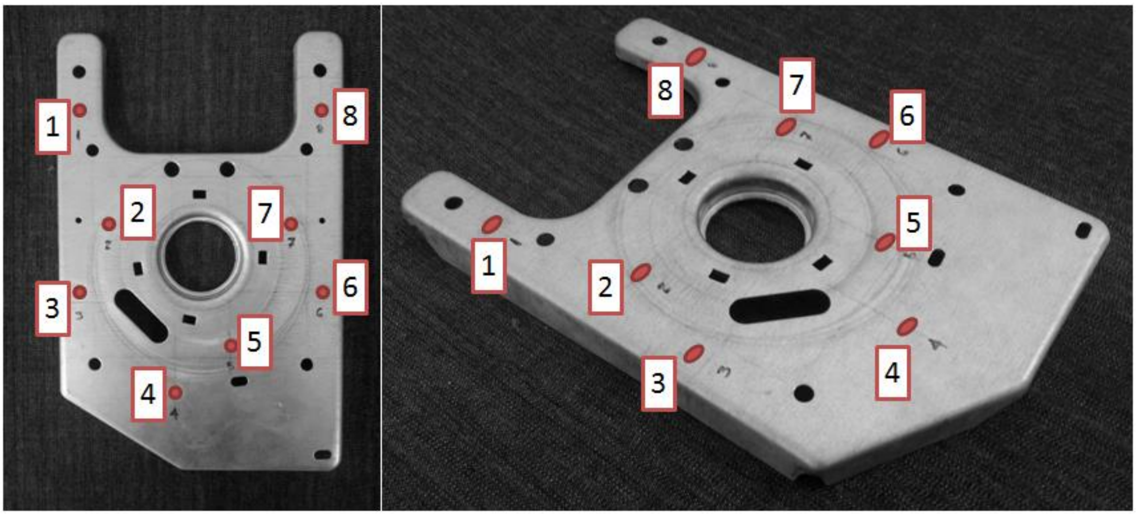

Figure 4 - Excitation and Measure Points in Sample

As a result of the aforementioned strategy, the Experimental Tree shown in Fig. 5, result in a data set of 320 experimental FRFs, extracted from 5 part samples, and tested by two operators with the prescribed number of replies. This data set will be used in the EMA process and treated in the MSE procedure, described in the following subsessions.

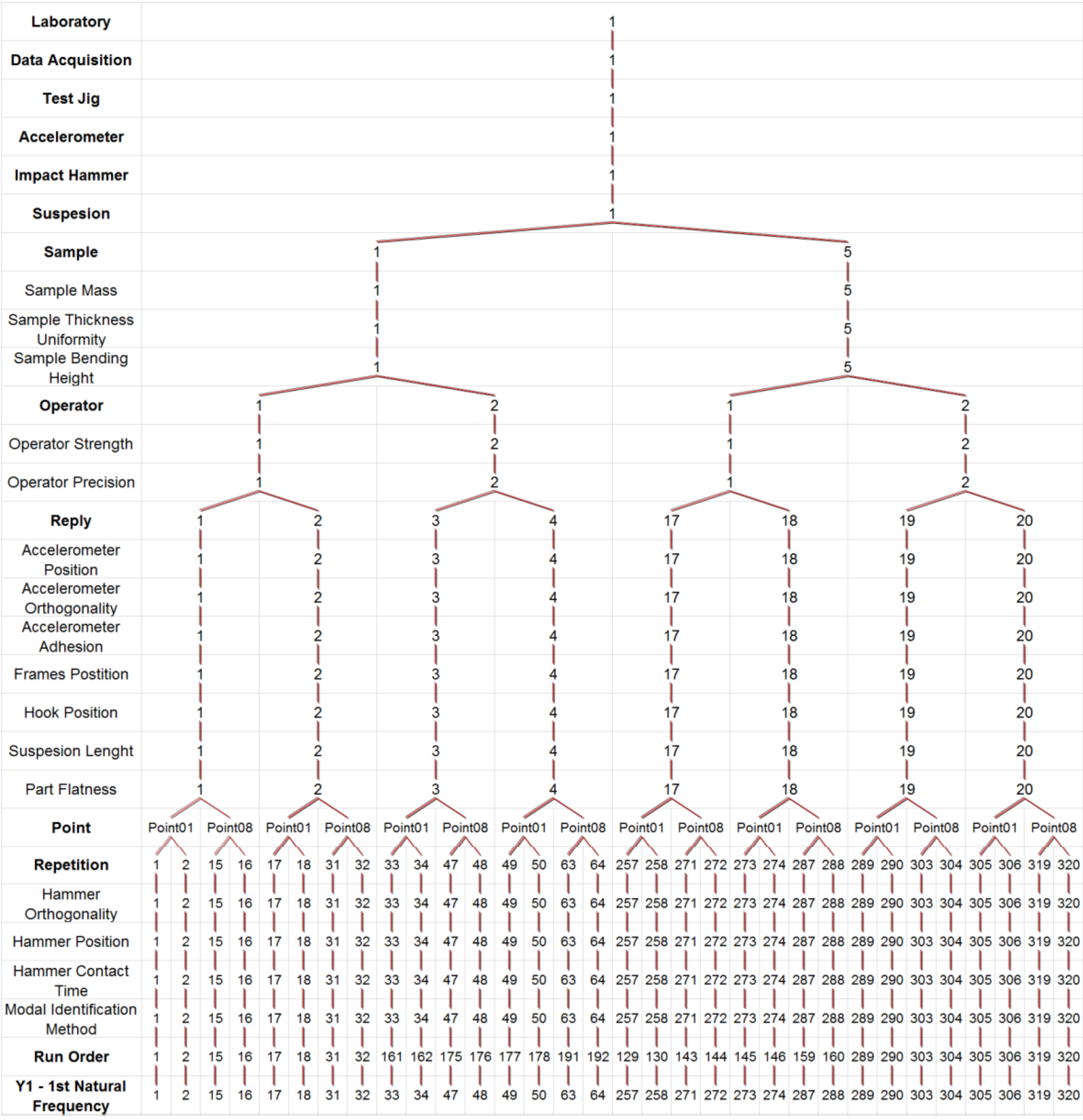

Figure 5 - Experimental Tree

Experimental Modal Analysis

The tests have been conducted as prescribed by the experimental tree shown in Fig. 5, using the Brüel & Kjær acquisition system Type 3032A with six input channels. Only two channels have been used in this test, one for the Brüel & Kjær impact hammer model 8206-002 and the other for the PBC Piezotronics uniaxial shear accelerometer model 352C33. Highly flexible rubber bands have been used as the component suspension, in order to simulate a free-free boundary condition, reducing the rigid body resonance frequencies and avoiding its interference with the flexible ones.

Even with the availability of extra channels in the acquisition system, decision was taken to use a single-input single output approach, once the mass of the accelerometers could modify the dynamic behavior of the rather light part. Point 01 (see Figure 4) was selected as the reference sensor position, whereas the impact hammer would travel through all the 8 highlighted points. Figure 4gure 4 shows one of the identified samples used in the test, with approximate dimension of 300 x 200 x 20 mm.

The test was performed on an experimental modal analysis bench. The test jig shown in Figure 6 was designed with the purpose of making the experiment setup easier and minimize the measurement errors. The test beach relies on rubber elements, with low stiffness, adjusted with endless worm, simplifying the balance and leveling of the components.

Figure 6 - Experimental Modal Analysis Bench

The modal identification method selected to perform this analysis was the circle fit method, which fits the points from the experimental Nyquist Diagram in a circle, resulting in an estimate of the resonance frequency and the damping factor (Ewins, 2000). Figure 7 shows an example of the results format, obtained from the test data post processing and plotted with MATLAB, as the operator will see it in the shop floor. Figure 7(a) shown the experimental points fitted in a circle form from the Nyquist Diagram for the inertance function, while Figure 7(b) shows the Bode Diagram for the same functions with the experimental data in blue and fitted curve (considering a one degree of freedom system) in red. Figure 7(c) groups the main information got from the test in a table and Figure 7(d) shows a color map of the loss factor evolution around the natural frequency.

After analyzing the test data, Point 02 (as in Fig. 4) turned out to be rather close to a nodal line of the first flexible mode, hence, providing no observability to the first resonance frequency. As the circle fit method applied to this single FRF would not show any reasonable results and lead to a high degree of uncertainty, the point was excluded from the MSE analysis. As a result, the data set further used in this analysis consists of 280, rather than 320 samples.

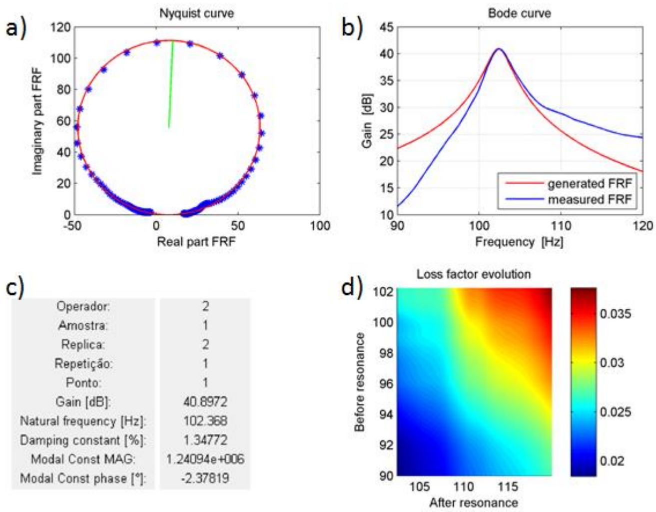

Figure 7 - Circle fit method: (a) Nyquist Curve, (b) Bode Curve, (c) test results summary and (d) Loss Factor evolution

Measurement System Evaluation

Analyzing any experiment begins with a critical evaluation of the obtained results. In a MSE, as proposed in this work, one way to perform this initial analysis is grouping the data as generated in chronological time, which allows for checking any tendency on the measurements, wither due to operators learning, tool wearing, etc. (Deming, 2003). Figure 1Figure 8 shows the obtained data in such temporal order.

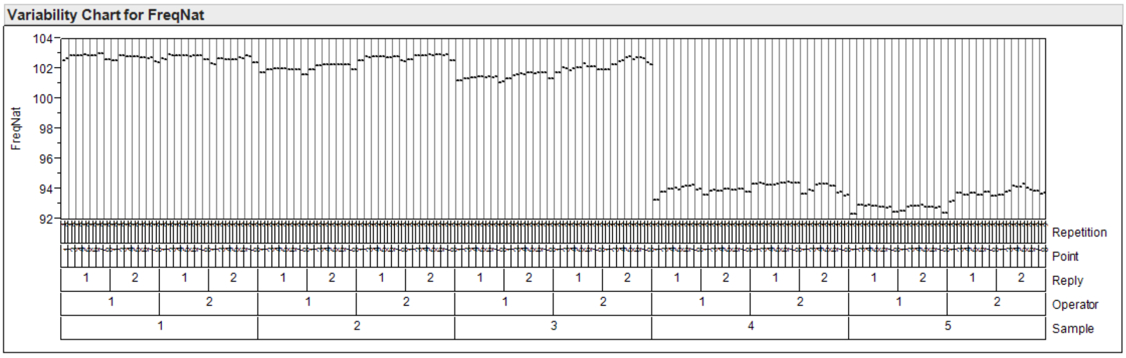

Figure 8 - Natural Frequency in Hz in chronological order

By analyzing Figure 8, no considerable temporal effects can be perceived among samples 1, 2 and 3. However, when compared with samples 4 and 5, a significant difference in the first natural frequency value is evident. If this variation is confirmed by analyzing $\bar{X}$- and R-Charts, it will be possible to affirm that the variation between the samples is larger than the variation between the measurements, what makes the measurement system robust to common cause variation.

Therefore, the next step is to create the $\bar{X}$- and R-Charts, beginning from the bottom levels shown in Fig. 5. It is important to remember that what varies inside this first subgrouping is *Repetition*, *Hammer Orthogonality*, *Hammer Position*, *Time of Contact* and the *Numerical Approximation* due to the modal identification method, while in between the subgrouping, the variations are *Points*, *Reply*, *Operator* and *Samples*, including its own source of variation.

Figure 9 shows the control charts for this subgrouping. Performing the analysis of the R-Chart, it is possible to conclude that the Discrimination Threshold of the experiment is very small, once this value is connected to the numerical error of the modal identification method. In the same chart, it is possible to identify 7 points above UCL, which can indicate the presence of special causes or imply that the measurement system is not SPC, however, the value defined by the UCL using Equation (**1**), is small enough (in the range of tenths of Hertz) much lower than the variation found between the parts, which is show in $\bar{X}$-Chart. This may be caused by the small number of repetitions performed in the test, given a few degrees of freedom to the variation source. Therefore, based in this analysis, on can define the system as SPC.

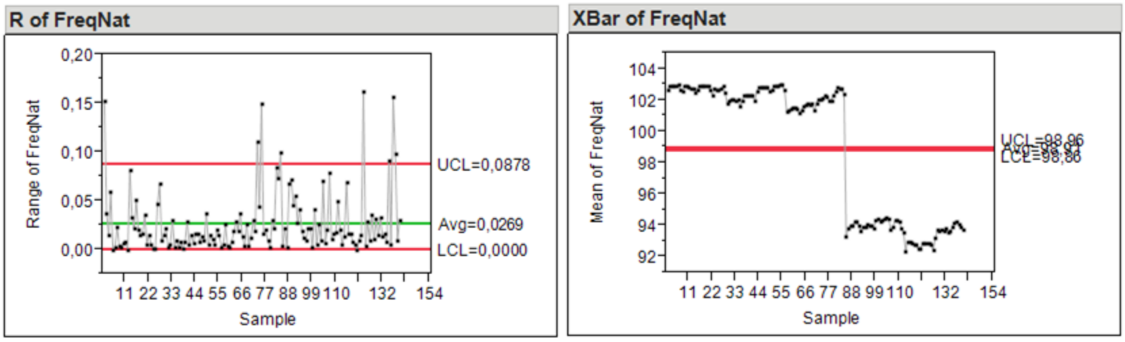

Figure 9 - R and $\bar{X}$ Charts to Repetition Subgroup

Also based in Figure 9, it is possible to conclude that the greatest share of variation is in between the subgroups, once all the points are outside the UCL and LCL in $\bar{X}$-chart. Therefore, the larger pieces of variation are due to *Points*, *Reply*, *Operator* or *Samples*.

Throughout the next subgrouping, the varying parameters inside are *Point* and *Repetition* with their sources of variation, and what is varying in between are *Reply*, *Operator* and *Sample* with their own sources of variation. Figure 10 shows the control charts to this subgroup.

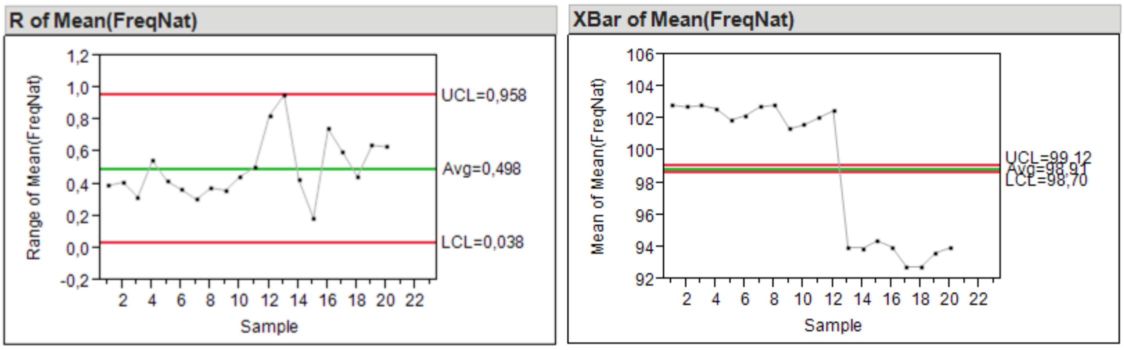

Figure 10 - $\bar{X}$ and R Charts to Point Subgroup

This subgrouping reveals that the 13th point in the R-Chart is marginal to UCL, which should be regarded with care. In future experiments, the test procedure should be reviewed to avoid new discrepant points, even so, in the current analysis its effect may be discarded once, according with Eq. (**3**), the main effect of the $\bar{R}$ in a MSE is to increase the UCL and LCL in the $\bar{X}$-Chart. Notice that, in this case, it is not significant, as all points are far from the limits in the $\bar{X}$-Chart, which renders the fair assumption that the larger variation is in between this subgroup.

In the next level of the experimental tree, what is varying inside subgroup is *Reply*, *Point* and *Repetition* and what is varying in between is *Operator* and *Sample*. Figure 11 shows the control charts for this subgrouping. Here again, the same reasoning can be used to outline point 9 in the R-Chart and conclude, based on the $\bar{X}$-Chart, that the system is SPC and the large variation is in between subgroups.

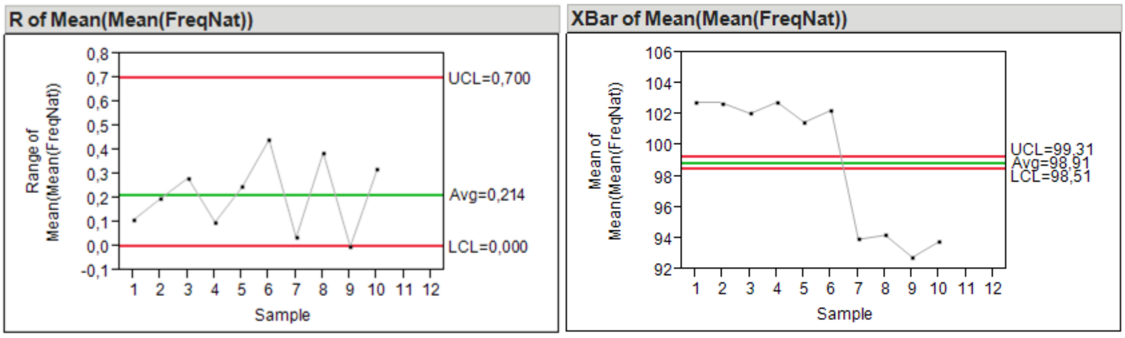

Figure 11 - $\bar{X}$ and R Charts to Reply Subgroup

At the last level of the experimental tree, what is varying inside the subgroup is *Operator*, *Reply*, *Points* and *Repetition* and what is varying in between is *Sample*. Figure 12 shows R and $\bar{X}$ Charts for this subgrouping. With the system shown as SPC, also in Figure 12, the analysis of the $\bar{X}$-Chart results in the conclusion that the large variation is in between *Operator*, *Reply*, *Point* and *Repetition* subgroup. Once there is only on level above this subgroup, the conclusion is that the largest variation is inside the *Sample* subgroup.

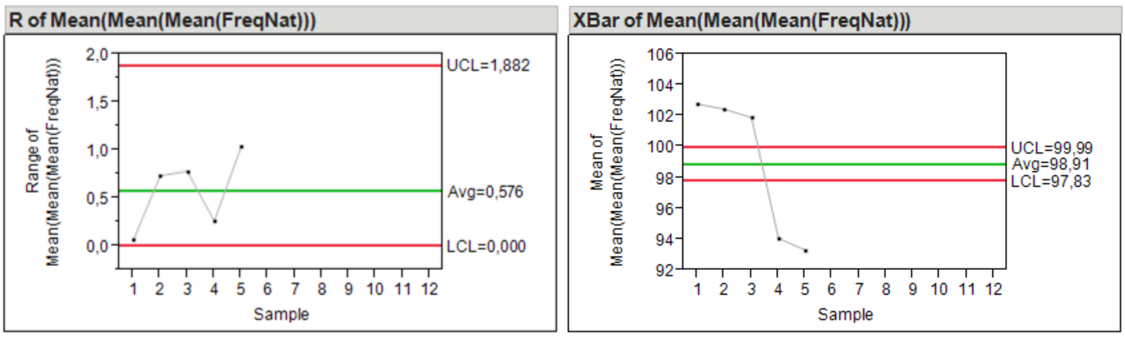

Figure 12 - $\bar{X}$ and R Charts to Sample Subgroup

The total variation inside each subgrouping can be obtained from Eq. (**4**). When calculating the variation from each subgroup, it is necessary to exclude the variation from previous subgroups and divide by the subgroup size, as shown in Table 2.

Table 2 - σ^2 for Each Subgroup

Subgroup	Subgroup Variation	Size of SG (N)
Repetition	$\hat{\sigma}^2_{Repetition} = \left(\frac{\bar{R}}{d_2}\right)^2 = \left(\frac{0{,}0269}{1{,}128}\right)^2 = 5{,}687 \times 10^{-4}$	2
Point	$\hat{\sigma}^2_{Point} = \left(\frac{\bar{R}}{d_2}\right)^2 - \frac{\hat{\sigma}^2_{Repetition}}{N_{Repetition}} = \left(\frac{0{,}498}{2{,}704}\right)^2 - \frac{5{,}687 \times 10^{-4}}{2} = 3{,}363 \times 10^{-2}$	7
Reply	$\hat{\sigma}^2_{Reply} = \left(\frac{\bar{R}}{d_2}\right)^2 - \frac{\hat{\sigma}^2_{Reply}}{N_{Reply} \times N_{Point}} - \frac{\hat{\sigma}^2_{Point}}{N_{Point}} = \left(\frac{0{,}214}{1{,}128}\right)^2 - \frac{5{,}687 \times 10^{-4}}{2 \times 7} - \frac{3{,}363 \times 10^{-2}}{7} = 3{,}115 \times 10^{-2}$	2
Operator	$\hat{\sigma}^2_{Operator} = \left(\frac{\bar{R}}{d_2}\right)^2 - \frac{\hat{\sigma}^2_{Reply}}{N_{Reply} \times N_{Point} \times N_{Reply}} - \frac{\hat{\sigma}^2_{Point}}{N_{Point} \times N_{Reply}} - \frac{\hat{\sigma}^2_{Reply}}{N_{Reply}} = \left(\frac{0{,}576}{1{,}128}\right)^2 - \frac{5{,}687 \times 10^{-4}}{2 \times 7 \times 2} - \frac{3{,}363 \times 10^{-2}}{7 \times 2} - \frac{3{,}115 \times 10^{-2}}{2} = 0{,}243$	2

The total variation which the system is subject to is the sum of variations in each subgroup, as shown in Table 2. This implies that the first experimental natural frequency obtained by this test jig, for this component, may vary ±0.308 Hz with a 95% confidence interval, as show below.

$$\hat{\sigma}^2_{MSE} = \hat{\sigma}^2_{Repetition} + \hat{\sigma}^2_{Point} + \hat{\sigma}^2_{Reply} + \hat{\sigma}^2_{Operator} = 5{,}687 \times 10^{-4} + 3{,}363 \times 10^{-2} + 3{,}115 \times 10^{-2} + 0{,}243 = 0{,}308\ Hz \tag{5}$$

Not only this value, per se, is of great importance for the validity of any quality control analysis based on such modal criteria, but also the way it breaks down into each contributor. The most sensitive source of variation, for the present test jig, is the *Operator;* one order of magnitude bellow, but sharing similar levels are *Reply* and *Point*. Finally, it is important to highlight that the value obtained for $\sigma^2_{\text{Repetition}}$ may be subjected to special case action and that its value may increase in case extra experiments would have been performed, however, once this is the smallest variation of the system, the overall $\hat{\sigma}^2_{MSE}$ is a reasonable estimate of the total variation in this measurement system.

CONCLUSIONS

This manuscript presented a methodology that differs from the usual MSE due the use of process mapping to organize the steps of the measurement process. By tracing the sources of variation, the analysis of the discrimination threshold, measurement precision, measurement reproducibility and stability of the measurement process have been assessed. The selected case study consists of the estimation of the first natural frequency of a washing machine stamped component, tested on an impact modal fashion in free-free boundary condition. The use of control charts with progressive subgrouping of the data set allowed for the evaluation of the variation in each step of the measurement process. As shown, the modal testing rig presets a total variability of ±0.308 Hz with a 95% confidence interval. The breakdown of this value ranks the *Operator* as the higher contributor to the overall variability ($\hat{\sigma}^2_{Operator} = 0{,}243Hz$) while *Repetition* as the lowest ($\hat{\sigma}^2_{Repetition} = 5{,}687 \times 10^{-4}Hz$). Future work should include a more careful sensor placement procedure, in order to avoid proximity with nodal lines, as well as include more complex assemblies and the study of the variability propagation in the machine assembly process.

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