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Stochastic Dynamic Analysis of a Tower Excited by Wind Forces

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Abstract: We present a frequency domain analysis of a tall variable section metallic tower under wind action. An equivalent one-degree-of-freedom model is generated via the classical Rayleigh's Method. We use the wind velocity Power Spectrum Density function of the Canadian Wind Forces on Structures National Code and reference velocity of Brazilian Wind Code. Standard deviation of displacement values is computed and Gaussian distribution is adopted. Thus, it is possible to make reliable engineering assessment of structural reliability.

Keywords: random vibrations, wind forces, Rayleigh's Method

INTRODUCTION

We present a frequency domain analysis of a tall variable section metallic cantilever tower under wind action. Recent work on the subject is to be found in, for example, Silva and Brasil, 2006 and 2013, and Blocken, 2014. An equivalent one-degree-of-freedom model is generated via the classical Rayleigh's Method, using a parabolic approximation to the first vibration mode. The used wind velocity Power Spectrum Density function is the one proposed in the Canadian Wind Forces on Structures National Code by Prof. Davenport (Isyumov, 2012), with reference velocity of the Brazilian Wind Code (Blessmann, 1998, ABNT, 1988). As wind pressure is linearly proportional to the square of the velocities, the PSD of the wind pressures, whose unities are (Pa^2)/(rad/s), is also proportional to the wind velocity PSD, whose unities are (m^2/s^2)/(rad/s). We consider our problem to be a stationary ergodic zero mean Gaussian random process (Newland, 1993). Variance of displacements values is computed and a probability density function, the Gauss distribution, is adopted. Thus, it is possible to make reliable engineering assessment of structural reliability.

This frequency domain analysis, on basis suggested by, among others, Clough and Penzien (1993) and Yang (1986), is possible in our case because linear behavior of the structure is adopted. If consideration of nonlinear structural behavior were necessary, an alternative time domain scheme would be preferable as the so-called "synthetic wind", a Monte Carlo type of analysis suggested by Franco (1993).

METHODOLOGY

Frequency Domain Dynamic Analysis of 1-DOF Structural System

The equation of motion of a1-DOF structural system is (Brasil and Silva, 2015),

$$m\ddot{u} + c\dot{u} + ku = p \tag{1}$$

where m is a lumped mass (kg), c is a damping coefficient (Ns/m), k is a stiffness coefficient (N/m), p a time dependent load function (N) and u the time dependent displacement (m). Superposed dots denote successive derivations in time.

For harmonic loading

$$p = Pe^{i\varpi t} \tag{2}$$

where P is the load amplitude (N), ϖ its frequency (rad/s) and $i = \sqrt{-1}$, the steady state response is also harmonic at the same frequency,

$$u = U e^{i\varpi t} \tag{3}$$

where U is the response amplitude (m). Thus, the complex frequency response function, or *admittance*, is

$$H(\varpi)=\frac{U}{P}=\frac{1}{k+ic\varpi-m\varpi^2} \tag{4}$$

Considering that the natural frequency of undamped free vibrations of the structure is

$$\omega=\sqrt{\frac{k}{m}} \tag{5}$$

and the damping ratio is

$$\xi=\frac{c}{2m\omega} \tag{6}$$

we have

$$H(\varpi)=\frac{1}{\omega^2-\varpi^2+2i\xi\varpi\omega} \tag{7}$$

whose absolute value squared is

$$|H(\varpi)|^2=\frac{1}{(\omega^2-\varpi^2)^2+4\xi^2\varpi^2\omega^2} \tag{8}$$

Random analysis in the frequency domain

We consider our problem to be a stationary ergodic Gaussian zero mean random process (Newland, 1993).

The PSD (power spectrum density function) of the input load is $S_p(\varpi)$, whose units are N²/(rad/s). For our case of wind effect on structures, it is possible to show that wind pressure q (Pa) is linearly proportional to the square of the velocities, as pressure is a consequence of the kinetic energy of air masses in motion (Blessmann, 1998). Thus, the PSD of the wind pressures, whose unities are (Pa²)/(rad/s), is also proportional to the wind velocity PSD, whose unities are (m²/s²)/(rad/s),

$$S_q(\varpi)=(\rho c V_0)^2 S_{\dot{u}}(\varpi) \tag{9}$$

where ρ is the air density (kg/m²), c an aerodynamic coefficient, and V_0 a reference wind velocity. In our case, we adopt the wind velocity Power Spectrum Density function proposed in the Canadian Wind Forces on Structures National Code by Prof. Davenport (Isyumov, 2012), with reference velocity of Brazilian Wind Code (Blessmann, 1998, ABNT, 1988), shown in Fig. 1.

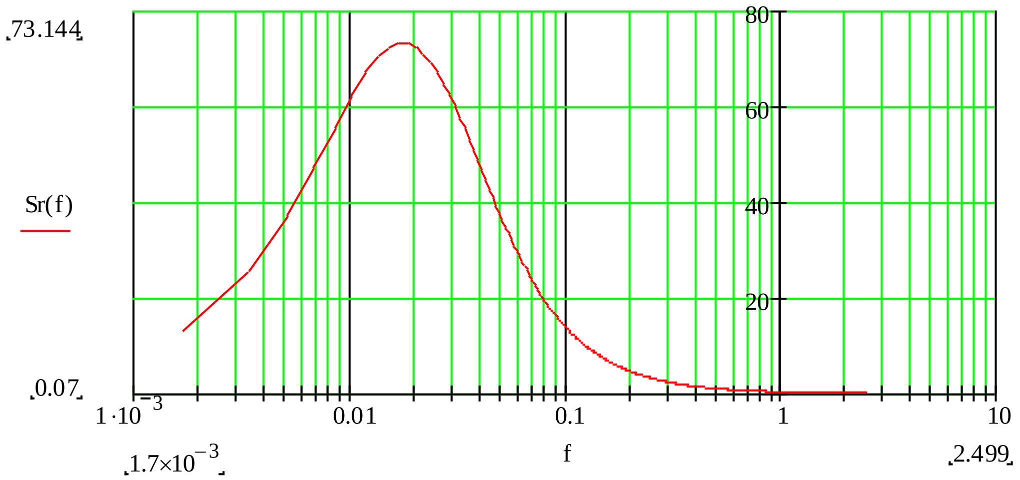

Figure 1 – Davenport’s Wind Velocity Reduced PSD (Blessmann, 1998)

It should be observed that Davenport's PSD of Fig. 1 is given as function of cyclic frequencies f in Hertz instead of rad/s, and is the one-sided PSD. Its expression is (Franco, 1993):

$$\frac{fS_{\dot{u}}(f)}{V_0^{\,2}} = 4\frac{x^2}{(1+x^2)^{4/3}} \tag{10}$$

with

$$x = \frac{1200\,f}{V_0} \tag{11}$$

The conversion is given by the relationship

$$S(f) = 4\pi\, S(\varpi) \tag{12}$$

The PSD of the response displacement is $S_u(\varpi)$, whose units are m²/(rad/s), and it is connected to the PSD of the response by

$$S_u(\varpi) = |H(\varpi)|^2\, S_p(\varpi) \tag{13}$$

The variance of the response is the mathematical expectation of its square value:

$$\sigma^2 = E\left[q^2\right] = \int_{-\infty}^{\infty} S_u(\varpi)\,d\varpi \tag{14}$$

or

$$\sigma^2 = E\left[q^2\right] = \int_{0}^{\infty} S_u(f)\,df \tag{15}$$

We can now use the Gaussian probability density function to compute probabilities of the values assumed by the response random process.

Rayleigh's Method

A very interesting way to model a continuous system as a 1-DOF oscillator is Rayleigh's Method (Clough and Penzien, 1993).

In our case, the tower is a cantilever varying section metallic beam. Our generalized coordinate will be the horizontal displacement of its top $a(t)$, and the horizontal displacements of the sections along its x axis is

$$u(x,t) = a(t)\,\phi(x) \tag{16}$$

where $\phi(x)$ is a shape function in such a way as to satisfy the geometric boundary conditions at the clamped base (zero displacement and tangent) and assume unity value at the top. We adopt $\phi(x) = x^2/L^2$, which is quite convenient but is a not a very accurate approximation to the actual first mode of vibration of a cantilever.

It can be shown that the equivalent mass and stiffness will be

$$m^* = \int_0^L \rho A(x)\,(\phi(x))^2\,dx, \quad k^* = \int_0^L E\,I(x)\,(\phi''(x))^2\,dx \tag{17}$$

where L is the length of the tower, ρ is the material density (kg/m²), E the elastic modulus (Pa). A is the area of the cross section (m²) and I its moment of inertia (m^4):

$$A(x) = \pi D t, \qquad I(x) = \frac{\pi D^3 t}{8} \tag{18}$$

where t is the wall thickness (m), and D is the diameter (m), given by

$$D(x) = D_{base} - \frac{D_{base} - D_{top}}{L} x \tag{19}$$

leading to an approximation of the natural frequency

$$\omega^* = \sqrt{\frac{k^*}{m^*}} \tag{20}$$

The equivalent load of a general loading distribution $p(x,t)$ is

$$p^*(t) = \int_0^L p(x,t)\, \phi(x) dx \tag{21}$$

In our case, supposing the wind pressure q (Pa) is constant along the height of the tower, the loading is

$$p(x,t) = q(t)\, D(x) \tag{22}$$

CASE STUDY

Problem Data

Our case study is a tall cantilever variable annular section steel tower, characteristics given in Table 1.

Table 1 – Tower Data

Height (m)	87,6
Base diameter (m)	6,00
Top diameter (m)	3,87
Base thickness (m)	0,027
Top thickness (m)	0,019
Elastic modulus (GPa)	210
Material density (kg/m^3)	8.500

Material density is considered larger than usual steel density to allow for contributions of mounted equipment such as ladders, electric cables, antennas etc.

Figures 1 and 2 show the considered structure.

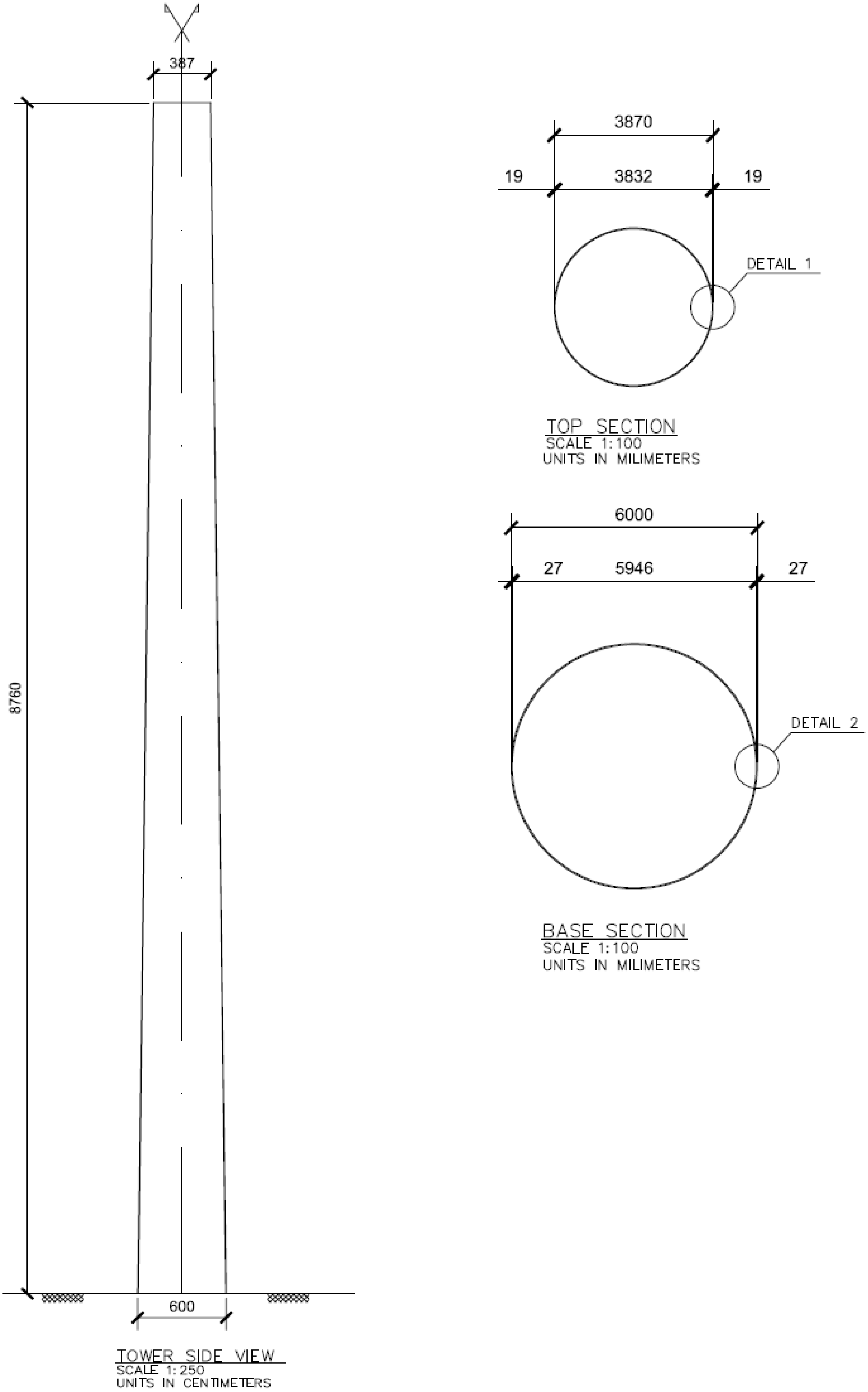

Figure 1: The tower model

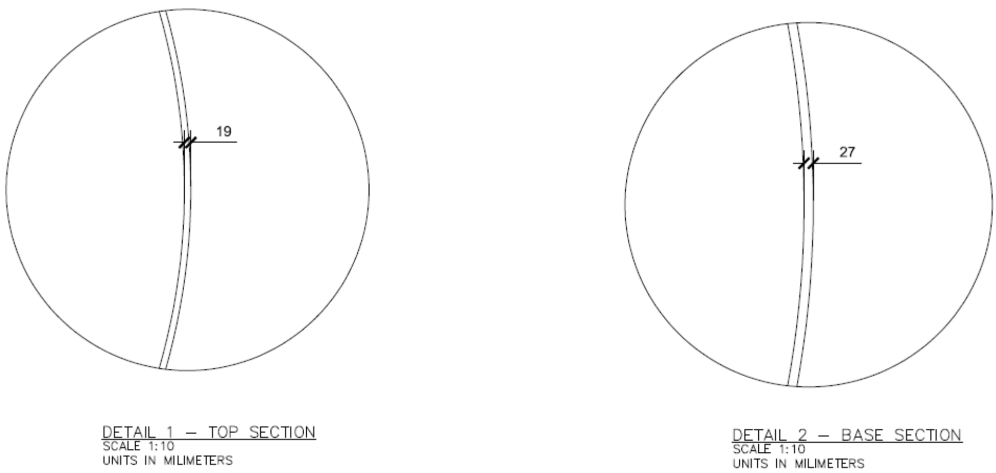

Figure 2: Zoom of tower sections

Applying Rayleigh's Method, as described in Eq. 17, we get dynamic properties given in Table 2

Table 2 – Dynamic characteristics via Rayleigh's Method

Equivalent mass (kg)	4.2983×10^5
Equivalent stiffness (N/m^2)	1.4726×10^6
Natural frequency (rad/s)	1,8509
Natural frequency (Hz)	0,2946
Natural period (s)	3,3946

In the computed natural frequency, a lumped mass of 400 kg was added at the top of the tower to simulate the presence of some heavy lumped equipment.

Results

Following the scheme of Eqs. (8) through (22), numerical values given in Tables 1 and 2, reference wind velocity set by the Brazilian Wind Code (Blessmann, 1998, ABNT, 1988) and considering a Gaussian probability density function, we computed the maximum displacement at the top of the structure to be $a_{\max} = 15\,\text{cm}$, with a 95% level of reliability.

The resulting shear force at base is given by

$$F = k^* a_{\max} \tag{23}$$

CONCLUSIONS

We presented a complete random dynamic analysis in the frequency domain of the wind effect upon a cantilever varying annular section steel tower. We considered our problem as a stationary ergodic and Gaussian zero mean random process. Davenport's Power Spectrum Density function of wind velocities (Isyumov, 2012) and Brazian Wind Code (ABNT, 1988) reference velocity were used. An equivalent one degree of freedom model was derived using Rayleigh's Method adopting a second-degree function as an approximation of the structure's first mode of vibration.

After getting the response random process variance, a Gaussian probability density function may be applied to access possible structural failure probability.

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REFERENCES

Blessmann, J., 1998, “Introdução ao estudo das ações dinâmicas do vento”, Ed. da Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil, 282 p.

Blocken, B., 50 years of Computational Wind Engineering: Past, present and future, J. Wind Eng. Ind. Aerodyn. 129 (2014) 69–102.

Brasil, R.M.L.R.F., Silva, M.A.,2015, “Introdução à dinâmica das estruturas para a engenharia civil, 2ª ed”, Ed. Edgard Blucher, São Paulo, Brazil, 268 p.

Brasilian Association for Standardization (ABNT - Associação Brasileira de Normas Técnicas), 1988, NBR 6123/88 - Forces due to wind on buildings (Forças devidas ao vento em edificações), Rio de Jnaeiro, Brazil.

Clough, R.W., Penzien, J., 1993,”Dynamics of Structures, 2nd Ed.”, Ed. McGraw-Hill, New York,

Franco, M., 1993, “Direct along-wind dynamic analysis of tall structures”, Boletim Técnico PEF/EPUSP, São Paulo, Brazil, 30 p.

Isyumov, N., Alan G. Davenport's mark on wind engineering, J. Wind Eng. Ind. Aerodyn. 104–106 (2012) 12–24.

Newland, D.E., 1993,”An introduction to random vibrations, spectral & wavelet analysis, 3rd Ed.”, Ed. Pearson, England, 477 p.

Silva, M. A., J.S. Arora, R.M.L.R.F. Brasil, Dynamic analysis of pre-cast RC telecommunication towers using a simplified model, Design and Analysis of Materials and Engineering Structures, Advanced Structured Materials 32 (2013) 97–116.

Silva, M. A., R.M.L.R.F. Brasil, Nonlinear dynamic analysis based on experimental data of RC telecommunication towers subjected to wind loading, Mathematical Problems in Engineering 2006 (2006) 1–10, Article ID 46815.

Yang, C., 1986, “Random vibrations of structures”, Ed. John Wiley, New York, 295 p.

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