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Effect of Bit-Rock Interaction Uncertainties on the Torsional Stability Regions of an Oil-Well Drilling System

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Abstract: Torsional vibrations due to stick-slip phenomena may lead to large variations in the drilling angular velocity which in turn may cause to a large number of failures in the drilling process such as damage in drill pipes, due to large torsional deformation, and in drill bits, due to large angular velocities. Previous works have dealt with the search for stable drilling regions, in terms of the oscillation of the drill-bit angular velocities. It has been shown that standard linear proportional-integral (PI) control laws, in which the control torque is proportional to the deviation of the drive table angular velocity and displacement from the reference/target ones (constant angular velocity), may lead to an oscillating angular velocity at the drill-bit due to stick-slip phenomena. This effect can be reduced using properly designed control parameters (control gains). Stability regions were defined for a given set of control parameters, and for a given drilling condition, using as a criteria the average deviation from the expected/target angular velocity. This was done using a finite element model for the drilling system combined with a dry friction model for the interaction between drill-bit and rock formation. However, in reality, the latter is much more complex and, thus, quite hard to predict using such a simple dry friction model. In order to provide robustness for the analysis, one possible approach is to consider uncertain parameters for the dry friction model. This can be done using a stochastic model for the bit-rock interaction torque which then leads to stochastic results for the angular velocity response, when subject to a linear PI control, and, thus, for the angular velocity deviation stability regions. In this work, confidence intervals for the stability regions are presented. This is done using a Monte Carlo simulation using the previously developed finite element model and considering parametric uncertainties for a dry friction model, intended to represent the interaction between drill-bit and rock formation for a given drilling condition. Results should lead to more robust criteria for the design of the control parameters. In addition, a similar analysis is performed for a linear PI control law combined to a varying weight-on-bit. Previous results have shown that a controlled weight-on-bit variation may help stabilize the angular velocity oscillations. However, a stochastic analysis is required to show that this is also true in the presence of uncertainties in the interaction between drill-bit and rock formation.

Keywords: Drilling oil wells, control torsional vibration, stick-slip, drill strings dynamics, uncertainty analysis.

INTRODUCTION

The process of oil well drilling consists, according to Jansen and van den Steen (1995), in drilling rock formations, with diameters of about 10 to 85 cm and depths of 0-5000 meters for extracting oil and gas. The rotary drilling occurs, according to Mihajlovic et al (2006), thanks to the drill bit rotation that crushes and expel the rock formation opening a well (borehole). This drill bit is one of the components of a rigid segment, called BHA (Bottom Hole Assembly), whose connection with the surface is done through a drill string that consists of a series of cylindrical steel tubes (Tucker and Wang, 1999a). The drill string and BHA form, basically, the drilling assembly, which is rotated by a torque provided by the rotary table at the surface.

During the drilling process, axial, torsional and lateral vibrations or combinations of these are generated, resulting in one of the major problems in the process of oil well drilling (Schlumberger, 2006). Experimental evidence shows that the drillstring vibrations is a major cause of deterioration in the drilling process and, if uncontrolled, may result in losses on efficiency, premature component failure, sudden changes in direction of drilling, and even breaking the drillstring, disabling that well (Khulief and Al-Naser, 2005).

The torsional vibrations can cause variation of the rotation along the drillstring, damaging the drill-bit and reducing efficiency of the drilling process (Alamo, 2004) and can generate the well-known stick-slip phenomenon, which is one of the major causes of failures in the drilling process (Plácido, Santos and Galeano, 2002). The stick-slip phenomenon is characterized by the extreme reduction of the angular velocity of drill-bit, and BHA assembly, while the rotary table at the surface continues rotating and transmitting power, increasing the energy stored in the drill string, on the form of torsional deformation, and therefore the applied torque on the drill bit. The drill-bit then overcomes the frictional resistance between drill-bit and rock formation, restarting the drilling process, but at much higher angular velocities than expected.

In field measurements, presented by Plácido, Santos and Galeano (2002), the stick-slip is observed by large amplitude oscillations in the drilling angular velocity. This self-excited oscillation becomes regular and stable once the mode is started and can only be reduced by increasing the angular velocity at the rotary table (Plácido, Santos and Galeano, 2002; Trindade and Sampaio, 2005). The frequency of these oscillations is typically slightly below the first natural frequency of torsional vibration, in the range between 0.05 and 0.5 Hz (Jansen and van den Steen, 1995). Although the propagation of torsional waves can be governed by linear wave equations, the interaction between the drill-bit and rock formation has a highly nonlinear behavior that may be a cause of a non-uniform penetration system (Spanos et al., 1995, Tucker and Wang, 2003). Richard and Detournay (2000) studied the self-excited response of such systems using a discrete model with two degrees of freedom (dof), demonstrating that the linear coupling between the normal and tangential modes in the drill-bit is sufficient to generate the stick-slip phenomenon. Ritto, Soize and Sampaio (2009) also studied the nonlinear interaction between drill-bit and rock formation, using a stochastic model for the main forces acting on the system.

Jansen and van den Steen (1995) adopted a simplified friction model that captures the essence of the nonlinear effect and seems to be sufficient to carry out linear analyses. Tucker and Wang (1999b) used a continuous approach to the drillstring combined with a continuous friction model between drill-bit and rock formation. Since vibrations induced by the stick-slip phenomenon have a key role in the drilling process, several different techniques have been proposed over the years to control this phenomenon. They can be summarized in the following categories: active damping (Jansen and van den Steen, 1995), torque correction (Tucker and Wang 1999a), soft torque (Tucker and Wang 1999a; Tucker and Wang, 2003), proportional-integral control (PI) (Christoforou and Yigit, 2003; Trindade and Sampaio, 2005) and dynamic variation of the weight on bit (Navarro-López and Suárez, 2004).

Generally speaking, two classes of mathematical models can be considered for the effective dry friction based (Coulomb friction) interaction between drill-bit and rock formation. Regularized (or continuous) friction models eliminate the discontinuity for zero or very low angular velocities at the cost of very low frictional force for this case. Non-regularized (or discontinuous) friction models treat differently the case of stick (zero or very low angular velocity) and slip (higher angular velocity). Manzatto (2011) used a non-regularized friction model and proposed several drilling performance criteria for oil-well drilling with a proportional integrative controller (PI). Coletti (2011) also considered a non-regularized friction model in order to study three different control techniques, the standard PI controller and the latter combined with two additional implementations: the dynamic variation of the weight on bit and the soft-torque proposed by Tucker and Wang (2003).

In this work, a finite element model for the drillstring is combined to a non-regularized friction model, adjusted to empirical data, and considering both PI control and PI control with dynamic weight-on-bit variation, with the objective of determining stable drilling regions. Then, the effect of parametric uncertainties of bit-rock interaction on these stable drilling regions is discussed.

MATHEMATICAL MODELLING

The physical-mathematical model proposed for the drilling system, considers the following elements: the rotary table, which applies the driving torque to the drilling system; the drillstring, which acts as the mechanical link between the table and the drill-bit; and the drill-bit, which is subjected to the reaction torque arising from the interaction of drill-bit and rock formation.

The system is described by coupled equations, in the form of a matricial differential equation in accordance with Eq. 1, where u is the vector of nodal angular displacements and M, C and K are, respectively, mass, damping and stiffness matrices. The vector F contains the torques applied by the rotary table (T_m), at the first node, and on the drill-bit (T_b), at the last node (Monteiro, 2012).

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F\} \tag{1}$$

Modeling of the torque on the rotary table

The torque T_m applied by the rotary table provides rotary motion to the drillstring and the BHA and reads

$$T_m = K_p(\omega_{ref} - \omega_m) + K_i(\omega_{ref}t - \theta_m) \tag{2}$$

The controller considered is proportional integrative with gain parameters K_p, proportional to the difference between the angular velocity at surface ω_m (rotary table) and the reference (target) angular velocity ω_{ref} of the drilling assembly, and the integrative K_i that multiplies the difference between the expected and the actual angular displacement. A schematic representation of the feedback control loop is shown in Fig. 1.

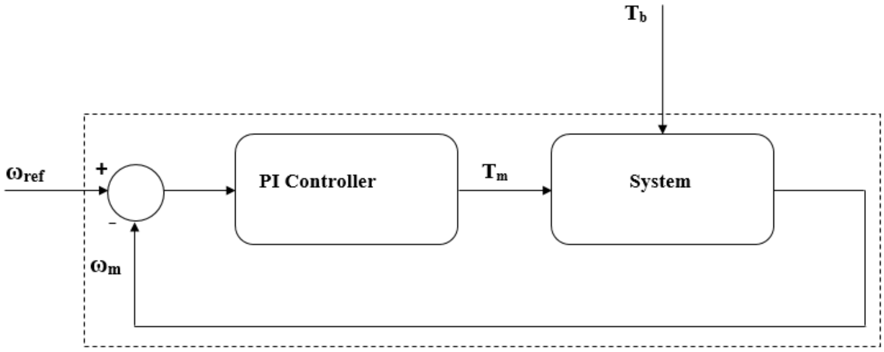

Figure 1 - Schematic model of a PI controller with constant weight on bit, adopted for the drilling system.

Modeling of the torque on the drill bit

To model the expected reaction torque on the drill-bit due to interaction with the rock formation, a Coulomb friction based non-regularized model proposed by Navarro-López and Suárez (2004) was considered, as shown in Fig. 2. The effective torque T_b is written as

$$T_b = K(\theta_{n-1} - \theta_n);\ |\omega_b| \le \delta,\ |T| \le a_2 F_b\ ; \tag{3}$$

$$T_b = a_2 F_b sign(T);\ |\omega_b| \le \delta,\ |T| > a_2 F_b\ ; \tag{4}$$

$$T_b = \left[a_1 + (a_2 - a_1)e^{-\beta|\omega_b|}\right] F_b sign(\omega_b);\ |\omega_b| > \delta\ ; \tag{5}$$

This function is defined in three ranges:

1. For angular velocities lower than δ and when the reaction torque does not exceed the maximum torque possible, the torque value is the stiffness constant of the finite element adjacent to the drill multiplied by the angular displacement difference between the ends of the element, according to Eq. 3.

2. For angular velocities lower than δ and when the reaction torque is greater than the maximum torque possible, the torque on the drill-bit becomes equal to the maximum torque (static friction) corresponding to the multiplication of the coefficient a_2 by the weight on bit F_b, according to Eq. 4.

3. For angular velocities with non-null absolute value (that is, larger than a tolerance δ = 0.0001), the reaction torque follows a function that decays exponentially from the static friction limit to a dynamic friction convergence value, according to Eq. 5.

In these equations, K is the drillstring torsional stiffness constant, GJ/L; θ_{n-1} is the angular displacement of the node adjacent to the drill-bit extremity of the drillstring; θ_n is the angular displacement of the drill-bit; $sign(\omega_b)$ represents the signal function of the drill-bit angular velocity; a_1 and a_2 are the adjustment coefficients, similar to Coulomb friction coefficients: dynamic and static, respectively; β is the decay exponent; and F_b is the weight on bit (WOB).

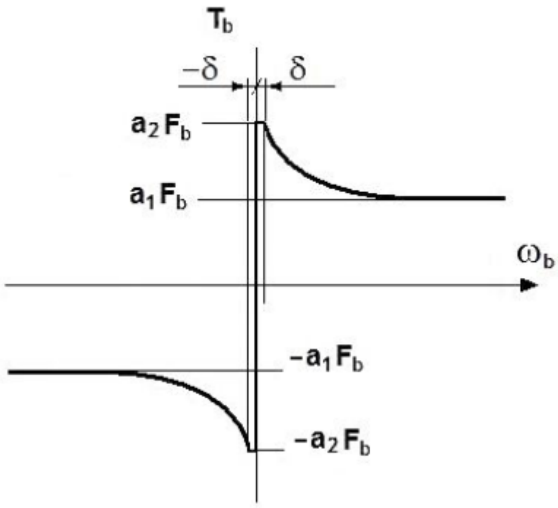

Figure 2 - Model of torque on the drill-bit due to interaction with rock formation proposed by Navarro-López and Suárez (2004).

Adjustment of coefficients a_1, a_2 and β

With typical values of the drilling parameters used by Tucker and Wang (2003), a typical drilling condition was simulated with: reference angular velocity ω_{ref} = 100 rpm (10.47 rad/sec); Weight On Bit (WOB) = 120 kN; control parameters: proportional K_p = 200 Nms and integrative K_i = 100 Nm; initial angular velocity of 70 rad/s throughout the drillstring, with an interval of time considered for analysis from 0 to 100 s.

The decay function described in Eq. 5 was adjusted to the curves obtained for the normalized friction function described by Trindade and Sampaio (2005), leading to a set of coefficients a_1 and a_2 and decay exponent β, for each weight on bit, which were recorded in Tab.1.

Table 1 – Coefficients a_1, a_2, β, for normalized non-regularized function of the torque on the drill-bit, for the following weight on bit values: 80, 100, 120, 140 and 160 kN.

	WOB (kN)				
Coefficients	80	100	120	140	160
a_1	0,037	0,032	0,029	0,026	0,025
a_2	0,057	0,070	0,079	0,085	0,089
β	0,082	0,093	0,097	0,098	0,099

With the values given in Tab. 1, for the coefficients a_1, a_2, and β, it is possible to obtain the curves describing the drill-bit torque behavior in terms of drill-bit angular velocity.

Proposed model

The proposed model is shown schematically in Fig. 3. Two inertias of the rotary table, J_m, and of the Bottom Hole Assembly (BHA), J_b, are connected by a flexible drillstring, whose motion was considered to be only rotation around its own axis. Only two active forces were considered in the system: the control or driving torque at the table, T_m, and the torque induced by the friction resistance in the drill-bit, T_b.

In summary, the drillstring is discretized by finite elements with linear Lagrange shape functions. The torque applied by the rotary table is a PI controller with constant weight on bit, while the torque at the drill bit is not regularized, as proposed by Navarro-López and Suárez (2004).

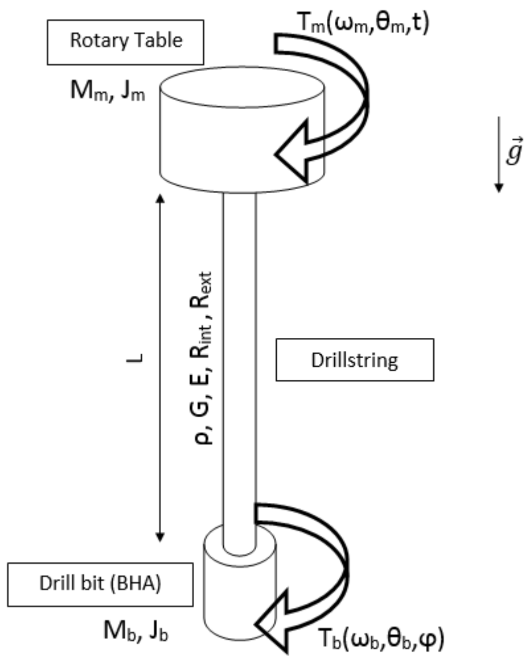

Figure 3 - Representative scheme of the model of the drilling system including the rotary table, the drill-bit assembly and the drillstring.

NOMINAL MODEL ANALYSIS

Several different criteria have been proposed for evaluating performance, viability and operational safety of the drilling process (Manzatto, 2011). It was found that the average deviation of drill-bit angular velocity is the most significant for the analysis of torsional vibration (or stick-slip) control performance.

Average deviation of the drill-bit angular velocity

The proposed average deviation of drill-bit angular velocity, J_1, quantifies the variation of the angular velocity ω_b in relation to the reference angular velocity ω_{ref} and consists of the sum of specific deviations at each time t. For initial and final time instants t_i and t_f, J_1 is represented by the integral described by Eq. 6,

$$J_1 = \int_{t_i}^{t_f} \frac{\left|\omega_b - \omega_{ref}\right|}{\omega_{ref}} dt \qquad (6)$$

Adjusting control parameters K_p and K_i

For each WOB, different pairs of parameters K_p and K_i were analyzed, and it was observed that for certain pairs, the angular velocity deviation was minimized, with a settling time to the reference value smaller than 100 seconds; for other pairs, the settling time was higher than 100 seconds leading to large angular velocity oscillations at both the rotary table and the drill-bit.

From the contour plots of this 2D parametric analysis, closed curves were observed (deformed ellipses) meaning that it is possible to define stability regions of the control parameters for a given angular velocity deviation threshold. Any pair inside a given closed curve leads to smaller angular velocity deviations than the one that defines the closed curve (the contour level).

RESULTS AND DISCUSSION

In Fig. 4, the graphs are obtained for WOB = 140kN with angular velocity deviation values in ascending order. It is possible to observe that the higher the values of the angular velocity deviation criteria J_1 the higher the amplitude oscillations and the stick-slip occurrences, which graphically characterizes the persistence of the drill-bit angular velocity. For values higher than J_1 = 33, the operating conditions are characterized as undesirable.

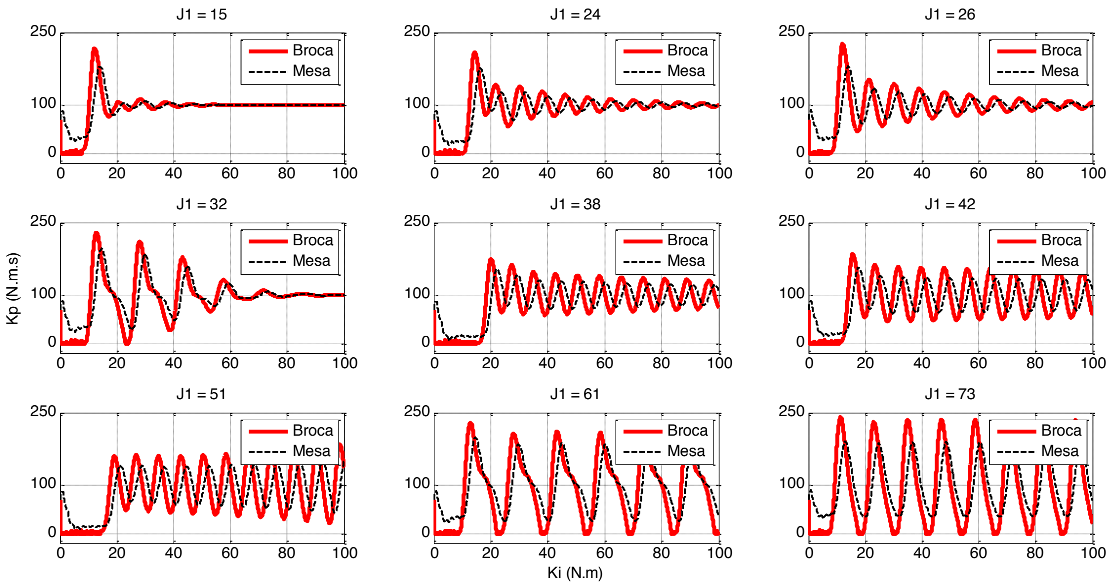

Figure 4 – Time response of angular velocities at drill-bit and rotary table for different values of J_1, with a PI controller with constant WOB equal to 140kN.

Fig. 5 presents the contour plot considering four angular velocity deviation levels, J_1 equal to 20, 30, 40 and 50, and for a given range of control parameters, K_p between 0 and 1000 Nms and K_i between 0 and 500 Nm. Notice that for this value of WOB = 140 kN, there is no control parameters pair able to yield a deviation of J_1 = 10 or lower. It is also noticeable that the different deviation levels lead to regular contours (closed curves) that contract for decreasing deviations, where clearly the contours for smaller deviation levels are contained in those for higher deviation levels. In the search for adequate value ranges for the control parameters K_p and K_i that yield smaller levels of angular velocity deviation, it is reasonable to try to set these parameters inside contour regions relative to smaller deviation levels.

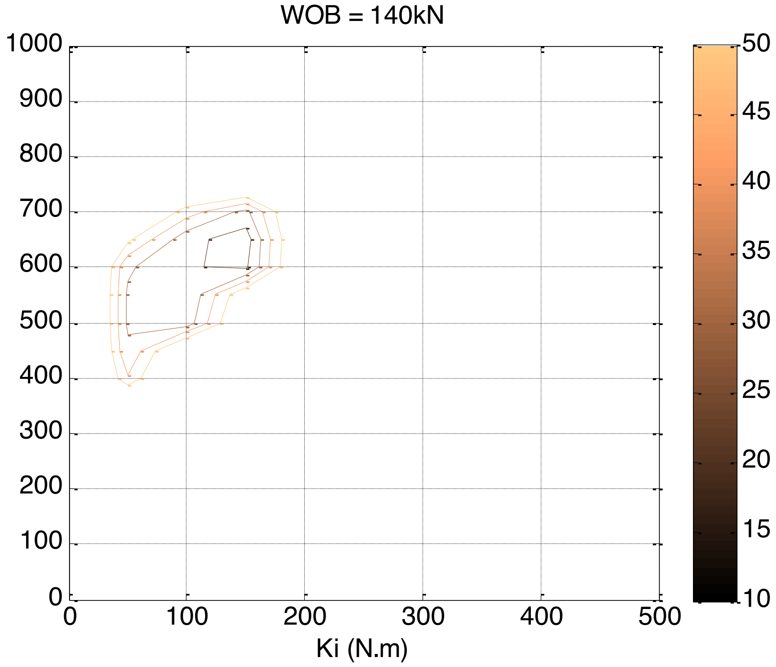

Figure 5 – Level curves for the J1 in relation with the values of parameters of Kp and Ki, with constant WOB and equal to 140kN.

One may conclude from Fig.5, that the ranges of K_p and K_i that yield better performance, in terms of angular velocity deviation, are 400 to 700 Nms and 50 to 200 Nm, respectively. These analyses, and the conclusions therein, are evidently highly dependent on the parameters considered in the drill-bit/rock formation interaction model. Therefore, it is worthwhile to evaluate how uncertainties of these parameters might affect the previous analysis.

Accounting for the uncertainties of the bit-rock interaction model

Considering that the bit-rock interaction model is highly empirical and subject to fast and dramatic changes due to rock formation natural variabilities, this section presents a preliminary analysis of the effect of uncertainties of the parameters a_1 and a_2, of the proposed bit-rock interaction model, on the stability regions, that is regions of adequate control parameters where smaller angular velocity deviations are possible, as discussed previously.

This was done considering a stochastic model for the interaction model parameters a1 and a2. Since they both must be positive, a lognormal probability density function was considered. The mean values were the ones adjusted from empirical observations for WOB=140 kN (see Tab. 1). The relative dispersions of both parameters were set to 5%, so that the most variations would lead to values still valid vin the range of the WOB considered. Tab. 2 presents the nominal (deterministic) and mean values for these parameters as well as their relative dispersion.

Table 2 – Lognormal parameters considered for the coefficients a_1 e a_2 (WOB = 140 kN).

	Parameters of Probability Density Function Log-Normal (PDF)		
Coefficients	Deterministic Value	μ	σ/ μ
a_1	0,026	0,026	0,05
a_2	0,085	0,085	0,05
β	0,098	-	-

Following the deterministic analysis performed previously, the contour plots were re-evaluated for each realization of the stochastic variables a_1 and a_2. To speed-up the computations, only a smaller range of control parameters near the best performing ones in the deterministic analysis was considered, that is K_p between 400 and 800 Nms and K_i between 0 and 200 Nm. Also due to the required computational effort, in this preliminary analysis, only 10 realizations were generated for the stochastic parameters. Fig.8 shows the contour plot for angular velocity deviation levels 20, 30, 40 and

50, where each nominal (deterministic) contour curve is replaced by two contour curves defining a confidence interval for the control parameters. It was observed that the parametric uncertainties tend to shrink (contract) the stability region (contour curve) and, thus, reduce the flexibility in the choice of adequate control parameters. Nevertheless, it seems that the region defined by Kp from 600 to 650 Nms and Ki from 100 to 150 Nm is still the best performing (or more stable) region.

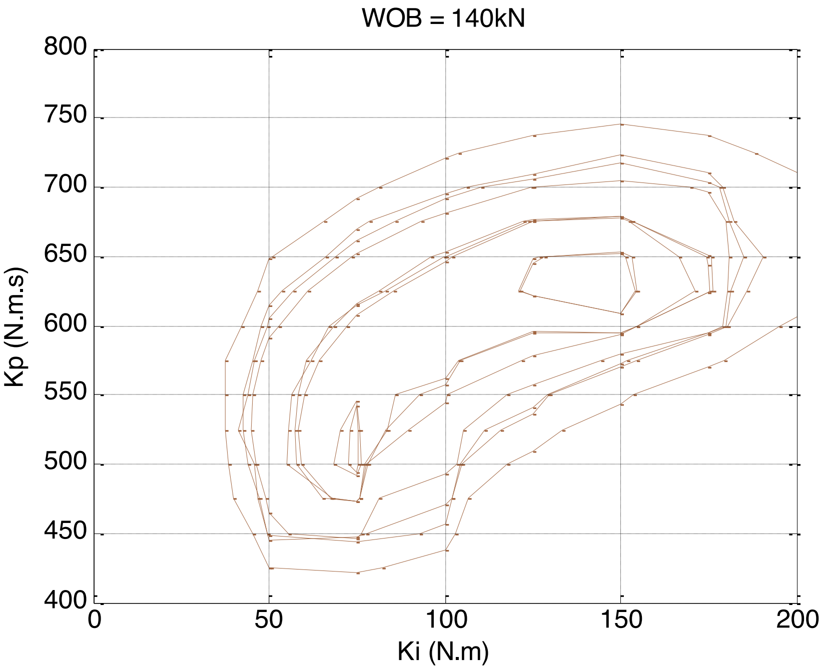

Figure 6 – Variation of the regions with angular velocity deviation levels 20, 30, 40 and 50, for WOB=140 kN, subjected to uncertainties on bit-rock interaction model parameters a_1 and a_2.

CONCLUSIONS

This work extended previous ones that have dealt with the search for stable drilling regions, in terms of the oscillation of the drill-bit angular velocities. Stability regions were defined for a given set of control parameters, and for a given drilling condition, using as a criteria the average deviation from the expected/target angular velocity. This was done using a finite element model for the drilling system combined with a dry friction model for the interaction between drill-bit and rock formation. In order to provide robustness for the analysis, uncertain parameters were considered for the bit-rock interaction model. Results should lead to more robust criteria for the design of the control parameters. The preliminary results presented have shown that indeed the considered uncertainties tend to shrink the stability regions and, thus, reducing the flexibility in the choice of control parameters. Future works will be directed to the reduction of the computational cost in order to allow larger number of stochastic realizations and, thus, more robust analysis.

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