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UNCERTAINTIES 2016–Fuzzy Analysis of the Unbalance Response of a Vertical Rotor

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Abstract: This paper is dedicated to the fuzzy uncertainty analysis of a vertical rotor supported by tilting-pad journal bearings, which is used to model a Kaplan turbine of an energy unit generator. The inherent uncertainties of the model are the nominal oil temperature of each bearing which is then considered as fuzzy variables for the uncertainty model. The analysis was limited to the orbits of the turbine guide bearing and generator guide bearing, and through the uncertainty analysis the uncertain envelopes of each guide bearing was generated. The fuzzy analysis was carried through the α-level optimization technique due to its mathematical simplicity and to its reliability.

Keywords: Fuzzy uncertainty analysis, Rotor dynamics, Journal bearings.

INTRODUCTION

According to Meggiolaro (1996), the mathematical-computational simulation of rotating machines represents an indispensable resource for the engineer, allowing a comprehensive understanding of the dynamic behavior of the system concerning the innumerous state variables involved, as well as forecast of undesired mechanical behavior of the equipment. The author reinforces that its common situations in which the obtainment experimental results or extracting information directly from measures signs are difficult and costly procedures; associated with the lack of access points for measuring and the conditions in which it is performed.

Therefore, a mathematical model representing accurately the dynamic behavior of a rotating machine is obtained considering different subsystems as follows: first the subsystems that can be defined by its geometry, like the rotor (usually modeled through Finite Element Method), discs and couplings; after, the subsystems that are frequency and/or state dependent, as the bearings; and at last, the gyroscopic effect (Cavalini Jr et al., 2012).

The bearings constitute the most critical of the rotor subsystems. Its influence over the performance, life and reliability of the machine cannot be ignored. According to Vance et al. (2010), many of the rotating systems problems can be attributed to the conception and application of the bearings. Thus, the comprehension of the physical phenomena that are related to the bearings is essential to aid the adequate choice of the suitable bearing for the rotating system. Even for equipments already in operation, the amendment or modification of the bearings are one of the most effective, direct and economical ways to improve the dynamic performance of the system.

Hydrodynamic bearings represents one very important class of bearings (Riul, 1988), due to its capacity in support the rotating machine load through a thin lubricant film that separates the rotor from the bearing (there is no contact among the metal parts). The film pressure is a result of the movement between the superficies in a speed sufficient to separate the parts. Due the high load capacity, the hydrodynamic bearings are commonly used in large rotating machines such as the key components of generator units in hydroelectric power plants.

There are several configurations for hydrodynamic bearings, used accordingly to the space availability and the rotating machine operational demands in which the bearings are going to be installed (Vance et al., 2010). The cylindrical bearings are the most easily fabricated when compared to other types of bearings. Besides, the cylindrical bearings possess consolidated mathematical models that are available in the specific literature (Childs, 1993). Such models are capable of generating the developed pressure field and, consequently, the hydrodynamic lifting forces, like for instance, the linear model of Sommerfel (Riul, 1988) and the nonlinear model by Capone (1986), both based in the Reynolds equation.

However, the cylindrical hydrodynamic bearings are subject to dynamic instability when the system operates in high velocities. The asymmetric pressure profile in the oil film is characteristic of these bearings (fixed geometry). Thus, according to Vance et al (2010), the tendency in the last decades is to apply variable geometry bearings, known as *tiltingpad*, especially in machines that operates in super critic velocities due to inherent stability of the *tiltinpad* bearings.

Nonetheless, as mostly mechanical system the *tiltinpad* bearings has inherent uncertainties regarding its geometric and operational parameters, either due to lack of knowledge or to operational fluctuations. So, in this context, this paper aims at evaluating the uncertainties impact on the dynamic behavior of a vertical rotor supported by *tiltingpad* hydrodynamic bearings. For such, a fuzzy uncertainty analysis was performed in the Generator Unit 01 (GU-01), located at the HEPP Álvaro de Souza Lima (Bariri-Brasil). In this work, besides the fuzzy uncertainty analysis performed by means of the α-level optimization technique; the formulation and incorporation of *tiltingpad* hydrodynamic bearings in the mathematical model of the GU-01 are addressed.

ROTOR MODEL

The differential equation that describes the dynamic behavior of a flexible rotor supported by hydrodynamic or ball bearings is given in Eq.(1).

$$\mathbf{M}\ddot{\mathbf{q}} + \left[\mathbf{D} + \Omega\mathbf{D}_g\right]\dot{\mathbf{q}} + \left[\mathbf{K} + \dot{\Omega}\mathbf{K}_{st}\right]\mathbf{q} = \mathbf{W} + \mathbf{F}_u + \mathbf{F}_f + \mathbf{F}_s \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{D}$ is the damping matrix (proportional damping $\mathbf{D}_p$ properly considering the bearings associated damping), $\mathbf{D}_g$ is the gyroscopic effect matrix, $\mathbf{K}$ is the stiffness matrix and $\mathbf{K}_{st}$ represents the system stiffening when in transient state. All this matrixes are associated to the rotating parts, such as the discs, the coupling and the rotor. The displacements vector is represent by $\mathbf{q}$ and the rotation speed is given by Ω. The weight force, W, considers only the rotating parts, $\mathbf{F}_u$ represents the unbalance forces, $\mathbf{F}_f$ are forces different from the unbalance that can be applied in the rotor (for instance, magnetic forces and the hydraulic stress) and $\mathbf{F}_s$ is the vector of forces produced by the bearings to support the rotor.

UNBALANCED MAGNETIC PULL

The magnetic attraction force arises in electric machines when the magnetic field generated between the rotor and the stator is not completely symmetric. Thus, the magnetic field acts as a spring with negative stiffness, i.e., with the radial translation of the rotor (approximation between the rotor and the stator) the magnetic force increases, encouraging the rotor to translate even more in this same direction.

Considering the displacements u and w of the rotor (in the generator position), the unbalanced magnetic pull (UMP) developed in the X and Z directions are given, respective, by Eq.(2) and Eq.(3) (Friswell et al., 2010).

$$F_{mX} = \frac{10^7}{8\pi}\frac{A_g B_{avg}^2 C_m}{g}u \tag{2}$$

$$F_{mZ} = \frac{10^7}{8\pi}\frac{A_g B_{avg}^2 C_m}{g}w \tag{3}$$

where $A_g = 2\pi L R_s$ (radial clearance area), L is the generator length measured in the Y direction, R is the stator radius and $g = R_s - R_g$ (R_g is the generator radius). C_m is known as the Carter coefficient, , $C_m = 1$ (Bettig and Han, 1999). B_{avg} represents the amplitude of the magnetic flow density main component and is given by Eq.(4) (Dorjee, 1994).

$$B_{avg} = \frac{\sqrt{2}}{\pi}\frac{E_a}{\Omega k N L R_g} \tag{4}$$

in which, E_a is the eletromotriz force (RMS value), k is known as winding factor and N is the number of turns per phase. The induced eletromotriz force (E_a) is determined through the relation presented in Eq.(5).

$$\vec{E}_a = \vec{V} + r_a\vec{I} + j\,x_d\vec{I}_d + j\,x_q\vec{I}_q \tag{5}$$

where $\vec{I}$ is the current and $\vec{I}_d$ and $\vec{I}_q$ are its components; r_a is the armor resistance and x_d and x_q are reactances.

The RMS values of $\vec{I}_d$ and $\vec{I}_q$ are obtained in Eq.(6) and Eq.(7).

$$I_d = I\,\mathrm{sen}(\theta_m + \delta) \tag{6}$$

$$I_q = I\cos(\theta_m + \delta) \tag{7}$$

where $\cos(\theta_m)$ is known as potency factor. The line tension V_l and the torque factor δ are given by Eq.(8) and Eq.(9), respective.

$$V = \frac{V_l\sqrt{3}}{3} \tag{8}$$

$$\tan\delta = \frac{x_q I\cos\theta_m - r_a I\,\mathrm{sen}\,\theta_m}{V + r_a I\cos\theta_m + x_q I\,\mathrm{sen}\,\theta_m} \tag{9}$$

HYDRAULIC RADIAL FORCE

The hydraulic forces developed through the radial directions of the turbine can be calculated using the Alford's equation, as shown in Eq.(10) and Eq.(11). According to Bettig and Han (1999), this equation considers the reduction of the energy generation in accordance with the increasing of the dimensions of the turbine blades. In addition, the energy generation is directly related with the rotor torque and the system rotation speed.

$$F_{HX} = \frac{T\beta_H}{D_p H} w \tag{10}$$

$$F_{HZ} = \frac{T\beta_H}{D_p H} u \tag{11}$$

where β_H is known as Alford's coefficient ($1 \le \beta_H \le 3$), D_p is the dimeter measured on the blades edges and H is the blades length measured in the radial direction. Note that the hydraulic force developed through the determined direction depends of the observed displacement in the perpendicular direction and from the force (coupling of the orthogonal directions). The torque T is defined by Eq. (12).

$$T = \frac{P}{\Omega} \tag{12}$$

in which P is the potency (Watts) produced by the generator unit.

GUIDE HYDRODYNAMIC BEARINGS

In the mathematical model of hydrodynamic segmented bearings, the lifting forces are determined from the Reynolds equation. For the analytical development of the Reynolds equation (segmented bearings), four reference systems are used (see Fig.1). Following the formulation presented by Russo (1999), the first system is placed in the center of the bearing (inertial system *I* XYZ). The second system indicates the position of the *j-th* pad in the bearing (auxiliary system *B* xyz). Each pad possess its own mobile auxiliary system (mobile system *B*' x'y'z'). The last system follows the inner surface of the pad (curvilinear mobile system *B*" x"y"z").

Taking the oil viscosity as constant, the Eq.(13) shows the Reynolds equation for the guide bearings segments in its dimensionless form.

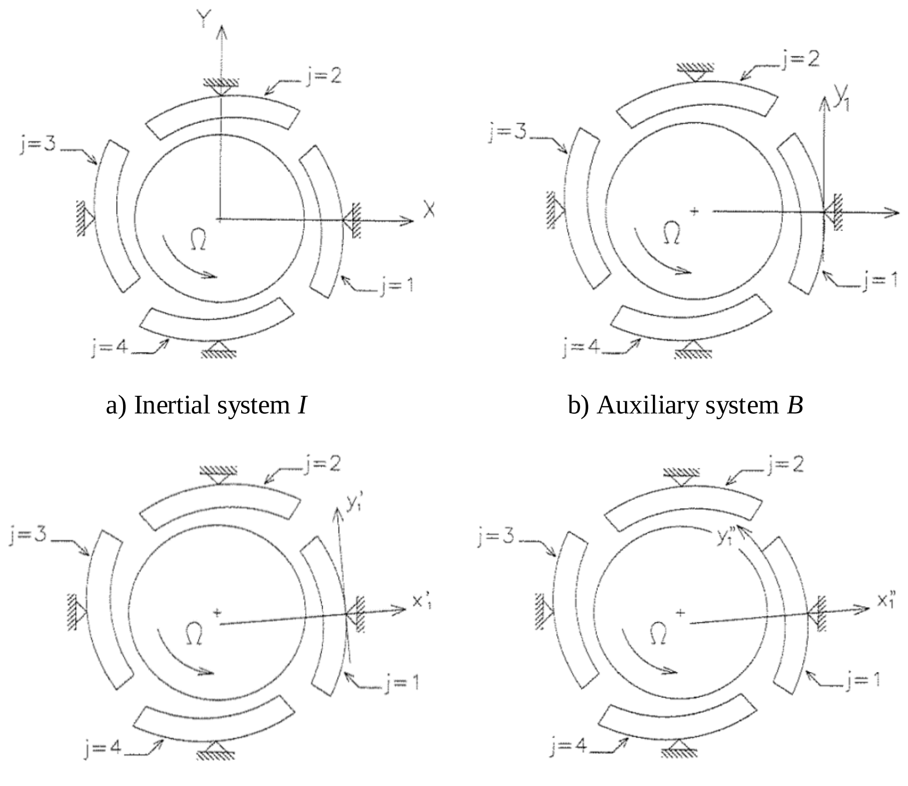
$$\frac{1}{\beta_s^2}\frac{\partial}{\partial \bar{y}''}\left(\bar{h}^3\frac{\partial \bar{p}}{\partial \bar{y}''}\right) + \left(\frac{R_s}{L}\right)^2\frac{\partial}{\partial \bar{z}''}\left(\bar{h}^3\frac{\partial \bar{p}}{\partial \bar{z}''}\right) = 6\frac{R}{\beta_s R_s}\frac{\partial \bar{h}}{\partial \bar{y}''} + 12\frac{\partial \bar{h}}{\partial \bar{t}} \tag{13}$$

where p is the pressure distribution over the bearing segment, y" is the curvilinear coordinate, z" is the coordinate through the length L of the bearing, β_s is the angular length of the segment, R_s is the segment radius, R is the rotor's radius at the bearing position, t represents time and h is the oil film thickness, shown in its dimensionless form in Eq.(14).

$$h = R_s - R - \left\{\mathrm{sen}(\beta)\left[y_R + \alpha(R_s + h_s)\right] + \cos(\beta)(x_R + R_s - R - h_o)\right\} \tag{14}$$

in which $-\beta_s/2 \le \beta \le \beta_s/2$, α is the angular position of the segment, x_R and y_R are the position coordinates of the rotor center at the auxiliary system B, h_s is the segment thickness and h_o the bearing radial clearance. The dimensionless parameters [$\bar{\cdot}$] are shown in Eq.(15).

$$\bar{y}'' = \frac{\beta}{\beta_s} \qquad \bar{z}'' = \frac{z''}{L} \qquad \bar{h} = \frac{h}{h_o} \qquad \bar{t} = t\Omega \qquad \bar{p} = \frac{p h_o^2}{\mu\Omega R_s^2} \tag{15}$$



a) Inertial system *I* b) Auxiliary system *B*

c) Mobile system *B*' d) Curvilinear mobile system *B*"

Figure 1 – Adopted coordinate system used in the tilting pad hydrodynamic bearings modeling (*j* = 1, ..., *N* with *N* = 4; Russo, 1999).

The oil viscosity μ is calculated as a temperature function using an approximation of Vogel's model, as shown in Eq.(16).

$$\mu = A_{bearing} \exp\left(\frac{B_{bearing}}{T_{bearing} + C_{bearing}}\right) \tag{16}$$

where $A_{bearing}$ = 4,98 *x* 10^{-5}, $B_{bearing}$ = 959,79 e $C_{bearing}$ = 98,62. All this coefficients are specific of the oil ISO VG 68. $T_{bearing}$ is the oil temperature in Celsius.

The solution of Eq.(13) is determined through an iterative process, in which Gauss-Seidel model is used to adjust the values of α, along with rotor center equilibrium position (directions x and y), until the conditions established by Eq.(17), Eq.(18) and Eq.(19) being satisfied. The Finite Differences method is used to determine the pressure field developed in each bearing segment. (Riul, 1999; Daniel 2012).

$$F_X = \sum_{j=1}^{N}\left[\int_0^L \int_0^{y''} p_j \cos\left(\beta_j\right) dy'' dz''\right]\cos\left(\varphi_{s\,j} + \alpha_j\right) = 0 \tag{17}$$

$$F_Y - F_w = \sum_{j=1}^{N}\left[\int_0^L \int_0^{y''} p_j \cos\left(\beta_j\right) dy'' dz''\right]\mathrm{sen}\left(\varphi_{s\,j} + \alpha_j\right) - F_w = 0 \tag{18}$$

$$M_{y_j''} = \left[\int_0^L \int_0^{y''} p_j \,\mathrm{sen}\left(\beta_j\right) dy'' dz''\right]\left(R_s + h_s\right) = 0 \tag{19}$$

where F_X and F_Y are resulting forces in the bearing (directions X and Y, respective), F_w is the force that the rotor applies in the bearing (estimated in an iterative process; vertical rotor) and $M_{y''j}$ is the momentum that acts over the segment around the pivoting point.

Finally, the stiffness and damping coefficients of each segment ate determined from the integration of the pressure fields *p* obtained. These were determined following the formulation presented by Russo (1999). Small displacements and velocities are imposed from the equilibrium position (position in which the conditions established by Eq. (17), Eq. (18) and Eq. (19) are satisfied).

Is important to reiterate that the results obtained through the developed computational codes were compared with the results presented by Daniel (2012) and Russo (1999), proving satisfactorily close.

THRUST HYDRODYNAMIC BEARINGS

According to Heinrichson (2006), there are innumerous mathematical models that can be applied to represent the dynamic behavior of thrust hydrodynamic bearings (each possessing distinguishes levels of detailing). Nevertheless, the mostly used models are the one dimension pressure field model considering a two dimensional isothermal distribution model, or the one dimension pressure field model including a two dimension temperature distribution calculus over the segments. Beside those models, the two dimension pressure field model can still take into account the deflections of the segments or incorporate temperature fields or tridimensional deflections (there are versions that considers both phenomena). The results presented by Heinrichson (2006) show that the two dimension isothermal model is capable of properly characterizing the hydrodynamic thrust bearings. So, this was the model adopted is this work.

Assuming a constant oil viscosity, Eq.(20) shows the Reynolds equation for the thrust bearing segments in its dimensionless form.

$$\frac{1}{12\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\frac{\bar{r}\,\bar{h}^3}{\bar{\mu}}\frac{\partial\bar{p}}{\partial\bar{r}}\right)+\frac{1}{12\theta_o^2\,\bar{r}^2}\frac{\partial}{\partial\bar{\theta}}\left(\frac{\bar{h}^3}{\bar{\mu}}\frac{\partial\bar{p}}{\partial\bar{\theta}}\right)=\frac{1}{2\theta_o}\frac{\partial\bar{h}}{\partial\bar{\theta}}+\frac{1}{\Omega}\frac{\partial\bar{h}}{\partial t} \tag{20}$$

in which p represents the pressure distribution in the segment, θ is the cylindrical coordinate, r is the radial coordinate, θ_o is the angular length of the segment and h is the film oil thickness, that is given in its dimensionless form by Eq.(21).

$$\bar{h}=1-\bar{\alpha}_r\left\{\bar{r}_{piv}-\bar{r}\cos\left[\theta_o\left(\bar{\theta}-\bar{\theta}_{piv}\right)\right]\right\}-\bar{\alpha}_p\bar{r}\,\mathrm{sen}\left[\theta_o\left(\bar{\theta}-\bar{\theta}_{piv}\right)\right] \tag{21}$$

where α_r is the roll angle, r_{piv} the pivoting position through the radial direction, θ_{piv} the pivoting position through the circumferential direction and α_p the tilt angle. The dimensionless parameters [$\bar{\cdot}$] are shown in Eq. (22). Some of these parameters can be observed in Fig. 2.

$$\bar{\theta}=\frac{\theta}{\theta_o}\qquad \bar{r}=\frac{r}{r_1}\qquad \bar{h}=\frac{h}{h_o}\qquad \bar{\alpha}_r=\alpha_r\frac{r_1}{h_o}\qquad \bar{\alpha}_p=\alpha_p\frac{r_1}{h_o}\qquad \bar{\mu}=\frac{\mu}{\mu_o}\qquad \bar{p}=\frac{ph_o^2}{r_1^2\Omega\mu_o} \tag{22}$$

in which r_1 is the inner radius of the bearing segment (r_2 represents the outter radius), h_o is the film oil thickness at the pivoting point and μ_o is the oil viscosity that is being injected in the segment (in this case, it is assumed that $\mu_o = \mu$).

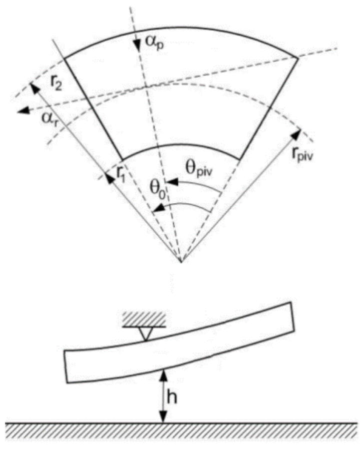


Figure 2 – Coordinate system adopted in the two dimension isothermal model (Heinrichson, 2006).

The solution of Eq. (20) is determined by an interactive procedure, in which the SIMPLEX method is used to adjust the values of α_r, α_p e h_o, until the conditions established by Eq. (23), Eq. (24) e Eq. (25) are satisfied. The Finite Differences method and the Successive Relaxation are used along with the described minimization procedure. Thus, a mesh that discretizes the pressure field is generated as proposed by Riul (1988).

$$F_{res}=\int_{r_1}^{r_2}\int_0^{\theta_o} pr\,d\theta\,dr-F_Z=0 \tag{23}$$

$$\bar{M}_X = \int_1^{\bar{r}_2}\int_0^1 \bar{p}\,\bar{r}^2 \,\mathrm{sen}\left[\theta_o\left(\bar{\theta}-\bar{\theta}_{piv}\right)\right] d\bar{\theta}\, d\bar{r} = 0 \tag{24}$$

$$\bar{M}_Y = \int_1^{\bar{r}_2}\int_0^1 \bar{p}\,\bar{r}\left\{\bar{r}\cos\left[\theta_o\left(\bar{\theta}-\bar{\theta}_{piv}\right)\right]-\bar{r}_{piv}\right\} d\bar{\theta}\, d\bar{r} = 0 \tag{25}$$

where F_{res} is the residual force, F_Z represents the force that acts over the segment (weigh of the generator unit); $\bar{M}_X$ and $\bar{M}_Y$ are the orthogonal moments that acts over the segment around the pivoting point.

Finally, the stiffness and damping coefficients K_{seg} and D_{seg}, respective, of each segment are determined from the integration of the obtained pressure field p. Small displacements and velocities are then imposed from the equilibrium position (position in which the conditions established by Eq. (23), Eq. (24) e Eq. (25) are satisfied).

TURBINE MATHEMATICAL MODEL

Figure 3 illustrates the formulated Finite Element model of the GU01 (Kaplan turbine) of the HEPP Álvaro de Souza Lima (Bariri). In this case, 29 beam elements to represent the rotor dynamic behavior. The formulated mathematical model contemplate a vertical steel rotor (E = 208 GPa, ρ = 7870 kg/m^3 e υ = 0,3) with length of 12,5 m, three rigid steel (ρ = 7870 kg/m^3) discs located at nodes #3 (representation of the turbine blades), #23 and #30 (representation of the generator distributed in two nodes), and two segmented hydrodynamic bearings located at nodes #7 and #20 (B_1 and B_2; turbine guide bearing and generator guide bearing, respectively). The body of the Kaplan turbine (the entire set disregarding the blades) was represented in the GU01 model by a localized rotor diameter increase (elements located between nodes #1 e #5). Thus, along with the disc used to represent the blades (node #3) a total mass of 126000 kg was found. Additionally, were included in the model the discs at nodes #7, #18 e #20 to represent the inertias of the turbine guide bearing track, the thrust bearing collar and from the generator guide bearing track, respectively. Table 1 and Table 2 shows the features of the discs and the hydrodynamic bearings, respectively. The oil temperature in the three bearings is considered constant and equal to 60°C. A proportional damping with coefficients $\lambda = 1\,x\,10^{-3}$ e $\beta = 1\,x\,10^{-7}$ was considered in the system. The sensors were positioned through the orthogonal directions of both bearings, measure plane S_{B1} e S_{B2}, respectively, denominated S_{B1X}, S_{B1Y}, S_{B2X}, e S_{B2Y}.

Table 1 – Discs features of the GU01 model.

Discs	$M_D\,x\,10^3$ (Kg)	$I_D\,x\,10^3$ (Kg.m^2)	$I_{Dz}\,x\,10^3$ (Kg.m^2)
#3	19,33	48,87	97,72
#7	1,28	0,24	0,45
#18	19,32	10,96	18,99
#20	0,49	0,09	0,18
#23	158,0	834,28	1666,0
#30	158,0	834,28	1666,0

The thrust hydrodynamic bearing (bearing with 12 segments) was included in the GU01 Finite Element Model through applied momentums in the *dof* θ and φ of node #18 (thrust bearing localization; see Fig. 3). For such, it was considered that the rotation of the *dof* θ implies that the restoring forces are produced for only 6 segments. The same hypothesis is also considered for the *dof* φ. The applied momentums in the Finite Element model are given by Eq.(26) and Eq.(27).

$$M_\theta = \sum_{i=1}^{6} K_{seg\ i}\, d_i\, \theta + \sum_{i=1}^{6} D_{seg\ i}\, d_i\, \dot{\theta} \tag{26}$$

$$M_\varphi = \sum_{i=1}^{6} K_{seg\ i}\, d_i\, \varphi + \sum_{i=1}^{6} D_{seg\ i}\, d_i\, \dot{\varphi} \tag{27}$$

where i = 1, 2, ..., 6 indicates the bearing segments and d_i is the distance between the rotor center and the pivoting point of each segment, taken parallel to the radial directions (Y for the *dof* θ and X for the *dof* φ). The approximations θ = sen(θ) and φ = sen(φ) are used.

Nonetheless, the resulting force sustained by the thrust hydrodynamic bearing (F_{thrust}; maximum force) is given by the sum of the applied load in the bearing (for instance, the weight of GU01 along with the force exercised by the water drop in the turbine) with the resulting applied momentums. The parcel of the resulting momentums is clearly divided by the largest value d_i, as shown by Eq. (28).

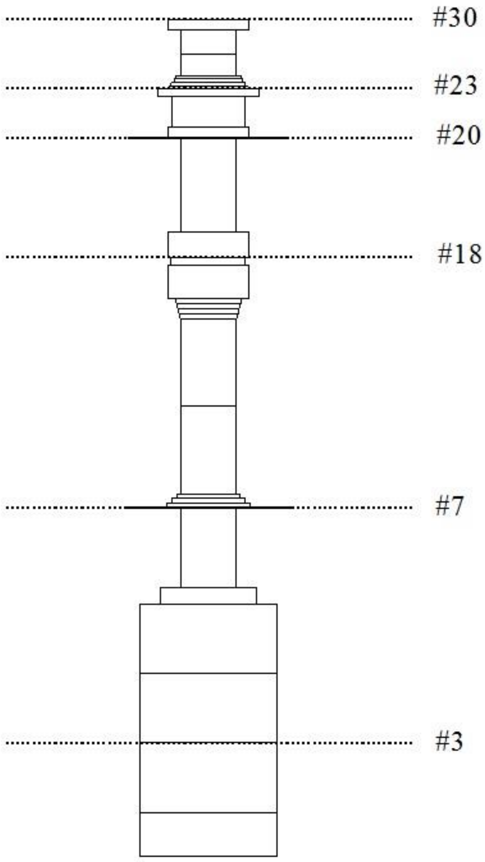


Figure 3 – Finite element model of turbine GU01 possessing cylindrical hydrodynamic bearings.

Table 2 – Segmented bearings dimensions of the GU01 model.

Bearing	#7 (turbine)	#20 (generator)
Length L (m)	0,210	0,305
Rotor radius R (m)	0,650	0,635
Radial clearance h_o x 10^{-6} (m)	160	180
Angular length β_s (degrees)	20	27
Segments radius R_s (m)	0,6504 (estimated)	0,63545 (estimated)
Segments thickness h_s (m)	0,047	0,048
Number of segments N	16	12
Position φ_s of segment 1 (degrees)	0	0
Distance between the pivots (degrees)	22,5	30
Position φ_s of segment N (degrees)	337,5	330
Discretization of the pressure field (y")	32	32
Discretization of the pressure field (z")	32	32

$$F_{thrust} = P_{thrust} + \left(M_\theta^2 + M_\varphi^2\right)^{1/2} d_i^{-1} \tag{28}$$

in which P_{thrust} is the load applied to the bearing. It was considered that the turbine weight is the sole responsible for the load exercised over the bearing (P_{UG} = 5,24 MN; mass 534,5 x 10^3 kg). Clearly, there are other hypothesis that can be incorporated in future works (such as the water drop effect). Table 3 presents the thrust bearing segments dimensions.

Table 3 – Segment dimensions of the hydrodynamic thrust bearing.

Parameter	Value
r_1	0,700 m
r_2	1,370 m
r_{piv}	1,035 m
θ_o	26^o
θ_{piv}	15^o

For the resolution of the Reynolds equation the pressure field was divided in 30 elements in each direction (θ and r). In this case, it was obtained K_{seg} = 1,57 x 10^{10} N/m and D_{seg} = 2,12 x 10^8 Ns/m, considering a displacement of 0,01 h_o around the equilibrium position (to determine the stiffness) and a time of 0,001 s (to determine the damping). The equilibrium position of the segment was h_o = 57,7 x 10^{-6} m, $\bar{\alpha}_r$ = 0,6569 and $\bar{\alpha}_p$ = 0,3754.

FUZZY UNCERTAINTY ANALYSIS

According to Lara-Molina et al. (2014), the random parameters of rotating systems can be modeled by using fuzzy theory as an alternative approach to the stochastic methods. By using the fuzzy theory, it is possible to describe incomplete and inaccurate information. The theory of fuzzy sets was initially formulated by Zadeh (1965) to characterize vague aspects of information. Thereafter, a different approach for fuzzy sets was developed; then it can be compared with the theory of possibilities to deal with the uncertainty of information Zadeh (1968). Both theories are connected, so that the uncertainties are modeled by means of the theory of fuzzy sets for the cases in which the stochastic process that describes the random variables is unknown.

In the fuzzy set theory, the uncertainties are described as fuzzy variables and the mathematical model is represented as a fuzzy function that maps those inputs. The α-level optimization (Möller et al., 2000) is a method for mapping fuzzy inputs, in which the fuzzy variables are discretized by means of α-cuts creating a crisp subspace (a subspace at the ℝ) that can be used as a search space for a two stage optimization problem. This optimization problem consists in finding the maximum or minimum value of the output, at each evaluated τ (the independent variable of the dynamic response that may represent time, frequency, or spacial coordinates) generatings thus the lower ($z_{\alpha kl}$) and upper ($z_{\alpha kr}$) bounds of the fuzzy resulting variable $\tilde{z}$ in the α-level α_k (see Fig. 4). The set of discretized intervals [$z_{\alpha kl}$, $z_{\alpha kr}$] for $\alpha_k \in$ (0,1] composes the whole fuzzy resulting variable.

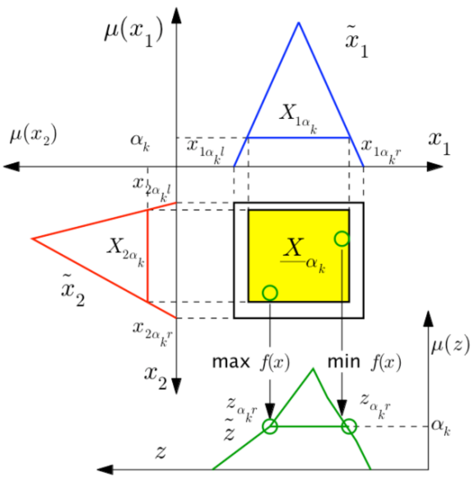


Figure 4 – The α-level optimization method.

The application of the fuzzy uncertainty analysis demands the solution of a large number of optimization problems regarding all the α-levels of interest and the time, frequency, or spacial steps considered. Each upper and lower bounds of the system analysis is obtained from an optimization procedure. The output value of the transient analysis at the evaluated time, frequency, or spatial-step constitutes the objective function. The inputs to this function are the uncertain parameters described previously as fuzzy variables.

NUMERICAL RESULTS

The uncertainty analysis was applied to the turbine model, treating the oil nominal temperature as an uncertain information. The oil temperature affects the oil viscosity influencing thus the pressure fields in the hydrodynamic bearings. In the uncertain scenario proposed in this word a fluctuation of ±6°C around the oil nominal temperature (54°C) was considered, and the impact of these fluctuations was evaluated in the time response of the system (the orbits at the bearings positions: node #7 (turbine guide bearing) and node #20 (generator guide bearing)).

Since this is a vertical rotor, the effect of the weight force is disregarded in the analysis. In all the simulations, the rotor operates at a constant speed of 112,5 RPM. An unbalance of 500 Kg.m with phase angle of 0° was considered in the turbine (simulation time t = 5 s; time step of Δt = 0,001 s). Table 4 presents the parameters adopted to generate the magnetic unbalance. For the hydraulic forces the following parameters were considered: Alford's coefficient β_H = 3, D_p = 5,988 m e H = 1,5 m. Is important to reiterate that the goal of this paper is to evaluate the model behavior when expose to the considered fluctuations, thus variations in the magnetic unbalance and the hydraulic unbalance were not considered. Figure 5 illustrates the model responses regarding the time domain.

Table 4 – Electric parameters adopted for the magnetic unbalance.

Parameter	Value	Parameter	Value
V_l	13800 volts	N	432
P	41,4 MW	k	0,949
I	1930 A	cos (θ_m)	0,9
R_s	4,65 m	x_d	0,7
R_g	4,55 m	x_d	0,5
L	1,13 m	r_a	0

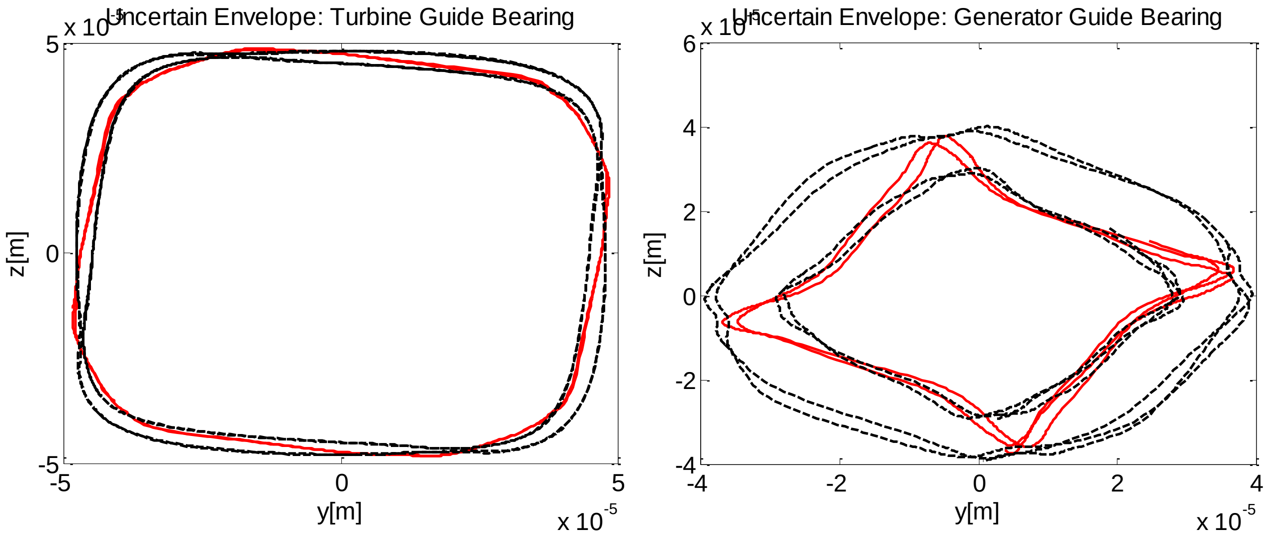


Figure 5 – Orbits Envelopes.

From Fig.5 it can be observed that the temperature fluctuation as a greater impact in the performance of the generator guide bearing causing its orbit to vary from a maximum amplitude of 20 μm to 40 μm in the horizontal direction; wherein for the turbine guide bearing the effects were less significant.

FINAL REMARKS

The goal of this paper was to evaluate the use of the fuzzy logic to perform an uncertainty analysis of a vertical rotor supported by tilting-pad journal bearings, which is used to model a Kaplan turbine of an energy unit generator. The uncertain oil temperature was treated as a fuzzy variable and the uncertainty analysis limited to the time domain (orbits of the turbine and generator guide bearings).

The proposed methodology successfully evaluate the influence of the oil temperature fluctuation in the dynamic behavior of the system; showing that the most sensible subsystem was the generator guide bearing, that presented a large variation in its performance, when exposed to the system fluctuations.

Despite the dynamic representativeness, the presented results were obtained from some dimensional and operational approximations, thus, it should be noted that for more refined results it is necessary to adjust the proposed Finite Element model regarding reliable experimental data. This procedure is essential to ensure the model representativeness.

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