Proceedings of the 3rd International Symposium on Uncertainty Quantification and Stochastic Modeling
D.A. Rade; A. M. G. Lima (Editors), ABCM, Maresias, SP, Brazil, February 15-19, 2016

STOCHASTIC COLLOCATION APPROACH FOR EVALUATION OF JOURNAL BEARINGS DYNAMIC COEFFICIENTS

Gabriel Yuji Garoli [1], Helio Fiori de Castro [2]

[1] University of Campinas, Faculty of Mechanical Engineering – Department of Mechanical Design

Abstract: Rotordynamics is an important field of study due to the vastly applicability of rotating machines. A crucial component of these machines is the bearing that connects the rotating shaft with the static foundation of the machine. Therefore, it is important this component does not transmit the shaft's vibration to the base. One way to minimize that transference is the use of fluid journal bearings, which absorbs part of the energy of the vibration. Mathematical models of rotor-bearing systems were develop to predict it behavior and design it. A common approach used for bearing modelling is the consideration of a linear model, which takes into account the damping and stiffness coefficients of each element. In order to evaluate these coefficients, a numerical solution of the Reynolds equation can be applied, considering bearing's geometry and the fluid properties. However, these parameters have some uncertainties, which can be modeled using probability distributions. Therefore, the deterministic problem to evaluate the damping and stiffness coefficients of the bearing should become a stochastic problem. Thus, this work presents a Stochastic Collocation method for performing damping and stiffness coefficients evaluation of a fluid journal bearing. Generalized polynomial chaos is adopt to approach the probability distributions functions of the random parameters. The Stochastic Collocation is a non-intrusive method, like the Monte Carlo Methods, so it needs a deterministic solver. To obtain the solution of the Reynolds equation, which model the fluid flow inside the bearing, a finite-volume method was used. Through the pressure distribution on the lubricant and the gap between the journal and the bearing, the damping and stiffness coefficients are evaluated. The application of the Stochastic Collocation method allows statistical post-processing, thus the uncertainties of the bearing can be propagated to the damping and stiffness coefficients and then, for example, can be used in the modelling of a rotor-bearing system. The combination of the Stochastic Collocation method and the generalized Polynomial Chaos allows a good convergence with computing time reduction, compared with Monte Carlo methods, due to the smaller number of samples used.

Keywords: Stochastic Collocation, Generalized Polynomial Chaos expansion, Journal Bearing, Rotordynamics

INTRODUCTION

Journal bearings are important components of rotating machine, because it allows the relative movement between the rotating part the foundation. Besides, they support the weight of the rotating part. To avoid the straight contact between these elements with relative motion, a lubricating fluid is used. Initially, there is a direct contact between the journal and the bearing. Then the fluid is forced into the region below the shaft due to the shear force. This phenomenon creates high pressure at the journal surface, which supports the shaft weight and prevent it contact with the bearing wall.

Stodola (1925) and Hummel (1926) aimed to improve the calculation of the critical speed of rotor, introduced the idea of representing the dynamics characteristics of journal bearing in stiffness e damping coefficients. Solutions for the pressure distribution, used to determinate the dynamics coefficients, were developed by various researchers, as Hummel (1926), for long bearing, Dubois and Orcvirk (1953), for short bearings, Pinkus (1958) and Strenlicht and Maginnis (1957), for finite bearings However, the assumptions used in each case make these models suitable for a small number of applied journal bearings. With the advent of the computer mechanics, more advanced models of journal bearings could be numerical calculated. Lund (1964) introduced a method to calculate the linearized dynamical coefficients.

The characteristics of the journal bearing, used in the solution of Reynolds equation (Reynolds, 1886) has some uncertainties, the addition of these uncertainties transform the problem into a stochastic problem. Probabilistic distributions to model these parameters are needed and it needs to be count in the solution, to propagate the uncertainties to the pressure distribution and then to the dynamic coefficients. In this case, the oil film viscosity and radial clearance are potential source of uncertainty. In the first case, due to the temperature variation in hydrodynamic model, with neglected thermal effects. Besides, as the bearing can suffer wear, the small dimension of the radial clearance can also presents some uncertainties.

Wiener (1938) defined the "homogenous chaos" as the span of Hermite polynomial functions of a Gaussian distribution. Ghanem and Spanos (1991) shows the construction of polynomial chaos expansion and Xiu and Karniadakis (2002) use the Askey scheme of hypergeometric orthogonal polynomials to determine the generalized polynomial chaos, to expand others probabilistic distributions.

The solution of the stochastic problem are obtained by using a stochastic collocation method. The solution is approximated by a series of orthogonal polynomial expansion, in this case generalized polynomial chaos expansion. This method is used in many currently works to solve stochastic problems. Babuska, Nobile and Tempone (2007) propose a method to solve elliptic partial differential equations with random variables and forcing terms. Xiu (2007) presents an algorithm to solve parametric uncertainty into mathematical models. Marzouk and Xiu (2009) presents a solution to Bayesian inference in inverse problems through a stochastic collocation method.

In the present work, a numerical stochastic collocation method, using generalized polynomial chaos expansion, is presented to evaluate the dynamical coefficients of journal bearing with uncertainties in the geometry and oil film temperature that influences the viscosity. A comparison with Monte Carlo simulation is accomplished to validate the results.

ROTOR SYSTEM MODELING

The Finite Elements Method (FEM) is extensively used in rotodynamics analysis. Through an energy method, the FEM transforms systems with complex modeling into smaller elements with simple equations. Each element is connected to each other through the boundaries conditions; nodes symbolize the connection between the elements. The movement equation of the rotor system in then obtained and has the following matrix form:

$$[M]\cdot\{\ddot{x}\}+([C]+\omega\cdot[G])\cdot\{\dot{x}\}+[K]\cdot\{x\}=\{F\} \tag{1}$$

Where *[M]*, *[C]*, *[G]*, and *[K]* are the mass, damping, gyroscopic and stiffness matrices, respectively, and *{F}* is the vector of external forces.

The effects of the journal bearings are included in the damping and the stiffness matrices, the dynamical coefficients, which are added in the respective positions in the matrices.

DYNAMICAL COEFFICIENTS OF JOURNAL BEARINGS

The theory of the evaluation of dynamical coefficients are based on the paper of Lund (1987) and Machado and Cavalca (2009). In a journal bearing, the weight of the rotor and its lateral vibration, compress the lubricant, which causes radial reaction forces on it. These reaction forces of a fluid film are given by the equation:

$$\begin{Bmatrix} F_z \\ F_y \end{Bmatrix} = -\int_{-\frac{L}{2}}^{\frac{L}{2}}\int_{\theta_1}^{\theta_2} p\cdot\begin{Bmatrix}\cos\theta \\ \sin\theta\end{Bmatrix}\cdot R\cdot d\theta\cdot dx \tag{2}$$

Where p is the pressure generated within the fluid, R is the journal radius, L is the axial length, x is the axial coordinate and θ is the circumferential angular coordinate from the negative z - axis. The film extends from θ_1 to θ_2 where both angles may be functions of z.

The pressure distribution is modeled by the Reynolds Equation, which is:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\cdot\mu\cdot U\cdot\frac{\partial h}{\partial z}+12\cdot\mu\cdot\frac{\partial h}{\partial t} \tag{3}$$

Where h is the film thickness, x and z are the Cartesian coordinates, t is the time, μ is the lubricant absolute viscosity and U is the peripheral velocity of the rotor. The film thickness given is a function of the radial clearance, Cr, and the coordinates of the bearing center, x and y.

$$h = C_r + z\cdot\cos\theta + y\cdot\sin\theta \tag{4}$$

Consequently, the reaction forces are function of x and y and the instantaneous velocity of the bearing center, and . For small deviations, Δx and Δy, measured form the static equilibrium position (x0 and y0). Hence, the first order Taylor series expansion is:

$$\begin{aligned} F_z &= F_{z_0} + K_{zz}\cdot\Delta z + K_{zy}\cdot\Delta y + C_{zz}\cdot\Delta\dot{z} + C_{zy}\cdot\Delta\dot{y} \\ F_y &= F_{y_0} + K_{yz}\cdot\Delta z + K_{yy}\cdot\Delta y + C_{yz}\cdot\Delta\dot{z} + C_{yy}\cdot\Delta\dot{y} \end{aligned} \tag{5}$$

The dynamic coefficients are evaluated by the partial derivatives of the kind:

$$K=\left(\frac{\partial F}{\partial z}\right)_0, C=\left(\frac{\partial F}{\partial \dot{z}}\right)_0 \tag{6}$$

A finite volume method is used to evaluate the pressure distribution and, then, is integrated to obtain the horizontal and vertical reaction forces. The dynamical coefficients are evaluated by a Taylor expansion, Eq. (4).

The absolute viscosity of the lubricant is a function of the operating temperature. Hence, in this work, the operating temperature and the radial clearance are the random variables of the dynamic coefficients evaluation and it are modeled as a Gaussian distribution with mean and standard deviation known.

GENERALIZED POLINOMIAL CHAOS EXPASION

Given a finite dimensional random space Ω and ω a random variable of Ω, with probability density function (pdf) ρ(ω). Lets $\{\Phi_m(\omega)\}$ be a set of orthogonal polynomial satisfying the orthogonality conditions:

$$\int_{\Omega} \rho(\omega)\cdot\Phi_m(\omega)\cdot\Phi_n(\omega)\cdot d\omega = h_m^2\cdot\delta_{mn} \tag{7}$$

Where δm is the Kronecker delta function and is a normalization factor:

$$h_m^2 = \int_{\Omega} \rho\cdot\Phi_m^2\cdot d\omega \tag{8}$$

The probability density function (pdf), ρ in eq.(7), works as the integration weight, which defines thee type of the set of orthogonal polynomials. For Gaussian distributed random variable, which will be used in this work, its pdf defines Hermite polynomials.

For a N-variables, the basis of M orthogonal polynomials in the random space Ω is defined as:

$$\Phi_N^P = \underset{|d|\le P}{\otimes}\Phi^{i,d_i} \quad , M = \dim(\Phi_N^P) == \begin{pmatrix} N+P \\ N \end{pmatrix} \tag{9}$$

Where the tensor product is over all possible combinations of the unidimensional polynomial of maximum degree P and the sum of the total degree at most P.

$$\Phi_m(\omega) = \Phi_{m_1}(\omega_1)\cdot\Phi_{m_2}(\omega_2)\ldots\Phi_{m_N}(\omega_N) \quad , \; m_1 + m_2 + \ldots + m_N \le P \tag{10}$$

The stochastic solution of the stochastic problem, now, can be projected to a polynomial basis and is given by

$$v(q,\omega) \cong v_N^P(q,\omega) = \sum_{m=1}^{M} \hat{v}_m(q)\Phi_m(\omega) \tag{11}$$

The generalized polynomial chaos can be used in the Stochastic Garlekin Method and stochastic collocation method for the solution of the stochastic problems, the last one is used in the present work.

Statistical Parameters

The mean and the variance of the stochastic solution can be easily evaluated through the Eq. 11 and Eq. 12.

$$\mathrm{E}[u] \approx \int\left(\sum_{m=1}^{M}\hat{v}_m\Phi_m(\omega)\right)\rho(\omega)d\omega = \hat{v}_1 \tag{12}$$

$$Var(v(q)) = \mathrm{E}[(u(q,\omega) - \mathrm{E}[u(q,\omega)])\wedge 2] \approx \sum_{m=2}^{M}\hat{v}_m^2(q) \tag{13}$$

STOCHASTIC COLLOCATION METHOD

Let D⊂ℝd, d⊂ℕ* with boundary ∂D, $q = (q_1,\ldots,q_N)$ be nonrandom variables and $\omega = (\omega_1,\ldots,\omega_N)$ be random variables mutually independent of each other. Considering a partial differential equations:

$$\begin{aligned} L(q,v;\omega) &= 0 \quad , in \quad D \\ B(q,v;\omega) &= 0 \quad , on \quad \partial D \end{aligned} \tag{14}$$

Where L is a differential operator and B is a boundary operator. The solution *u(x,ω)* of the Eq. 14 is approximate as the form of generalized polynomial expansion as the Eq. 11. A set of nodes is chosen by a quadrature rule, in this work by the sparse grid method with Chenshaw-Curtis rule. The Eq. 14 is solved for each node and the expansion coefficients can be determine by the least square method.

$$\begin{Bmatrix} u(q,\omega^1) \\ \vdots \\ u(q,\omega^Q) \end{Bmatrix} = \begin{bmatrix} \Phi_0(\omega^1) & \cdots & \Phi_M(\omega^1) \\ \vdots & \ddots & \vdots \\ \Phi_0(\omega^Q) & \cdots & \Phi_M(\omega^Q) \end{bmatrix} \cdot \begin{Bmatrix} \hat{u}_1 \\ \vdots \\ \hat{u}_M \end{Bmatrix} \tag{15}$$

Where $u(q,\omega^i)$ is the solution of Eq 14 and $\Phi_j(\omega^i)$ is the polynomial calculated on the node ω^i. This way the expansion coefficients are calculated.

There is other solutions to the stochastic collocation method, like the pseudo-spectral approach and usage of the Lagrange polynomial. Each one has its advantages and difficulties, but, in general, is a straight forward implementation.

Sparse Grid

The sparse grid is constructed by the Smolyak algorithm, which is a linear combination of product formulas, and the lineal combination is chosen in such a way that an interpolation property for *N=1* is preserved for *N>1*. Only products with a relatively small number of points are used and the resulting nodal set has significantly less number of node compared to the tensor product rule. This method is interesting to the presented paper, due to the considerable time used by the deterministic solver. A Chenshaw-Curtis quadrature rule to determine the points for a unidimensional interpolation and the Smolyak algorithm does the combination to construct the nodes for the stochastic collocation method. The sparse grid is proposed in the work of Xiu (2009) and the sparse grid with the Chenshaw-Curtis in used by Bernadá (2011).

RESULTS

The random variables should be already modeled, so a correlation to a generalized Polynomial chaos expansion can be done. A Gaussian distribution was considered to model the uncertainties of the variables, the oil film operating temperature and the radial clearance. Its means and standard deviations are presented in the Tab. 1:

Table 1 - Mean and standard deviation considered of the random variables

Parameter	Mean	Standard Deviation
Operating Temperature	90.00 µm	2.50 µm
Radial Clearance	25.00 °C	2.00 °C

A program implemented by Tuckmantel (2010) was used as the deterministic solver to evaluate the pressure distribution and then calculate the dynamical damping and stiffness coefficients of the journal bearing. This program calculated the damping and the stiffness coefficients and the bearing locus for 13 operating rotation speeds. The dynamical coefficients were evaluated for each point chosen by the sparse grid method. Then, the expansion coefficients were accomplished. An expansion of degree five and grid level of six was made and then the expansion coefficients were calculated.

The mean and variance of stiffness and damping coefficients were evaluated according to Eqs. 12 and 13. The rate between standard deviation (variance root square) and mean were calculated and are shown in Figs. 1 and 2.

In the case of stiffness coefficients, is possible to observe that the cross coupling coefficients converges to the same value in higher rotation speed. Besides, the direct coefficients presents lower rates than the cross coupling coefficients.

For the damping coefficients, the direct coefficients rates tend to be equal. On the other hand, the cross coupling coefficients rates are close, because in the deterministic solution the coefficients values are coincident.

In order to validate the convergence of the method, the same problem was solved by a Monte Carlo simulations. Table 2 presents the time of processing and the number of samples used in each method.

The best fitted distributions of the resulted dynamical coefficients has the form of a generalized extreme value distribution, which is given in Eq. 16

$$f(\theta \mid k,\eta,\delta) = \left(\frac{1}{\delta}\right)\exp\left(-\left(1+k\frac{\theta-\eta}{\delta}\right)^{-\frac{1}{k}}\right)\left(1+k\frac{\theta-\eta}{\delta}\right)^{-1-\frac{1}{k}} \tag{16}$$

The distribution parameters for each coefficient, considering both methods, are present in the Tab. 3.

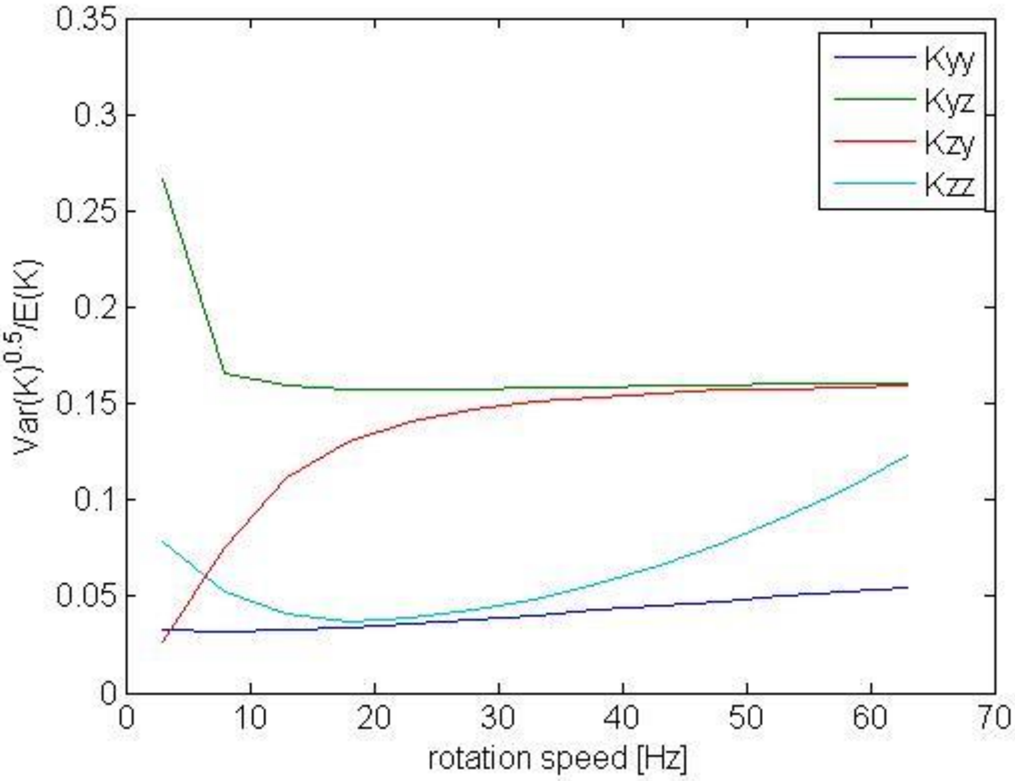

Figure 1 – Rate between standard deviation and mean for stiffness coefficients

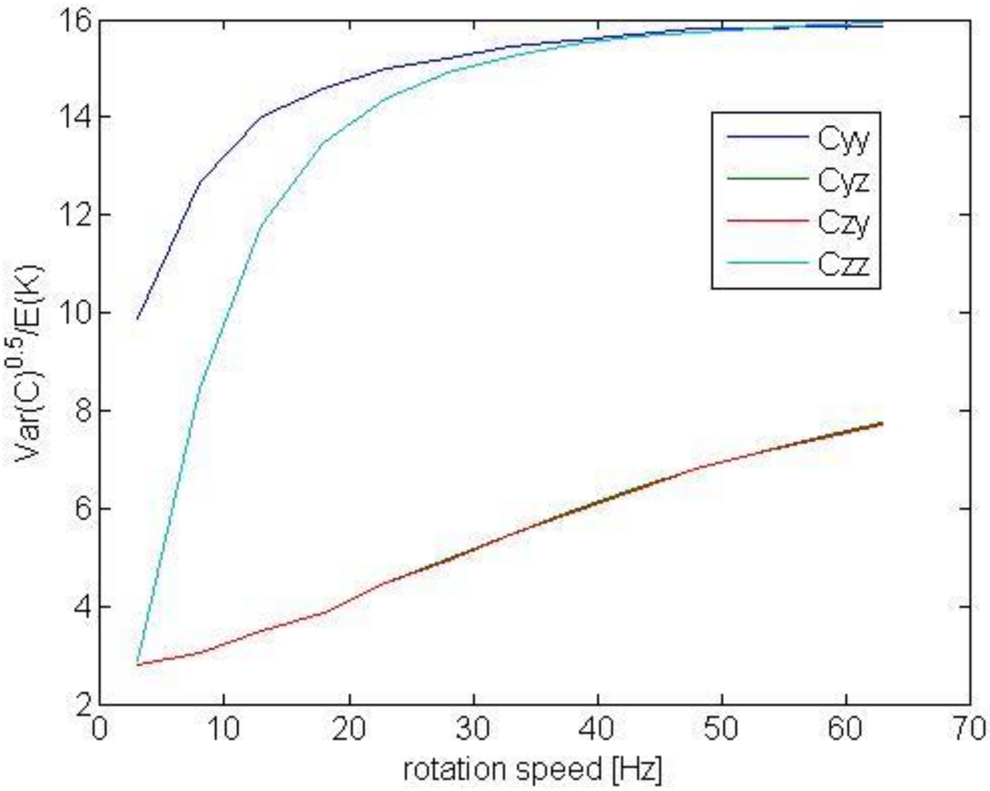

Figure 2 – Rate between standard deviation and mean for damping coefficients

Table 2 – Number of samples and computing time of each stochastic method

Method	Number of samples	Time
Stochastic Collocation	321	146.43 min
Monte Carlo	10000	$4.26 \cdot 10^3$ min

Table 3 – Parameters of the generalized extreme value distribution for each dynamical coefficient and stochastic method

Method	Coefficient	Shape parameter k	Scale parameter δ	Location parameter η
Stochastic Collocation	K (33 Hz)	-0.3697	$1.0488 \cdot 10^4$	$2.0315 \cdot 10^5$
	C (33Hz)	-0.1298	$2.9502 \cdot 10^3$	$1.7790 \cdot 10^4$
	K (53 Hz)	-0.6212	$1.8693 \cdot 10^4$	$1.5355 \cdot 10^5$
	C (53Hz)	-0.1355	$3.0278 \cdot 10^3$	$1.7062 \cdot 10^4$
Monte Carlo	K (33 Hz)	-0.2649	$9.0194 \cdot 10^3$	$2.0338 \cdot 10^5$
	C (33Hz)	-0.1101	$2.5520 \cdot 10^3$	$1.7829 \cdot 10^4$
	K (53 Hz)	-0.3293	$1.5093 \cdot 10^4$	$1.5421 \cdot 10^5$
	C (53Hz)	-0.1146	$2.6202 \cdot 10^3$	$1.7102 \cdot 10^4$

Despite of some difference between the distribution parameters of the Stochastic Collocation method and the Monte Carlo simulations, these results are satisfactory. Therefore, to corroborate with his concordance, the probability density functions for two rotation speeds are compared, as shown in Figs. 3 to 6.

Figure 3 presents the probability density function of the stiffness coefficients for a rotation speed of 33 Hz, considering the polynomial chaos approach. Besides, Fig. 4 shows the probability density function for the same situation, taking into account the Monte Carlo simulations. The difference between the correspondent pdfs are not significant. That means the stochastic collocation is an interesting possibility for uncertainty quantification for stiffness and damping coefficients of journal bearings.

A similar analysis can be pointed out for a different rotation speed, as considered in Figs. 5 and 6. As the previous analysis is also valuable for this situation, it is clear there is no influence of the rotation speed variation on the convergence.

Due to your minor of computing and satisfactory results, the stochastic collocation method is a good method to substitute the Monte Carlo method. With a powerful choosing rule of the nodes, the method should be more precise and faster.

The Figures 7 and 8 present the dynamical coefficients obtained by the stochastic collocation method by the rotating speed of the rotor. In both case, the full line represents the dynamic stiffness value for a cumulative probability function (cdf) of 50%. The range between both dot lines gives a confidence bound of 50%. Besides, the confidence bounds of 95% is provided by the range between the dashed lines.

In general, the stiffness coefficients presents larger confidence bounds for higher rotating speeds, the exception is the direct coefficient k_{zz} which presents the smallest confidence bounds with a discreet increment of it at higher speeds.

It is noticeable that the direct damping coefficients presents higher dispersion and almost constant with the increase of the rotating speed. The damping coefficients presents smaller dispersion than the stiffness coefficients.

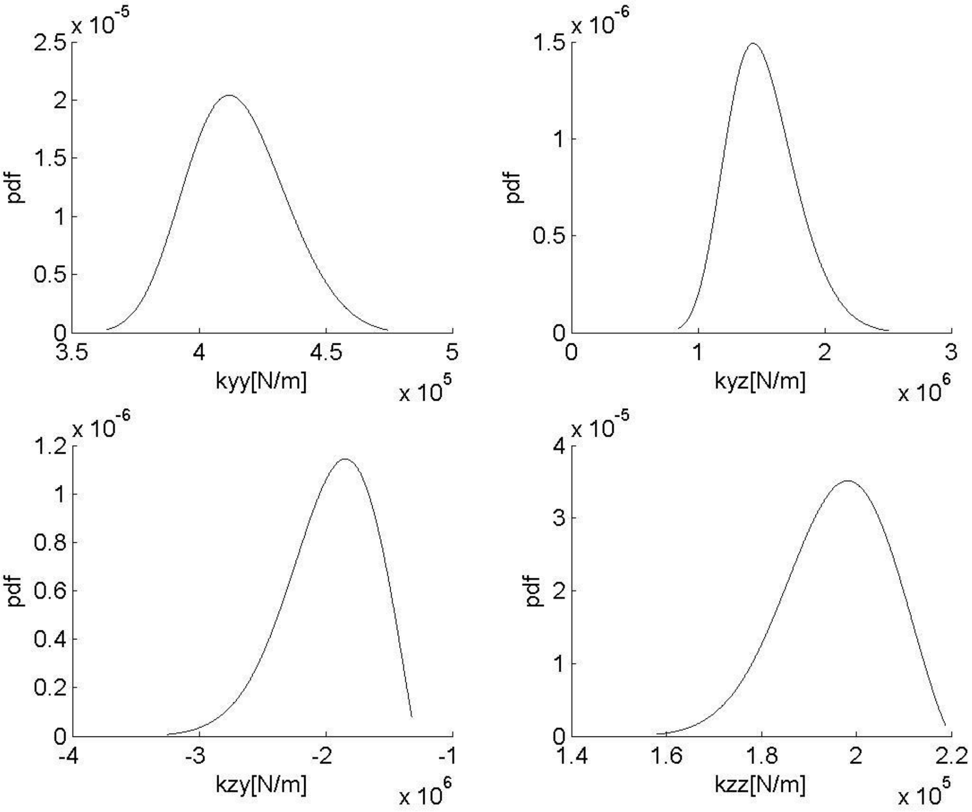

Figure 3 - Stiffness coefficients at the 7th operation speed (33 Hz)obtained with stochastic collocation method

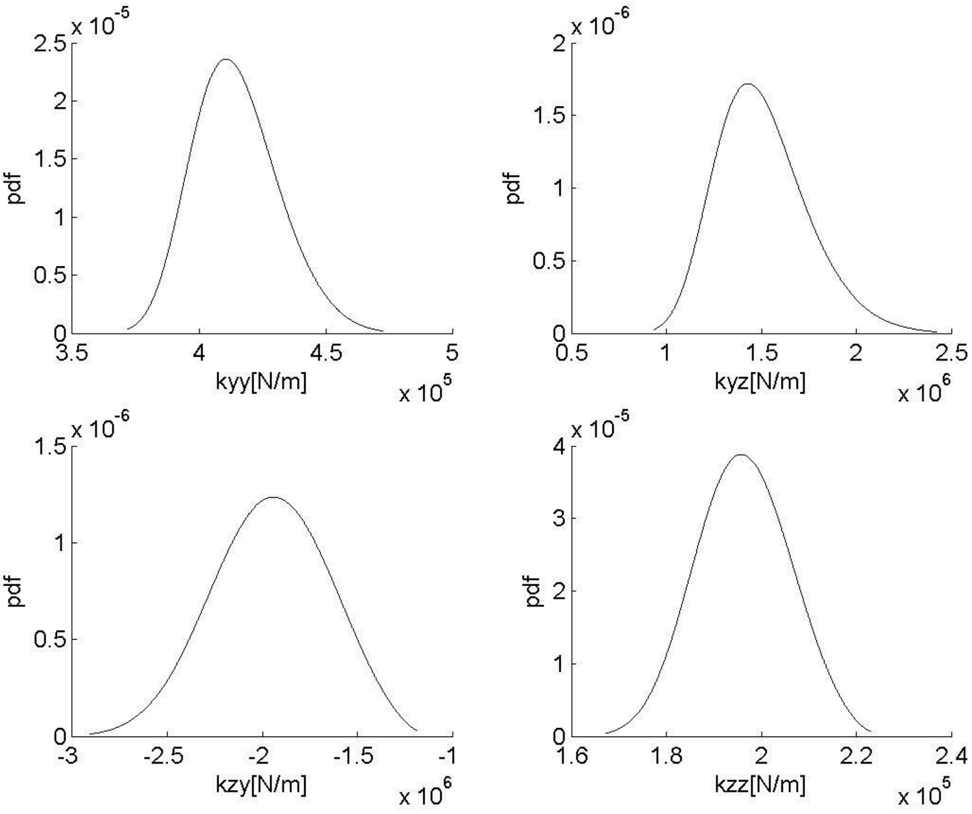

Figure 4 – Stiffness coefficients at the 7th operation speed (33 Hz) obtained with Monte Carlo method

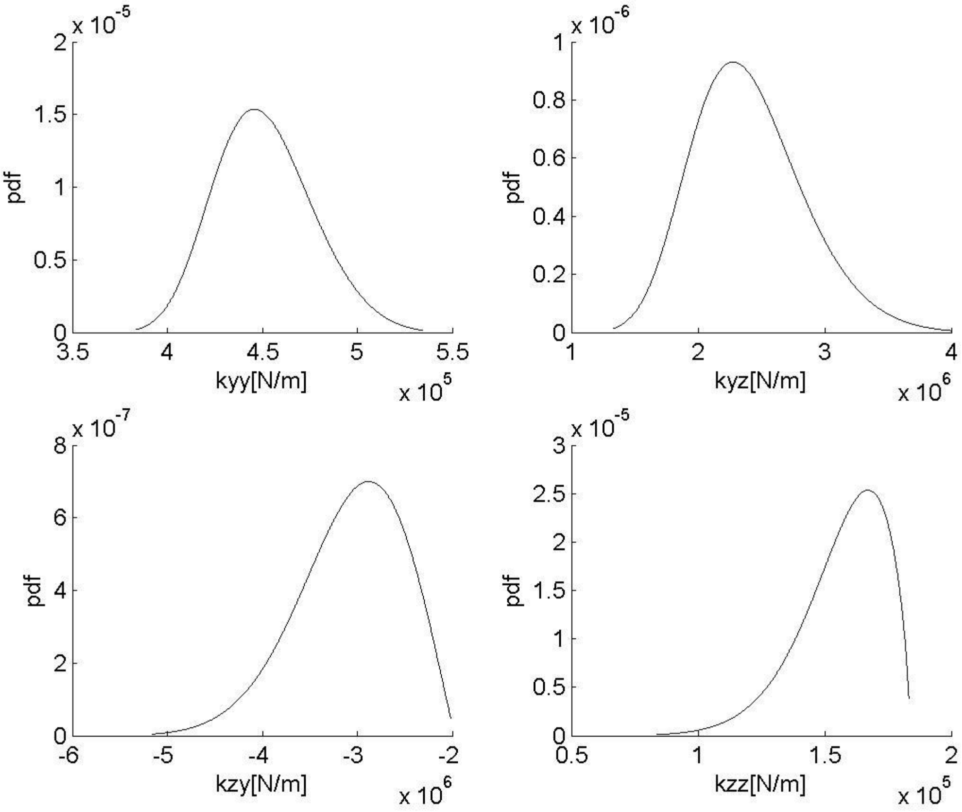

Figure 5 – Stiffness coefficients at the 11th operation speed (53 Hz) obtained with stochastic collocation method

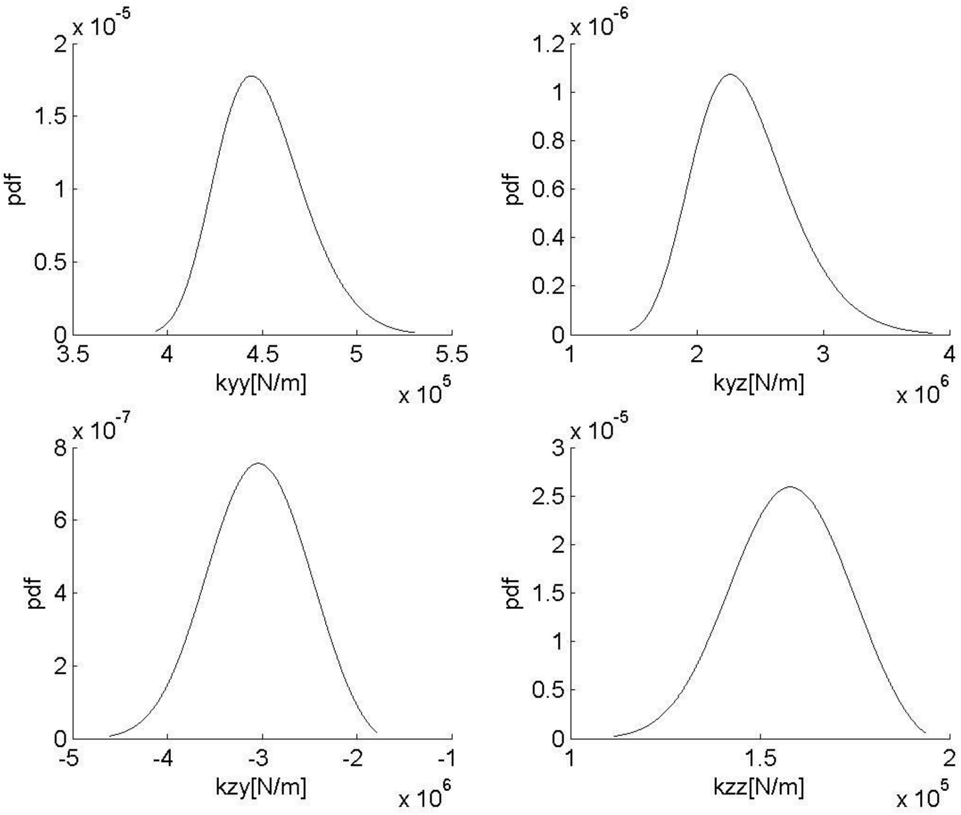

Figure 6 – Stiffness coefficients at the 11th operation speed (53 Hz) obtained with Monte Carlo method

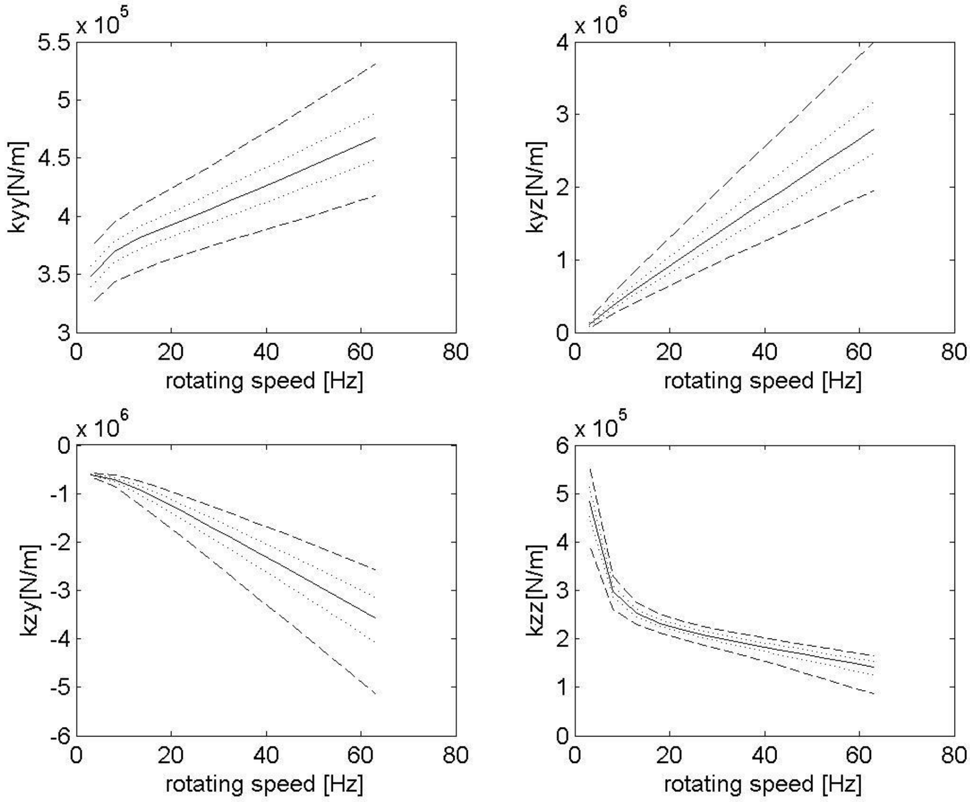

Figure 7 – Stiffness coefficients by the rotating speed

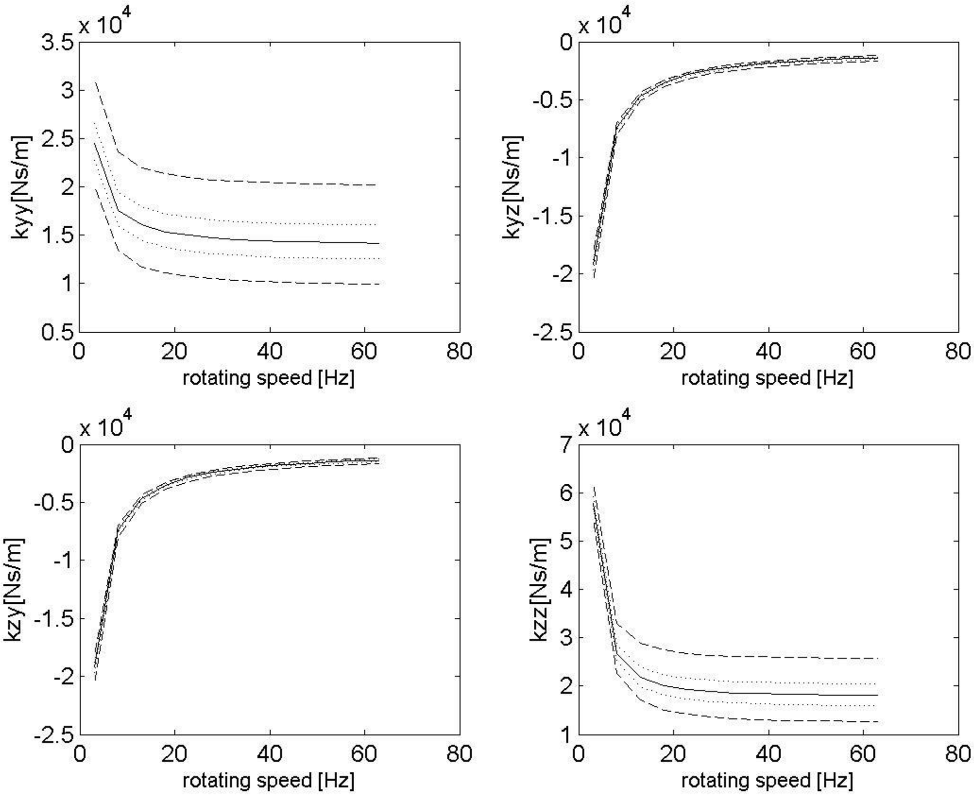

Figure 8 – Damping coefficients by the rotating speed

CONCLUSIONS

The damping and stiffness dynamical coefficients of journal bearing were evaluated, considering the uncertainties of the operating temperature and the radial clearance. To determine the stochastic solution the stochastic collocation method was used and a Monte Carlo method was used, as well, to compare with the first one. The uncertainties were modeled by a generalized polynomial chaos expiation to make possible the use of the collocation method.

The stochastic collocation method presents satisfactory results for the distributions of the dynamical coefficients. Its presents good correspondence the Monte Carlo simulation output, but its computation time is considerable shorter.

In future works, others quadrature rules can be used, like the Gauss-Hermite rule, to refine the stochastic collocation results. This method allows different picking rules of the nodes and solutions, like the pseudo-spectral approach or the usage of Lagrange polynomial, so it can adapt to the problem and statistical information needed. Another consideration to be made is the estimation of the stochastic response of the rotor-bearing model, which also take into account parameters' uncertainties of the rotating part.

ACKNOWLEDGMENTS

The authors thanks FAPESP, which foments the research of the authors.

REFERENCES

Bernadá, G.M.G., 2011, "quantificação de incertezas em problemas de interação Fluido estrutura via método de colocação estocástica", Doctorate's thesis, COPPE-UFRJ, Rio de Janeiro

Dubois, G.B., Ocvirk, F.W., 1953. "Analytical derivation and experimental evaluation of short bearing approximation for full jounrnal bearing", NACA, report 1157

Ghanem,R.G., Spanos, P., 1991. "Stochastic Finite Elements: a Spectral Approach". Springer-Verlag

Hummel, C., 1926, "Kristische Drehzahlen als folge der Nachgiebigkeit des Schmiermittels im Lager", VDI-Forschungsheft 287

Lund, J. W, 1964, Spring and damping coefficients for the tilting-pad journal bearing. ASME Transactions, n.7, p.342-352,

Lund, J., 1987, "Review of the Concept of Dynamic Coefficients for Fluid Film Journal Bearings", ASME Journal of Tribology, vol. 109, pp. 37- 41.

Machado, T.H., Cavalca, K.L, 2009. "Evaluation of dynamic coefficients for fluid journal bearings with different geometries", In Proceedings of the 20st International Congress of Mechanical Engineering -COBEM2009. Gramado, Brazil.

Mukherjee, A., Rao, J.S., 1977. "Stiffness and damping coefficients of an inclined journal bearing", Vol. 12, p. 339-355

Pinkus, O., 1958, " Solution of Reynolds Equation for Finite Journal Bearings", Trans. ASME, v. 80, pp. 858-864

Reynolds, O, 1886, On the theory of lubrication and its application to Mr. Beauchamp Tower's experiments, including an experimental determination of the viscosity of olive oil. Philosophical Transactions of Royal Society of London, Series A, v. 177, Part 1, p.157-234

Smith, R. C., 2014. "Uncertainty quantification: theory, implementation, and applications". SIAM, North Carolina, 1st edition

Strenlicht, B., Maginnis, F.J., 1957. "Application of digital computer of bearing design", Trans. ASME, Vol. 75

Stodola, A., 1925, "Kristische Wellenstörung infolge der Nachgiebigkeit des Oelpolsters im Lager", Schweizerische Bauzeiting, vol. 85, pp. 265-266

Tuckmantel, F.W.S., 2010, "Integração de sistemas rotor-mancais hidrodinâmicos-estrutura de suporte para resolução numérica", Master's thesis, Faculdade de Engenharia Mecânica-UNICAMP,Campinas

Xiu, D., 2009. "Fast numerical methods for stochastic computations: A review", Communications in Computational Physics, Vol. 5, pp. 242-272

Xiu, D., Karniadakis, G., 2002. "The Wiener–Askey polynomial chaos for stochastic differential equations", SIAM Journal on Scientific Computing, Vol. 24, pp. 619-644

Wiener, N., 1938. "The homogeneous chaos", American Journal of Mathematics, Vol. 60, pp. 897-936

RESPONSIBILITY NOTICE