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Robust, Risk and Robust Risk Optimization under Uncertainties

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Abstract: Robust optimization aims at producing designs which are less sensitive to uncertainties. Risk optimization looks for designs with optimal balance between performance and safety. This paper explores the similarities and differences between these formulations. The strong similitude between robust and risk-based optimizations has not been thoroughly explored before. In this paper, it is shown that the alpha factors, which lead to the compromise solutions in robust optimization, are equivalent to the costs of failure in risk-based optimization. Moreover, it is shown that the robust objective function is often non-convex, with results being given by (often arbitrary) design constraints. In some sense, the usual robust objective function lacks objectiveness, with results becoming largely dependent on arbitrary normalizing constants. On the other hand, when there is a critical limit to performance, which characterizes system failure, and when costs of failure can be defined, the risk-based optimization yields consistent results; no normalizing constants needed. The paper also addresses a combined formulation, the robust risk optimization. In this formulation, the objective, intrinsic uncertainties, which can be modelled in terms of probabilities, are considered in the risk optimization problem. The subjective, epistemic uncertainties are described by fuzzy variables, leading to a fuzzy risk optimization problem. The robust risk optimization is obtained by making the risk optimization problem less sensitive to the fuzzy epistemic uncertainties. The design of a concrete gravity dam is addressed for illustration purposes.

Keywords: structural optimization; optimum design; robust optimization; risk optimization; robust risk optimization; reliability analysis; epistemic uncertainties.

INTRODUCTION

Results of structural optimization should be robust with respect to the uncertainties inherently present in structural loads, material resistances and engineering models. This notion has led to the development of different approaches to structural optimization under uncertainties: stochastic or robust optimization (Kall & Wallace 1994, Birge & Louveaux 1997, Beyer & Sendhoff 2007, Schuëller & Jensen 2009), fuzzy optimization (Möller & Beer 2004) and reliability-based structural optimization (Aoues & Chateauneuf 2010, Valdebenito & Schueller 2010, Lopez & Beck 2012). The robust formulation looks for designs which are less sensitive to inherent parameter variability, without removing the sources of uncertainty. Robust Design Optimization (RDO) is by default a multi-objective optimization problem (Beyer & Sendhoff 2007), where the systems mean performance should be maximized, whereas performance variance should be minimized. The balance between these objectives remains a subjective choice of the analyst. In reliability-based design optimization, a probabilistic measure of system performance, also subjectively chosen by the analyst, is employed as design constraint. Consequences of failure are not explicitly taken into account in these formulations. However, one of the main consequences of uncertainties is the possibility of reaching a state of undesirable system performance. This possibility can be measured in terms of probability, and then multiplied by the cost of failure (monetary measure of the consequences). The resulting term, also known as expected cost of failure, can be incorporated in the objective function, leading to an unconstrained optimization problem: minimization of total expected costs. This formulation, which has been called risk optimization (Beck & Verzenhassi 2008, Beck & Gomes 2012, Beck et al. 2012, Gomes et al. 2013, Gomes & Beck 2013, 2014a, 2014b), allows one to find the optimum point of compromise between (better) performance and the consequences of losing performance due to failure.

In structural engineering design, economy and safety are competing goals. Generally, increasing safety implies higher costs, and reducing costs may require a compromise in safety. Hence, designing structural systems involves a tradeoff between safety and economy. In common engineering practice, this tradeoff is addressed subjectively. In codified design, the issue is decided by code committees, which define safety coefficients to be used in design, and basic safety measures to be adopted in construction and operation. In general, the tradeoff between economy and safety in structural design can be addressed by structural optimization.

Comprehensive literature reviews (Beyer Sendhoff 2007, Schuëller & Jensen 2009, Aoues & Chateauneuf 2010, Valdebenito & Schueller 2010) critically describe the different formulations for structural optimization under uncertainties. Nevertheless, such overviews do not have the required depth for the similarities and differences between the different formulations to become explicit. In a previous article by the authors (Beck & Gomes 2012), the differences between deterministic, reliability-based and risk-based optimization were thoroughly explored. Deterministic Design Optimization (DDO) allows one to find the configuration of a structure that is optimum in terms of mechanics, but grossly neglects parameter uncertainty and their effects on structural safety. By definition, the result of DDO is a structure with more failure modes designed against the limit: hence safety is largely compromised. With Reliability-Based Design Optimization (RBDO), one can ensure that a minimum (and measurable) level of safety is achieved by the optimum structure. However, since failure probability is a constraint and not part of the objective function, RBDO does not address the safety-performance tradeoff. Risk-based Optimization (RO) increases the scope of the problem, by including the (expected) cost of failure in the performance balance (Beck & Verzenhassi 2008, Beck & Gomes 2012, Beck et al. 2012, Gomes et al. 2013, Gomes & Beck 2013, 2014a, 2014b). Hence, RO allows one to find the optimum tradeoff point between the competing goals of performance and safety. RO aims at finding the optimum performance of a given structural system, in order to minimize the total expected cost or maximize the expected profit. It is a tool for decision making in the presence of uncertainties.

The present paper thoroughly addresses the differences and similarities between the robust and risk-based formulations of optimization under uncertainties. The article presents new insights into these formulations, showing that: a) the traditional Pareto front, which is obtained by a compromise solution between maximizing mean performance and minimizing performance variance, is associated to the cost of failure in the risk-based optimization; b) the robust formulation often leads to non-convex problems, where the optimal solution is given by (often arbitrary, user chosen) design constraints; c) the objective function in robust optimization may lack objectiveness, with results being dependent on arbitrary normalizing constants; d) when a critical response level leads the system to failure, and when failure costs can be quantified, consistent results are obtained in a risk-based optimization.

STRUCTURAL RELIABILITY PROBLEM

Let **X** and **d** be vectors of structural system parameters. Vector **X** represents all random system parameters, and includes geometric characteristics, resistance properties of materials or structural members, and loads. Some of these parameters are random in nature (physical uncertainty); others cannot be defined deterministically due to limitations in measurement and in sampling. Typically, resistance parameters are represented as random variables and loads are modelled as random processes of time. Vector **d** contains all (deterministic) system design parameters, like nominal member dimensions, partial safety factors, design life, parameters of the inspection and maintenance programs, etc.

The existence of uncertainty implies risk, that is, the possibility of undesirable structural responses. The boundary between desirable and undesirable structural responses is given by limit state functions g(**d**,**x**)=0, such that:

$$\begin{aligned} \Omega_f &= \{\mathbf{d},\mathbf{x} \mid g(\mathbf{d},\mathbf{x}) \leq 0\} \text{ is the failure domain} \\ \Omega_s &= \{\mathbf{d},\mathbf{x} \mid g(\mathbf{d},\mathbf{x}) > 0\} \text{ is the safety domain} \end{aligned} \tag{1}$$

Each limit state describes one possible failure mode of the structure, either in terms of serviceability or ultimate capacity. The probability of undesirable structural response, or probability of failure, is given by:

$$P_f(\mathbf{d}) = P[\mathbf{X} \in \Omega_f] = \int_{\Omega_f} f_{\mathbf{X}}(\mathbf{x},\mathbf{d})d\mathbf{x} \tag{2}$$

where $P[.]$ stands for *probability* and $f_{\mathbf{X}}(\mathbf{x},\mathbf{d})$ is the joint density function of the random variables in **X**. The probabilities of failure for individual limit states and for system failure are evaluated using traditional structural reliability methods such as FORM, SORM or Monte Carlo simulation (Ang & Tang 2007, Melchers 1999).

EXPECTED COST OF FAILURE

The life-cycle cost of a structural system subject to failure can be decomposed in an initial or construction cost, cost of operation, cost of inspections and maintenance, cost of disposal and expected costs of failure. For each possible failure mode of the system, the expected cost of failure (C_{EF}), or failure risk, is given by the product of a cost of failure (*cf*) by a failure probability:

$$C_{EF}(\mathbf{d}) = cf(\mathbf{d})P_f(\mathbf{d}) \tag{3}$$

Failure costs include the costs of repairing or replacing damaged structural members, removing a collapsed structure, rebuilding it, cost of unavailability, cost of compensation for injury or death of employees or general users, penalties for environmental damage, etc. In Eq. (3), all failure consequences have to be expressed in terms of monetary units. This can be a problem when dealing with human injury, human death or environmental damage. Evaluation of such failure consequences in terms of the amount of compensation payoffs may allow the problem to be formulated, without directly addressing matters about the value of human life. In other problems, costs of failure may be intangible and difficult to translate to monetary units; for such problems the risk optimization formulation may not apply. Failure costs could also depend on some random variables x. In such situations, one would either work with expected failure costs, or solve Eq. (3) as an integral over the random variables domain.

For each system failure mode, there is a corresponding failure cost term. The total or life-cycle expected cost (C_{ET}) of a structural system becomes:

$$\begin{aligned} C_{ET}(\mathbf{d}) = \; & C_{\text{initial}}(\mathbf{d}) \\ & + C_{\text{operation}}(\mathbf{d}) \\ & + C_{\text{inspection and maintenance}}(\mathbf{d}) \\ & + C_{\text{disposal}}(\mathbf{d}) \\ & + \sum_{\text{failure modes}} cf(\mathbf{d}) P_f(\mathbf{d}) \end{aligned} \tag{4}$$

The initial or construction cost increases with the safety coefficients used in design and with the practiced level of quality assurance. More safety in operation involves more safety equipment, more redundancy and more conservatism in operation. Inspection cost depends on intervals, choice of inspection method and quality of equipment. Maintenance costs depend on maintenance plan, frequency of preventive maintenance, etc. When the overall level of safety is increased, most cost terms increase, but the expected costs of failure are reduced.

Any change in **d** that affects cost terms is likely to affect the expected cost of failure. Changes in **d** which reduce costs may result in increased failure probabilities; hence increase expected costs of failure. Reduction in expected failure costs can be achieved by targeted changes in **d**, which generally increase costs. This compromise between safety and costs is typical of structural and mechanical systems. Generically, one can also see it as a compromise between safety and performance.

PERFORMANCE MEASURES

There are infinite ways in which performance can be measured, and this can be largely problem and user-dependent. For mechanical systems/devices, performance can be measured by means of physical quantities such as speed, acceleration, weight, horse-power, capacity, etc. As better performance usually comes at a cost, efficient performance measures are given as a relation between input and output, as km/liter, hp/kg, capacity/kg, etc.

For structural engineering systems, performance is a relatively new attribute. For many years, performance of engineering structures was basically a synonym to lightness. With the advent of structural optimization, function/kg or function/volume became measures of performance, with the best performing structure (the optimum) being that which could perform the intended function with the least spending of materials. In these approaches, weight and volume were frequently confused with costs, with expected costs of failure being largely neglected (Beck & Gomes 2012). More recently, the concept of Performance-Based Engineering (PBE) has emerged, associating performance to the spectrum of amplitudes of structural responses (Augusti & Ciampoli 2008, Ghobarah 2001, Ciampoli et al. 2011, Tubaldi et al. 2012, Barbato et al. 2013, Beck et al. 2015). According to this concept, the amplitudes of structural responses are associated to a transition from fully functional to completely failed (collapse). The idea of a crisp passing from functional to failure, such as stated in Eq. (1), is replaced by a smooth and gradual transition into failure. Usually, increasing costs (of failure) are associated to the larger response amplitudes.

As observed in the above, (reduced) cost is a frequently used measure of performance in structural engineering. Hence, often maximizing performance is equivalent to minimizing costs:

$$\max[perf(\mathbf{d},\mathbf{X})] = \max[-\text{cost}(\mathbf{d},\mathbf{X})] \Leftrightarrow \min[\text{cost}(\mathbf{d},\mathbf{X})] \tag{5}$$

where $perf(\mathbf{d},\mathbf{X})$ stands for performance. Note that both performance and costs may be functions of the random vector **X**, hence performance and costs can be random as well.

RELIABILITY-BASED DESIGN OPTIMIZATION (RBDO)

One popular formulation to take account of uncertainties in design optimization is known as Reliability-Based Design Optimization or RBDO. A typical formulation of RBDO reads:

$$\mathbf{d}^* = \arg\max[perf(\mathbf{d}) : \mathbf{d} \in \text{S},\; P_f(\mathbf{d}) < P_{f_{\text{admissible}}}] \tag{6}$$

or

$$\mathbf{d}^* = \arg\min[\text{volume}(\mathbf{d}) : \mathbf{d} \in \text{S},\; P_f(\mathbf{d}) < P_{f_{\text{admissible}}}] \tag{7}$$

where S=[$\mathbf{d}_l$, $\mathbf{d}_u$] is the vector of design constraints, with $\mathbf{d}_l$ and $\mathbf{d}_u$ the lower and upper bounds on the design variables. In RBDO, the performance is usually assumed independent of the random vector **X**. The objective function in RBDO generally involves volume (or cost) of materials or manufacturing costs. Expected costs of failure are not considered. RBDO allows finding a structure which is optimal in mechanical sense, and which does not compromise safety. Results, however, depend on the admissible failure probability used as constraint in Eq. (7). The measure of performance is related to the cost of the structure. The balance between safety and performance cannot be addressed, because failure probability is not a design variable but a constraint.

RISK-BASED OPTIMIZATION (RO)

When system response can be associated to a critical level, which causes the system to fail, as in Eq. (1), and when costs of failure can be quantified, the risk-based formulation can be employed. Increasing the performance of a system very often implies pushing it against such a critical level; hence, there is a compromise between performance and safety. A structure should not be made arbitrarily light, as to collapse under sustained loads. An aircraft should not be made so fast as to approach collapse under aerodynamic flow pressures. Risk optimization (reliability-based) searches for the optimal point of balance between the conflicting goals of performance and safety:

$$\mathbf{d}^* = \arg\min[\mathrm{C}_{\mathrm{ET}}(\mathbf{d}) \colon \mathbf{d} \in \mathrm{S}] \tag{8}$$

where $\mathrm{C}_{\mathrm{ET}}(\mathbf{d})$ is the total expected cost given by Eq. (4). In comparison to RBDO, risk optimization is by default an unconstrained optimization problem, since failure probability “constraints” are included in the objective function. Note that solution of Eq. (8) yields $\mathbf{d}^*$, which is associated to the optimal level of safety $P_f^*(\mathbf{d}^*)$ through Eq. (2). Some of the cost terms in Eq. (4) may be assumed independent of random vector **X**, but expected costs of failure already account for uncertainty through Eq. (2).

Risk optimization can be achieved by controlling failure probabilities and/or the consequences of failure. Risk mitigation through preparation, education, training, and so on, is out of scope of this investigation. Although failure costs are usually considered constant, it is important to note that the present formulation does allow for a trade-off between "competing" failure modes with distinct failure costs. Hence, serviceability and ultimate limit states, with their intrinsically different costs of failure, are readily accounted for in the RO formulation (Papadopoulos & Lagaros 2009).

If social, non-monetary or intangible consequences of failure are involved (e.g. human death or environmental damage), failure probability constraints may also be included in the RO formulation:

$$\mathbf{d}^* = \arg\min[\mathrm{C}_{\mathrm{ET}}(\mathbf{d}) \colon \mathbf{d} \in \mathrm{S},\ P_f(\mathbf{d}) < P_{f_{\mathrm{admissible}}}] \tag{9}$$

Note that in this case the failure probability appears in the objective function and in the constraints. Also in this case, only quantifiable costs of failure can be included in the objective function.

Consideration of costs of failure may also introduce associated epistemic uncertainties into the problem. When significant epistemic uncertainty is present, in particular in costs of failure, the robust risk optimization formulation of (Beck et al. 2012) can be employed. Epistemic uncertainty and robust risk optimization will be addressed in the presentation of this paper, but are not included here due to page count constraints.

ROBUST OPTIMIZATION

Another approach to address structural optimization in presence of uncertainties is robust optimization (Zang et al. 2005, Papadopoulos & Lagaros 2009, Marano et al. 2008, 2010, Ritto et al. 2011). Typical objectives for robust optimization are maximization of mean performance and minimization of performance variance. Generally, these are multi-objective problems, whose solution involves weighted sums, compromise programming or preferential aggregation. A typical formulation of robust optimization reads:

$$\mathbf{d}^* = \arg\max[\mathrm{stat}[\mathit{perf}(\mathbf{d},\mathbf{X})] \colon \mathbf{d} \in \mathrm{S}] \tag{10}$$

or

$$\mathbf{d}^* = \arg\min[\mathrm{stat}[\mathrm{cost}(\mathbf{d},\mathbf{X})] \colon \mathbf{d} \in \mathrm{S}] \tag{11}$$

where stat[.] represents some statistic of the performance function *perf*($\mathbf{d}$,$\mathbf{X}$). Possible statistics are:

$$\begin{aligned} \text{stat}[.] &= P_k[.] : \text{the } k^{th} \text{ percentile;} \\ &= E[.] : \text{the expected value;} \\ &= Var[.] : \text{the variance;} \\ &= \alpha E[.] + (1-\alpha) P_k[.] : \text{a multi-objective problem;} \\ &= \alpha E[.] + (1-\alpha) Var[.] : \text{the conventional multi-objective problem.} \end{aligned} \tag{12}$$

The lasts two lines of Eq. (12) are multi-objective problems, where $0 \leq \alpha \leq 1$ is a constant. The first and fourth lines of Eq. (12) involve the k^{th} percentile of the performance function; when this percentile is associated to the failure probability, strong similarity is observed between the robust and risk-based optimizations. The risk optimization formulation, however, avoids the arbitrary choice of percentile value k. In this article, the multi-objective approach will be targeted:

$$\mathbf{d}^* = \arg\min[\alpha E[.] + (1-\alpha) f(Var[.]) : \mathbf{d} \in \text{S}] \tag{13}$$

where $f(\ )$ is just some function of the performance variance. Usually, function $f(\ )$ is simply the variance or its square root, the standard deviation. $f(\ )$ is introduced here to facilitate the following generalizations.

THE "PARETO FRONT" FOR RISK-BASED OPTIMIZATION

Consider a simplified version of the risk-based optimization in Eq. (8). The most relevant cost terms in this formulation are the initial cost and the expected cost of failure. Assume that there is only one significant mode of failure, and that the cost of failure cf is independent of the design vector. The simplified version of Eq. (8) becomes:

$$\mathbf{d}^* = \arg\min[\text{C}_{\text{initial}}(\mathbf{d}) + cf \ P_f(\mathbf{d})] \tag{14}$$

The problem is not changed when the objective function is multiplied by the constant $1/(cf+1)$, hence:

$$\mathbf{d}^* = \arg\min[\frac{1}{cf+1} \text{C}_{\text{initial}}(\mathbf{d}) + \frac{cf}{cf+1} \ P_f(\mathbf{d})] \tag{15}$$

Now make a change of variables, and call $1/(cf+1)=\alpha$. It turns out that $cf/(cf+1)=(1-\alpha)$, hence one obtains:

$$\mathbf{d}^* = \arg\min[\alpha \text{C}_{\text{initial}}(\mathbf{d}) + (1-\alpha) \ P_f(\mathbf{d})] \tag{16}$$

Hence, under very general conditions (cf independent of $\mathbf{d}$), the risk-based formulation leads to something very similar to the multi-objective robust optimization of Eq. (13), or the fourth line of Eq. (12). In Eq. (16), the expected value of the cost (or performance) is the initial cost. This term could include the other costs in Eq. (4) with similar results. The most drastic difference is that, in the risk-based formulation, function $f(Var[.])$ is equal to the failure probability, which is a function of the performance variance. The robust formulation in Eq. (16) is a compromise solution between the expected value and percentile objectives, hence in Eq. (11) one could have:

$$\text{stat}[.] = \alpha E[.] + (1-\alpha) P_k[.] \tag{17}$$

The main conclusion of these observations is that the arbitrary compromise solution factor α of the robust formulation, which is subjectively chosen by the analyst, can be associated to the cost of failure in the risk-based formulation. In our opinion, the cost of failure can be a more objective measure; hence, when both are applicable, the risk-based formulation in Eqs. (8) or (14) should be favored over the robust formulation. These results also show that the risk-based problem can be solved for different costs of failure, yielding the equivalent of a "Pareto front". This is done in the examples section. Of course, such a solution is only for illustration purposes, as we understand that the cost of failure can be objectively chosen, in contrast to the arbitrary factor α.

Similar results are obtained if more than one failure mode exists. Consider the problem:

$$\mathbf{d}^* = \arg\min[\text{C}_{\text{initial}}(\mathbf{d}) + cf_1 \ P_{f1}(\mathbf{d}) + cf_2 \ P_{f2}(\mathbf{d})] \tag{18}$$

where sub-indexes $_1$ and $_2$ refer to two failure modes. Now, multiply the objective function by the constant $1/(cf_1+cf_2+1)$:

$$\mathbf{d}^* = \arg\min[\frac{1}{cf_1 + cf_2 + 1}\mathrm{C}_{\text{initial}}(\mathbf{d}) + \frac{cf_1}{cf_1 + cf_2 + 1}\, P_{f1}(\mathbf{d}) + \frac{cf_2}{cf_1 + cf_2 + 1}\, P_{f2}(\mathbf{d})] \tag{19}$$

Introduce a change of variables $1/(cf_1 + cf_2 + 1) = w_0$ such that:

$$\mathbf{d}^* = \arg\min[w_0 \mathrm{C}_{\text{initial}}(\mathbf{d}) + w_1\, P_{f1}(\mathbf{d}) + w_2\, P_{f2}(\mathbf{d})] \tag{20}$$

and it turns out that $w_0+w_1+w_2=1$, which is the usual way of combining multiple objectives in robust design optimization.

SLIDER-CRANK MECHANISM EXAMPLE

This problem consists in the design of the two-bar slider-crank mechanism, illustrated in Figure 1. The crank rotates at a constant angular velocity ω=100 rad/s. The design variables are the lengths of crank (r) and connecting rod (l), hence $\mathbf{d}=\{r,l\}$. The performance objective is to maximize the slider velocity when $\theta=30^o$. In order to ensure a 360^o rotation of the crank, the mechanism must satisfy Grashof's criterion: $l \geq 2.5r$. The random variables are the lengths of crank and rod, hence $\mathbf{X}=\{R,L\}$. Such uncertainty reflects the manufacturing tolerances for the slider-crank mechanism. The design variables are the means of the random variables: $E[\mathbf{X}]=\mathbf{d}=\{r,l\}$. Table 1 shows the parameters of random vector $\mathbf{X}$.

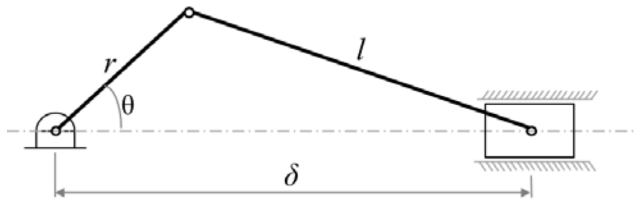

Figure 1 – Slider-crank mechanism problem.

Table 1 - Random variables of slider-crank mechanism problem.

Parameter	Distribution	Mean (μ)	St. Dev. (σ)	units
R	Gaussian	r	0.1	mm
L	Gaussian	l	0.1	mm

The speed of the slider is given by $speed=\omega r[\sin(\theta)+0.5r\sin(2\theta)/l]$. The limit state is related to the maximum elongation δ, which is obtained for $\theta=0^o$ (Fig. 1), and is given by $\delta=r+l$. The limit state function is written as:

$$g(\mathbf{d},\mathbf{X}) = \delta_{CRIT} - (R + L) = 0 \tag{21}$$

where δ_{CRIT}=20mm is the critical elongation. It is assumed that, for $\delta \geq \delta_{\text{CRIT}}$, the slider impacts a wall or sensitive equipment, leading to system failure. The failure probability can be obtained in closed form and is given by:

$$P_f(\mathbf{d}) = \Phi(-\beta),\ \text{with}\ \beta = \frac{\delta_{\text{CRIT}} - r - l}{\sqrt{r^2\sigma_R^2 + l^2\sigma_L^2}} \tag{22}$$

The risk-based optimization can be stated as:

$$\{r^*, l^*\} = \arg\min\left[speed(\mathbf{d})\left(-1 + cf\,\Phi(-\beta)\right) : l \geq 2.5r\right] \tag{23}$$

In Eq. (23), the cost of failure is assumed proportional to speed of the slider, therefore the term $speed(\mathbf{d})$ multiplies both terms of the objective function. The risk optimization problem is solved for cf varying from 2 to 20, and results are presented in Figure 2 (left). The Figure shows the "Pareto" front, or the compromise solutions balancing the two parts of the objective function in Eq. (23): maximization of the speed and minimization of expected costs of failure. This is not truly a Pareto front because one understands that the costs of failure are known; for a known cost of failure, solution is a single point in Figure 2 (left). The optimal solution changes little w.r.t. cf, with $\mathbf{d}$={5.583,13.8875} for cf=2 and $\mathbf{d}$={5.555,13.9575} for cf=20. However, such small change is significant in terms of reliability, with β_{RO}=3.055 for cf=2 and β_{RO}=3.735 for cf=20. For all optimal solutions the constraint $l \geq 2.5r$ is active.

Let us now deal with the robust optimization formulation of this problem. The standard deviation for the maximum elongation is given by:

$$\sigma_\delta(\mathbf{d}) = \sqrt{r^2\sigma_R^2 + l^2\sigma_L^2} \,. \tag{24}$$

Thus, the robust optimization problem can be stated as:

$$\{r^*, l^*\} = \arg\min\left[\frac{speed(\mathbf{d})}{speed(\mathbf{d}_0)}\left(-1 + cf\,\frac{\sigma_\delta(\mathbf{d})}{\sigma_\delta(\mathbf{d}_0)}\right) : l \geq 2.5r, \beta(\mathbf{d}) \geq \beta(\mathbf{d}_{\mathrm{RO}}^*)\right] \tag{25}$$

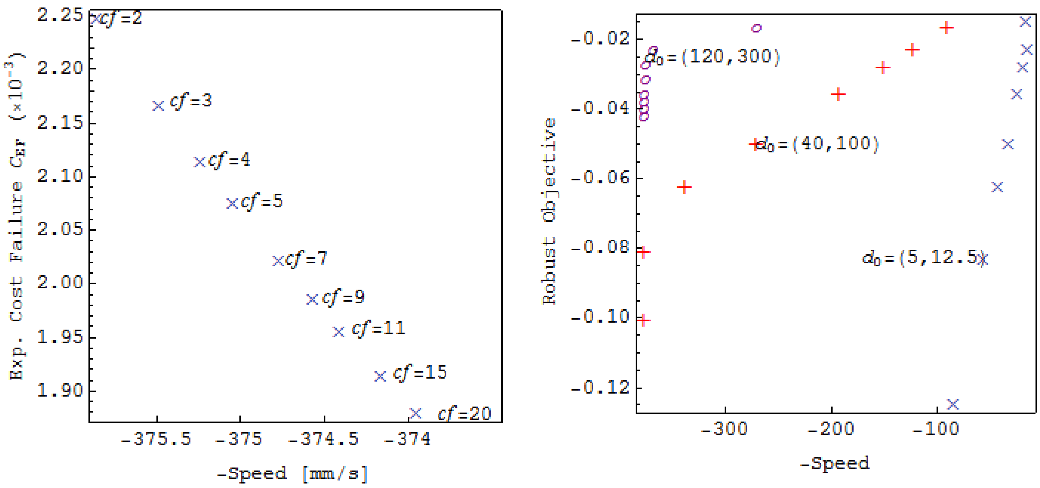

Figure 2 - Pareto fronts for risk-based (left) and robust (right) optimization, slider-crank mechanism problem; each point in the risk-based "Pareto" front corresponds to one cost of failure (*cf*); each sequence in the robust optimization Pareto front corresponds to a normalizing vector; the individual points correspond to the equivalent weights α.

where $\mathbf{d}_0$ is a normalizing vector. The reliability constraint is the reliability index found in the RO solution. Solutions are computed for the same values of *cf* and for different values of the normalizing vector. Results can be observed in Figure 2 (right). For large values of the normalizing vector ($\mathbf{d}_0$={40,100} and $\mathbf{d}_0$={120,300}) and for small values of *cf*, the same results as the RO are obtained, but only because the optimal design is controlled by the reliability constraint. For small values of the normalizing vector ($\mathbf{d}_0$={5,12.5}, for all *cf*), and for large *cf* with other normalizing vectors, the standard deviation term dominates the objective function and the optimal speed results very small.

As a conclusion, one notes that the robust solution is, again, dependent on the values of normalizing vectors. The risk-based optimization yields consistent results, with an objective solution for each value of the cost of failure parameter. The risk-optimization is independent of arbitrary normalizing vectors.

CONCLUSIONS

In this paper, the robust and risk-based approaches to optimization under uncertainties have been thoroughly compared. It was shown that the differences between these approaches are subtle, but they lead to radically different results. In a robust compromise solution between expected value and variance of performance, the α factor, which is subjectively chosen by the analyst, can be associated to the cost of failure in the risk-based formulation. Since the cost of failure may be a more objective measure, the risk-based formulation should be preferred when both apply. Often, the robust objective function is non-convex, with optimal designs being determined by design variable constraints. The objective function in risk optimization is more complex but convex, at least by part, leading to unconstrained, hence more relevant optimal solutions. In some sense, the objective function for robust optimization lacks objectiveness, with results being largely dependent on arbitrary normalizing constants. On the other hand, when there is a limit to performance, or a compromise between (better) performance and (reduced) safety, and when costs of failure can be quantified, the risk optimization formulation yields consistent results. The only parameters to be established are the limiting response, which characterizes the boundary between safe and failure domains, and the associated cost of failure. Even when the cost of failure cannot be precisely established, its choice is potentially less arbitrary than a normalizing constant in robust optimization. Hence, it is our opinion that the risk-based formulation should be preferred when both are applicable. In this oral presentation of this article, a robust risk optimization formulation will be discussed, combining the two formulations discussed herein.

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