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Analysis of Uncertainties in Nonlinear Vibration of Viscoelastic Sandwich Beams

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Abstract: This paper proposes a finite element modeling procedure of sandwich beams with a viscoelastic core, taking into account the dependence in frequency of its properties, with the eigenvalues calculated using the inverse iteration algorithm. The linear and nonlinear vibration analysis is done combining the harmonic balance technique with the complex mode Galerkin's procedure, generating an amplitude-frequency relation. Finally, uncertainties are introduced in parameters of the structure and of the environment, modeled via Maximum Entropy Method, and numerical tests are done in order to obtain envelopes of responses and see the level of influence of each variable in the structure's response.

Keywords: Nonlinear vibration, Viscoelastic structure, Uncertainties, Vibration Control

INTRODUCTION

Studies about vibration control have attracted the researcher's attention due to the search for safety, economy, acoustic insulation and better working conditions in sectors like the aeronautic, aerospace, automotive and other ones. One of the strategies applied to reduce vibration amplitudes is called passive control, usually by introducing in the structure, materials with damping properties, without any external intervention. Among these materials, one of the most employed is the viscoelastic material. The damping effect is brought by the shear of the viscoelastic layer due to the difference between the longitudinal displacements of the elastic layers, and also by the low stiffness of the viscoelastic core.

In applications of industrial interest, the structures are subject to large displacements, becoming necessary the nonlinear vibration analysis. Moreover, a small number of works have been dedicated to the study of nonlinear structures incorporating viscoelastic materials in order to reduce the nonlinear vibrations, which brings the interest to the present contribution.

This paper is dedicated to the development of a numerical and computational methodology dealing with linear and nonlinear vibration of sandwich beams, coupling the harmonic balance technique with the Galerkin's method, assuming a frequency- and temperature-dependent constitutive law for the viscoelastic material. For the exact solution of the nonlinear eigenvalue problems, it has been used the inverse iteration method. In order to obtain a major knowledge about the structure's response considering uncertainties in physical and environmental parameters, the Maximum Entropy Method is used to obtain the Probability Density Function of each variable. Numerical tests are performed to obtain the envelopes of responses to evaluate the level of influence of each variable in the system's response.

Kinematic Formulation and Constitutive Behaviour

Consider a symmetric rectangular sandwich structure composed by three layers shown in Fig. 1, where L represents the length, l is the width and h is the thickness. By convention, $i = 1,3$ along with the index f refers to the external elastic layers, and $i = 2$ along with the index c refers to the central viscoelastic layer. In this study, the sandwich structure is assimilated to a beam, with the zigzag model kinematic description proposed by Rao (1978), where the elastic layers are modeled by the Euler-Bernoulli theory, while the viscoelastic layer uses the Timoshenko theory. The analysis is done using the performance conditions established by Hu et al. (2008).

The sandwich structure is considered in the case of low disturbances, and the following hypothesis common to many authors are considered:

- All points on a normal to the plate undergo the same transverse displacement.
- There is no slipping between the three layers of the plate.
- The constitutive materials of the structure are homogeneous and isotropic. The Young/shear modulus of the viscoelastic core is complex frequency dependent, but the Poisson's ratio is assumed constant.
- The top and bottom elastic layers have the same Young modulus and the same mass density.
- The shear effect is taken into consideration only in the viscoelastic layer.

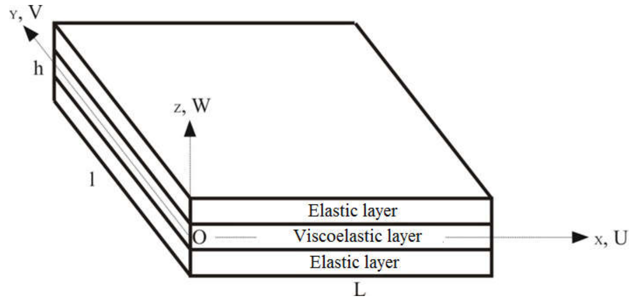

Figure 1 - Viscoelastic Sandwich Structure

Based on these hypotheses, the longitudinal displacement U and the transverse displacement W for the elastic layers i=1,3 can be expressed in the form:

$$\begin{aligned} U_i(x,z,t) &= u_i - (z - z_i)w_{,x} \\ W_i(x,z,t) &= w(x,t) \end{aligned} \tag{1}$$

where u_i is the midplane longitudinal displacement of the ith layer and w is the common transverse displacement. The coordinates z_i of the midplanes are given by $z_1 = -z_3 = (h_f + h_c)/2$, where h_f is the thickness of the elastic layers and h_c the thickness of the central layer. For the viscoelastic layer, the longitudinal displacement U_2 and the transversal displacement W_2 are expressed by:

$$\begin{aligned} U_2(x,z,t) &= u(x,t) + z\beta(x,t) \\ W_2(x,z,t) &= w(x,t) \end{aligned} \tag{2}$$

where u and β are the midplane longitudinal displacement and rotation of the central layer, respectively. Thus, the nonlinear deformation field is expressed for the three layers as following:

$$\begin{aligned} \varepsilon_i(x,z,t) &= u_{i,x} + \frac{1}{2}(w_{,x})^2 - (z - z_i)w_{,xx} \quad i = 1,3 \\ \varepsilon_2(x,z,t) &= u_{,x} + \frac{1}{2}(w_{,x})^2 + z\beta_{,x} \\ \xi_2(x,z,t) &= \beta + w_{,x} \end{aligned} \tag{3}$$

where ε_i is the normal deformation of the ith layer and ξ_2 the shear deformation of the central layer, obtained from the Green-Lagrange formulation.

By using the generalized Hooke's law, the behavior laws can be obtained for the elastic layers by calculating the normal and bending efforts Eq. (4)), and for the viscoelastic layer, they can be obtained by calculating the normal, bending and shear efforts (Eq. (5)):

$$\begin{aligned} N_i(x,t) &= E_f S_f \left[u_{i,x} + \frac{1}{2}(w_{,x})^2 \right], \quad i = 1,3 \\ M_i(x,t) &= E_f I_f w_{,xx} \end{aligned} \tag{4}$$

$$\begin{aligned} N_2(x,t) &= S_c R(t) * \left[\dot{u}_{,x} + \frac{1}{2}(w_{,x})^2 \right] \\ M_2(x,t) &= I_c R(t) * \dot{\beta}_{,x} \\ T &= \frac{S_c}{2(1 + v_c)} R(t) * (\dot{w}_{,x} + \dot{\beta}) \end{aligned} \tag{5}$$

where E_f represents the Young modulus for the elastic layers, S_f and I_f are, respectively, the cross section area and the cross quadratic moment of the ith layer, S_c and I_c are, respectively, the cross are and the cross quadratic moment of the central layer and $R(t)$ represents the relaxation function of the viscoelastic material.

The equations of motion are obtained through the virtual work principle. The continuity hypothesis of the displacement fields in the interfaces allows defining the generalized fields $\{u, w, \beta\}$ of the central layer's midplane. Through the internal, external and inertial components of the virtual work principle, using the effort components calculated in Eqs. (4) and (5), and considering an external effort $F = F(x,t)$, it's possible to obtain the general equation that rules the nonlinear vibration of the sandwich beam. This equation is the coupling of the bending and shear effects. Through neglecting the axial inertia and axial components of the external effort, the in-plane equilibrium solution can be obtained using Eq. (6), and the bending response of the sandwich beam is obtained by solving Eq. (7):

$$\int_0^L N\delta u_{,x}dx = 0 \tag{6}$$

$$\int_0^L \left[Nw_{,x}\delta w_{,x} + M_\beta \delta\beta_{,x} + M_w \delta w_{,xx} + T(\delta w_{,x} + \delta\beta) + (2\rho_f S_f + \rho_c S_c)\ddot{w}\delta w\right]dx = \int_0^L F\delta w dx \tag{7}$$

where N, M_β and M_w are the resultant normal and bending moments, ρ_c and ρ_c are the densities for the elastic and viscoelastic materials, respectively. Equation (7) has no analytical solution, and an approximated solution will be searched, using the finite element discretization and the one mode Galerkin's method.

Finite Element Dicretization Procedure

The finite element proposed for the discretization of the Eq. (7) is one-dimensional, with two nodes, each node having the following degrees of liberty: transversal displacement (*W*), the inclination ($W_{,x}$) and the rotation (*B*). Considering L^e as the finite element length, and that $\xi = (2x/L^e) - 1$, for $x \in [0 \quad L^e]$ and $\xi \in [-1 \quad 1]$, the polynomial shape functions are given by:

$$\begin{gathered} N_1(\xi) = \frac{(1-\xi)^2(2+\xi)}{4}, N_2(\xi) = \frac{L^e(1-\xi)^2(1+\xi)}{8}, N_3(\xi) = \frac{(1+\xi)^2(2-\xi)}{4}, \\ N_4(\xi) = -\frac{L^e(1+\xi)^2(1-\xi)}{8}, N_5(\xi) = \frac{1-\xi}{2}, N_6(\xi) = \frac{1+\xi}{2} \end{gathered} \tag{8}$$

The interpolation matrix containing the polynomial shape functions are:

$$\mathbf{N}_w = [N_1(\xi) \quad N_2(\xi) \quad 0 \quad N_3(\xi) \quad N_4(\xi) \quad 0], \quad \mathbf{N}_\beta = [0 \quad 0 \quad N_5(\xi) \quad 0 \quad 0 \quad N_6(\xi)] \tag{9}$$

By using Eq. (9) to solve the problem in Eq. (7), the mass and stiffness matrix are obtained:

$$\begin{gathered} \mathbf{M}^e = (2\rho_f S_f + \rho_c S_c)\mathbf{m}^e \\ \mathbf{K}^e(\omega) = \mathbf{K}_0^e + E(\omega)\mathbf{K}_v^e \\ \mathbf{K}_0^e = \mathbf{K}^e(0) = \left(I_c E_c^*(0) + \frac{E_f S_f h_c^2}{2}\right)\mathbf{k}_1^e - \frac{E_f S_f h_f h_c}{2}\mathbf{k}_2^e + \left(2E_f I_f + \frac{E_f S_f h_f^2}{2}\right)\mathbf{k}_3^e + \frac{S_c E_c^*(0)}{2(1+\nu_c)}\mathbf{k}_4^e \\ \mathbf{K}_v^e = I_c \mathbf{k}_1^e + \frac{S_c}{2(1+\nu_c)}\mathbf{k}_4^e \end{gathered} \tag{10}$$

The matrix $\mathbf{m}^e$ and $\mathbf{k}_i^e$ $_{1 \le i \le 4}$ are obtained integrating the shape functions, in the following way:

$$\begin{gathered} \mathbf{m}^e = \int_{L^e} \mathbf{N}_w{}^T \mathbf{N}_w dx \\ \mathbf{k}_1^e = \int_{L^e} \mathbf{N}_{\beta,x}{}^T \mathbf{N}_{\beta,x} dx \\ \mathbf{k}_2^e = \int_{L^e} \left(\mathbf{N}_{\beta,x}{}^T \mathbf{N}_{w,xx} + \mathbf{N}_{w,xx}{}^T \mathbf{N}_{\beta,x}\right)dx \\ \mathbf{k}_3^e = \int_{L^e} \mathbf{N}_{w,xx}{}^T \mathbf{N}_{w,xx} dx \\ \mathbf{k}_4^e = \int_{L^e} \left(\mathbf{N}_{w,x}{}^T \mathbf{N}_{w,x} + \mathbf{N}_\beta{}^T \mathbf{N}_{w,x} + \mathbf{N}_{w,x}{}^T \mathbf{N}_\beta + \mathbf{N}_\beta{}^T \mathbf{N}_\beta\right)dx \end{gathered} \tag{11}$$

By ignoring the geometrical nonlinear terms and the external effort, it's possible to set up the following nonlinear complex eigenvalue problem:

$$\left(\mathbf{K}(\omega)-\omega^2\mathbf{M}\right)\mathbf{U}=0 \tag{12}$$

where $\mathbf{M}$ and $\mathbf{K}$ are global mass and stiffness matrix, the complex vibration mode $\mathbf{U}$ and ω the natural vibration frequency. The solution of this problem generates the Gakerkin's base, necessary to solve the nonlinear vibration problem. However, this eigenvalue problem can't be solved by a classic solver. In order to solve this nonlinear eigenvalue problem, we propose the use of the Inverse Iteration Method, showed in the next section.

Resolution of the Nonlinear Eigenvalue Problem

Considering a nonlinear eigenvalue problem to determine the scalar values λ and nonzero vectors $\mathbf{x}$, where $\mathbf{T}(\lambda)$ is a square matrix with elements that are analytical functions of λ. λ and $\mathbf{x}$ refers to eigenvalues and eigenvectors, respectively.

The Inverse Iteration algorithm is a simple method that looks for an eigenvalue/eigenvector pair with an initial appropriated approximation. It requires the solution of different equation systems of linear equations with the same size of the original problem for each iteration. This algorithm is originated from Newton's method to solve nonlinear systems. The pseudo code of this algorithm is as following

1. Choose an approximated eigenvalue/eigenvector pair.
2. For j= 0, 1, ..., until convergence
   a. Solve $\mathbf{T}\left(\lambda^{(j)}\right)u=\mathbf{T}'\left(\lambda^{(j)}\right)\mathbf{x}^{(j)}$ for $\mathbf{u}$.
   b. Compute the next approximated eigenvalue/eigenvector pair.
   $$\begin{cases}\lambda^{(j+1)}=\lambda^{(j)}-\left[\left(\mathbf{r}^H\right)\mathbf{x}^{(j)}\right]/\alpha \\ \mathbf{x}^{(j+1)}=\mathbf{u}/\alpha\end{cases}, \text{ where } \alpha=\mathbf{r}^H\mathbf{u} \text{ and } \mathbf{r}^H=\mathbf{x}^{(0)}\Big/\left|\mathbf{x}^{(0)}\right|^2.$$

In practice, the approximated eigenvalue/eigenvector pair is the solution of the linear eigenvalue problem when $\lambda=0$. Besides that, the convergence is declared when the residual norm defined is smaller than a given tolerance.

Amplitude Equation for the Nonlinear Problem

The analyzed model of linear and nonlinear vibration of sandwich beams is based on free or forced periodic response with harmonic excitations in the form $F(x,t)=f(x)e^{i\omega t}$. In this sense, the in-plane problem in Eq. (6) must be solved to determine the normal effort and the longitudinal displacement that allow to calculate the nonlinear answer, solution of the problem in Eq. (7). This procedure is detailed by Bilasse (2010). Substituting each member in Eq. (7) by its respective expression and retaining the time independent terms, one can obtain the equation that relates the amplitude A with the frequency ω:

$$-\omega^2 mA+kA+k_{nl}\overline{A}A^2=Q \tag{13}$$

where m, k, k_{nl} and Q are, respectively, the modal mass, linear modal stiffness, nonlinear modal stiffness and modal excitation. $\overline{A}$ is the conjugate of the complex amplitude A. These coefficients can be calculated numerically using the finite element approximation:

$$\begin{aligned} m&=\overline{\mathbf{U}}^T\mathbf{MU} \\ k&=\overline{\mathbf{U}}^T\mathbf{K}(\omega)\mathbf{U} \\ k_{nl}&=k_{nl}(\omega) \\ Q&=\overline{\mathbf{U}}^T\mathbf{F}\end{aligned} \tag{14}$$

The nonlinear modal stiffness can be calculated using the expression:

$$k_{nl}(\omega)=\frac{2E_f S_f+S_c E_c^*(0)}{L}\left(\overline{\mathbf{U}}^T\mathbf{k}_5\mathbf{U}\right)^2+\frac{2E_f S_f+S_c E_c^*(2\omega)}{2L}\left|\overline{\mathbf{U}}^T\mathbf{k}_5\mathbf{U}\right| \tag{15}$$

where the matrix $\mathbf{k}_5$ is obtained by the global assembling of the following elementary matrix:

$$\mathbf{k}_5^{\mathrm{e}}=\int_{L_e}\mathbf{N}_{w,x}{}^T\mathbf{N}_{w,x}dx \tag{16}$$

Considering the decomposition of the linear and nonlinear modal stiffness, such that $k(\omega)=k^R(\omega)+ik^I(\omega)$ and $k_{nl}(\omega)=k_{nl}^R(\omega)+ik_{nl}^I(\omega)$, and assuming that $r=a^2$, with $a=|A|$, the general amplitude equation is:

$$f_3(\omega)r^3+f_2(\omega)r^2+f_1(\omega)r+f_0=0 \tag{17}$$

where the coefficients f_0, f_1, f_2 and f_3 are given as:

$$\begin{gathered}
f_3(\omega)=k_{nl}^{R\,2}(\omega)+k_{nl}^{I\,2}(\omega)\\
f_2(\omega)=2\left[k_{nl}^R(\omega)\left(-\omega^2 m+k^R(\omega)\right)+k^I(\omega)k_{nl}^I(\omega)\right]\\
f_1(\omega)=\left(-\omega^2 m+k^R(\omega)\right)^2+k^{I\,2}(\omega)\\
f_0=-|Q|^2
\end{gathered} \tag{18}$$

Equation (17) is of third order and can be solved for each frequency ω, and by any resolution method.

Maximum Entropy Method

A random variable is a variable subject to variations due to randomness, and conceptually hasn't a fixed or single value. It can assume a group of possible different values, each one with an associated probability.

A probability density function (PDF), of a random continuous variable, is a function that describes the probability that a random variable may assume a determined value. Using the Maximum Entropy Method (JAYNES, 1957), the construction of the probabilistic model of each parameter consists in obtaining the PDF which best describes a random variable considering the available information about it (SOIZE, 2010). From a set of distributions that satisfy the restrictions of a given random variable, e.g. mean and variance, the method allows the choice of the distribution that presents the maximum of uncertainties.

Thereat, given a PDF $p(x)$, of a random variable x, the entropy is measured using the following equation (SHANNON, 1948):

$$S(p_x(x))=-\int p_x(x)\ln(p_x(x))dx \tag{19}$$

Through the knowledge of some information about these random variables, that are the restrictions mentioned earlier, described as statistical moments, these may be calculated in the form:

$$\int x^i p_x(x)dx=d_i \tag{20}$$

where $i=0,1,\ldots,n$, $d_0=1$ as the PDF itself and d_i the known statistical moments.

Through the method of Lagrange multipliers, a function for the entropy's PDF $p(x)$ can be built, that when maximized, allows to obtain the expression for the most probable PDF of the random variable:

$$p(x)=\exp\left[-1+\sum_{i=0}^{n}\lambda_i x^i\right] \tag{21}$$

where λ_i are the Lagrange multipliers, that can be obtained from the equations of restriction, due to the $n+1$ known statistical moments of the random variable x.

Numerical Results

The structure adopted is modeled using 100 finite elements in the beam's length direction. The geometric nonlinearities are fixed in the two extremities, with $u(0)=u(L)=0$. The results are calculated using the one mode Galerkin's method, the vibration modes **U** and bending modes **W** are normalized w.r.t the transversal displacement:

$$\begin{cases} \mathbf{U} = \dfrac{\mathbf{U}}{|\mathbf{W}(x_0)|} \\ \mathbf{W} = \dfrac{\mathbf{W}}{|\mathbf{W}(x_0)|} \end{cases} \tag{22}$$

where $\mathbf{W}(x_0)$ represents the transversal displacement of the point x_0 in the beam's length. For the condition simple supported and clamped-simple supported, $x_0 = L/2$, and for clamped-free, $x_0 = L$. The structure's dimension, even as the materials properties are given in Tab. 1.

Table 1 – Material and structural data of the sandwich beam.

Structure's dimensions		Aluminum elastic faces		3M ISD112 Core	
Length	L = 177.8 mm	Young modulus	$E_f = 6.9 \times 10^{10}$ Pa	Shear modulus	Eq. (23)
Width	l = 12.7 mm	Poisson ratio	$v_f = 0.3$	Poisson ratio	$v_c = 0.5$
Faces' thickness	$h_f = 1.524$ mm	Mass density	$\rho_f = 2766$ kg.m^{-3}	Mass density	$\rho_c = 1600$ kg.m^{-3}
Core's thickness	$h_c = 0.127$ mm				

The viscoelastic material used in this structure is the 3M ISD112. The shear modulus law and the shift factor are given by (Guedri et. al, 2010):

$$\begin{aligned} G(\omega_r) &= 430700 + \frac{1200\times10^6}{1+3.241(i\omega_r/154300)^{-0.18} + (i\omega_r/154300)^{-0.6847}} \\ \log(\alpha_T) &= -3758.4\left(\frac{1}{T}-\frac{1}{290}\right) + 225.06\times\log\left(\frac{T}{290}\right) + 0.23273(T-290) \end{aligned} \tag{23}$$

where $\omega_r = \alpha_T\omega$ represents the reduced frequency, ω is the excitation frequency and is T the temperature.

Table 2 presents the damping properties (damped natural frequency and loss factor) for the first three modes of vibration of the beam for the simple supported boundary condition. These results are in accordance with other results available in literature, as in Bilasse (2010).

Table 2 – Damping properties for first three modes of the beam under various boundary conditions.

Mode	Ω_l [rad/s]	η_l
1	998.08	0.306
2	3426.12	0.392
3	7229.53	0.336

Figure 2 shows the linear and nonlinear vibration responses of the first mode of the beam in the condition Simple Supported. Through Fig. 2, it's possible to see the effects from the geometric nonlinearities in the system's response. One can see that, in the nonlinear responses, the amplitude is smaller and the resonance frequency is much higher, when compared with the linear response.

The evaluation of the influence of uncertainties in the system's response is considered in three parameters: the face's thickness, core's thickness and temperature. It's already known that these parameters are the ones with most influence in the linear vibration case (de Lima, Rade, Bouhaddi, 2010). The objective here is to investigate if this influence level exists also in the nonlinear vibration case.Through the known information about these variables, the PDF set for the face's thickness and core's thickness is the *Gamma* function, and for the temperature, the *Gaussian* function. The samples for each variable are generated using the Latin Hypercube.

The number of samples necessary to obtain a convergence in the results was obtained through the Least Squares Method. With it, it became established a number of 150 samples for the face's thickness, 100 samples for the core's thickness and 200 samples for the temperature.

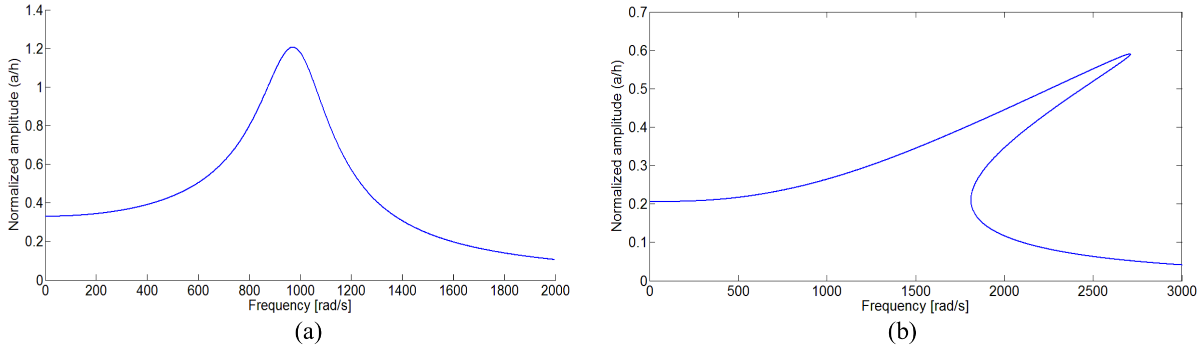

Figure 2 - Beam's responses for the condition Simple Supported: (a) Linear, (b) Nonlinear

The envelopes of responses were obtained considering uncertainties in one variable at a time, with three levels of uncertainties: σ = 1%, 2% and 5%, considering a 95% confidence level. The tests were performed considering linear and nonlinear vibration in the simple supported beam. Figures 3, 4 and 5 show the envelopes of responses considering uncertainties, respectively, in the core's thickness, face's thickness and temperature.

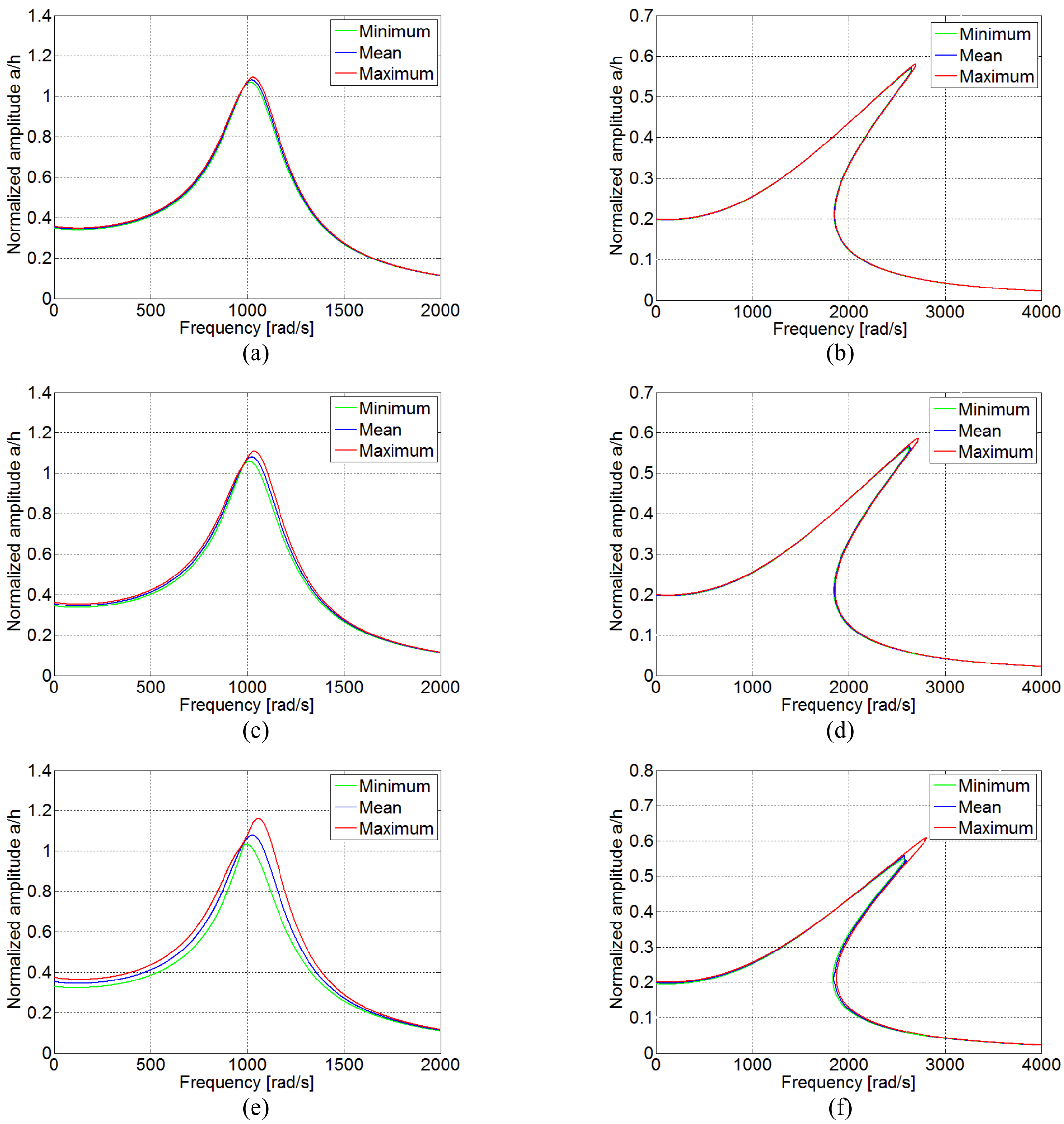

Figure 3 – Envelopes of responses considering uncertainties in the core's thickness: (a), (c), (e) Linear vibration; (b), (d), (f) Nonlinear vibration; (a), (b) σ = 1%; (c), (d) σ = 2%; (e), (f) σ = 5%.

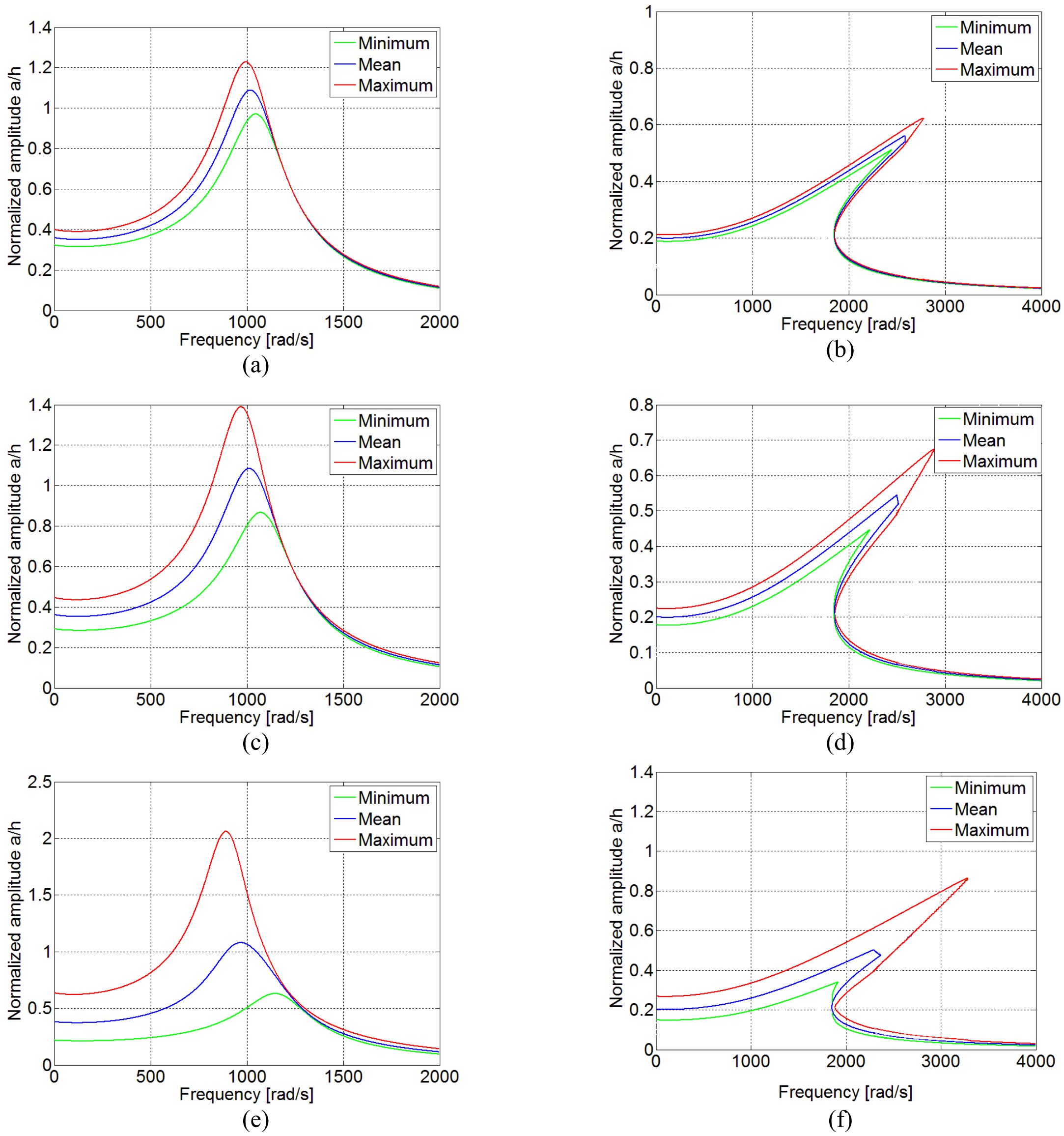

Figure 4 – Envelopes of responses considering uncertainties in the face's thickness: (a), (c), (e) Linear vibration; (b), (d), (f) Nonlinear vibration; (a), (b) σ = 1%; (c), (d) σ = 2%; (e), (f) σ = 5%.

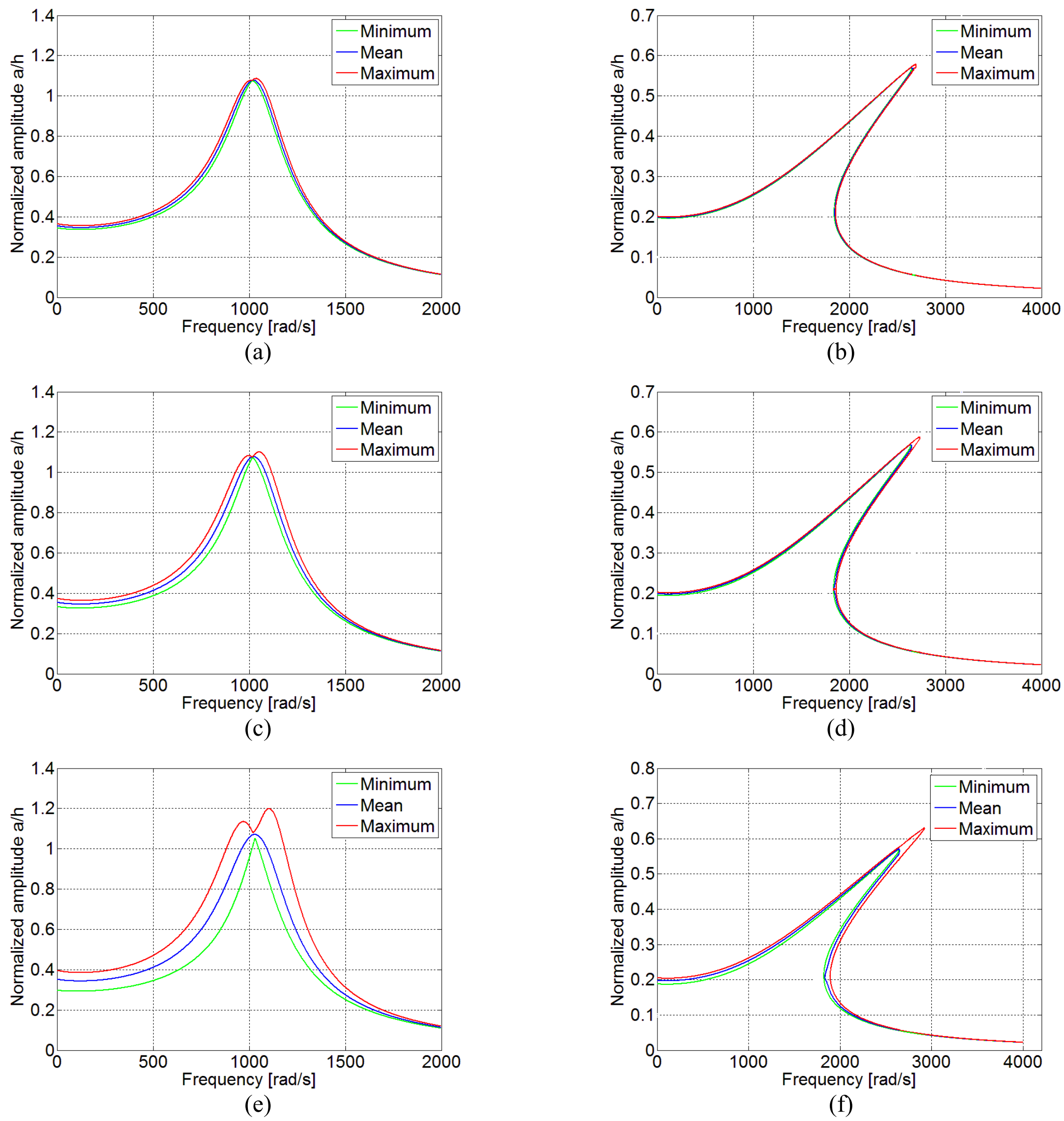

Figure 5 – Envelopes of responses considering uncertainties in the temperature: (a), (c), (e) Linear vibration; (b), (d), (f) Nonlinear vibration; (a), (b) σ = 1%; (c), (d) σ = 2%; (e), (f) σ = 5%.

Concluding Remarks

In this work, it has been developed a numerical method for linear and nonlinear vibrations analysis of viscoelastic sandwich beams. The equations are formulated using the virtual work principle and discretized using the finite element framework. Using the harmonic balance method and the one mode Galerkin's procedure, an amplitude–frequency equation relationship for nonlinear vibration is established. An efficient Galerkin's basis is calculated using the inverse iteration method. Using the present methodological approach, numerical simulations are performed on viscoelastic sandwich beams. The obtained results show that the present methodological approach is efficient to characterize the linear and nonlinear response of frequency dependent viscoelastic sandwich beams in a desired frequency range whatever the constitutive viscoelastic law. Lastly, the analysis of the influence of uncertainties in some physical and environmental parameters showed the relevance of this study in the search for safety, reliability and comfort. From the parameters considered uncertain, the temperature, and primarily, the face's thickness showed great influence in the system's response, being that this influence is more visible in the linear vibration case than in the nonlinear vibration case.

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