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DYNAMIC BEHAVIOR OF TIMBER FOOTBRIDGES WITH STOCHASTIC MECHANICAL PROPERTIES

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Abstract: A dynamic study of timber footbridges with stochastic mechanical properties is presented in this paper. These structural systems made of timber are increasingly employed due to the high relation stiffness/weight that this material exhibits. More, the development and implementation of laminated beams permits larger spans. The sources of uncertainty of this structural model are the timber mechanical and physical properties. Also, the geometrical design of the laminates that compose the laminated timber beams supporting the floor involves variability in the distances between finger joints. Probability Density Functions (PDFs) of the timber properties are formulated from the Principle of Maximum Entropy (PME) and visual surveys of structural size $Eucalyptus\ grandis$ laminated beams are used to obtain statistical parameters. The influence of these stochastic variables in the structural response on a forced vibration problem that includes a deterministic model of the load induced by the human walking is assessed. The PDFs of the natural frequencies of the structure, the mode shapes and the structural response are numerically obtained through the Finite Element Method (FEM) and Monte Carlo Simulations (MCS). In order to carry out this analysis, plate elements and laminated beam elements derived starting from the First Order Shear Deformation Theory (FSDT) are employed. The present stochastic model contributes to obtain a more realistic description of the response of this type of structures applicable for the study of the human comfort conditions.

Keywords: timber structures, footbridge, uncertainty, human walking

INTRODUCTION

Footbridges (or pedestrian bridges) are one of the most common timber structures, due to the high relation stiffness/weight that this material presents in comparison to other construction materials and mainly, for the possibility of covering long spans owing to the develop and implementation of laminated beams. This type of beam is extensively employed and has allowed the construction of slender timber structures. The footbridges are composed of a desk and longitudinal laminated beams. In this work, the complete structure is made of $Eucalyptus\ grandis$, one of the most important renewable species cultivated in Argentina. A simple method for visually strength grading sawn timber of these species has been developed by Piter (2003). As reported in this paper, the presence of pith and knots are considered the most important visual characteristic for strength grading this material by the Argentinian standard IRAM 9662-2 (2006). Experimental studies related with the bending strength and stiffness in $Eucalyptus\ grandis$ laminated beams are reported by Piter et al. (2007) and Saviana, Sosa Zitto and Piter (2009).

The stochastic approaches employed for the modeling of timber mechanical properties are derived from the probabilistic theories of random variables and processes. Köhler, Sørensen and Faber (2007) present a probabilistic model of timber structures where the Modulus of Elasticity (MOE) is represented as a random variable with a lognormal PDF, assuming a homogeneous value within a structural element. Fink and Köhler (2015) report a probabilistic approach for modeling the tensile strength and stiffness properties of timber boards and finger joint connections.

Timber footbridges must satisfy strength and serviceability requirements. Generally, due to its low weight, the serviceability requirements in terms of peak acceleration constitutes the most restrictive condition in the design. The control of the system maximum acceleration can be performed, in general, by increasing the structural damping or mass. The natural frequency is also relevant since the harmonic force intensity generally decreases with increasing harmonics. Nowadays, the design criteria for floor and footbridge vibration analysis due to people walking are based on these principles. An extensive literature review and state-of-the-art report of the dynamic behavior of these structures is included in Živanović, Pavic and Reynolds (2005). Among other topics, the load models, standards requirements and studies of the walk of people and crowns are reported. Segundinho (2010) studies the vibrations induced by pedestrian in timber footbridges carried out in a parametric way. Numerical results are compared with experimental ones obtained from a scaled model. Studies of vertical vibrations in existing footbridges of concrete and steel simulated in a Finite Element Method (FEM) software are reported in Da Silva et al. (2007) and Figueiredo et al. (2008) . The present work includes the load models used in the last reference.

In the present study, the sources of uncertainty in the structural model derive from the timber mechanical and physical properties. Also, the geometrical design of the laminates that compose the laminated timber beams is assumed stochastic regarding the distances between finger joints. A visual survey of structural size $Eucalyptus\ grandis$ laminated beams yields statistical data. Then, the Probability Mass Function of the distance between finger joints is constructed. Between the pieces of timber, defined by the distance among fingers joints, the properties vary stochastically and in a non correlated way. According to the standard IRAM 9662-2 (2006) each board of the laminated beams comes from a specific strength class. Within this quality class, the properties vary stochastically. The propagation of these sources of uncertainty in the first natural frequency of the footbridge is studied. This is one of the parameters useful to assess the serviceability performance of the structure. To accomplish this, the PDF of the first natural frequencies is found via Monte Carlo simulations (Rubinstein, 1981). The PDFs of the MOE and the mass density are obtained by means of the Principle of Maximum Entropy (Shannon, 1948) and the parameters by means of the maximum likelihood method (MLM) applied to MOE values obtained experimentally. In order to measure the fit between the experimental and theoretical PDFs of the MOE and the mass density, the Kolmogorov-Smirnov (K-S) test of fit is used.

Other important parameter in the study of the serviceability performance is the acceleration. The footbridge is excited by two walking load models. The first one is a model employed in the standard codes and the second is a more realistic representation that includes the heel impact effect. The results of these two models are compared in order to choose one for the stochastic serviceability study. Two limit values of peak accelerations according to the standards EUROCODE 5 (2004) and ISO 10137 (2005) are employed. Also, two values of damping ratio are used. For certain range of the first natural frequencies, the excitation condition could change and some results with higher values of acceleration appear. Hence, the traditional analysis with a mean model would be inadequate in the frequency zone in which the footbridge could be excited with a fast walking with the third harmonic or for a normal walking with the fourth harmonic of the force in accordance with the first natural frequency of the footbridge. The variation of the material properties can lead to an unacceptable serviceability performance of the structure in some scenarios.

PROBLEM STATEMENT

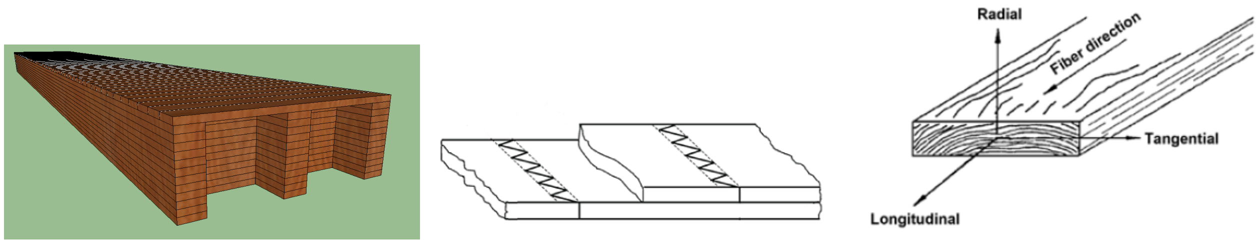
The study of the vertical dynamical behavior of a timber footbridge with stochastic mechanical properties under a walking load is presented. The structure is composed of three longitudinal laminated beams with a length of 13.2 m and a separation of 0.6 m in the transversal direction of the structure, five transversal laminated beams with a separation o 3.3 m in the longitudinal direction of the structure and a deck of timber boards. The width of the laminated beams and the timber boards of the deck is fixed in 0.15 m, and the height of each lamina of the beams and of the timber boards in 0.0375 m. The total height of the beams was designed for resistance according to the Argentinean standard CIRSOC 601 (2013) and for serviceability requirements. The latter is the main factor of the analyzed structural design. The number of laminates of the beams was considered between 12 and 16, laminated beams with 0.45 m and 0.6 m of height respectively, (see Fig. 1a).

Laminated beams are composed of several laminates formed by the union of boards with different mechanical properties. Upper and lower faces of the boars are glued to the superior and inferior continuous board. Previously the boards of each lamina are assembled with finger joints. The influence of the finger joints configuration in the natural frequencies will be studied stochastically in this work. Distances between finger joints obtained from a visual survey of laminated beams (Torrán, 2013) were employed in order to simulate the different boards that conform a laminated beam. With the results of this survey, the Probability Mass Function (PMF) of the distance between fingers joints was constructed. The mean value and standard deviation of the distance between joints are 0.865 m and 0.247 m, respectively (Fig. 1b).

TIMBER MATERIAL

The material model is based on the general assumption of orthogonal orthotrophy. Each board has a longitudinal direction parallel to the main material fibres direction that coincides with the x axis, $L = x$, a tangential direction with respect to the growth rings of the transversal section that coincides with the y axis, $T = y$ and finally a radial direction with respect to the growth rings of transversal section that coincides with the z axis, $R = z$. Figure 1c shows the orthogonal orthotrophy assumption. In Eq. (1) the stress-strain relations are presented.

$$\begin{Bmatrix} \varepsilon_L \\ \varepsilon_R \\ \varepsilon_T \\ \gamma_{LR} \\ \gamma_{LT} \\ \gamma_{RT} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{RL}}{E_R} & -\frac{\nu_{TL}}{E_T} & 0 & 0 & 0 \\ -\frac{\nu_{LR}}{E_L} & \frac{1}{E_R} & -\frac{\nu_{TR}}{E_T} & 0 & 0 & 0 \\ -\frac{\nu_{LT}}{E_L} & -\frac{\nu_{RT}}{E_R} & \frac{1}{E_T} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{LR}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{RT}} \end{bmatrix} \begin{Bmatrix} \sigma_L \\ \sigma_R \\ \sigma_T \\ \tau_{LR} \\ \tau_{LT} \\ \tau_{RT} \end{Bmatrix} \quad (1)$$



(a) Transversal section of the timber footbridge. (b) Finger joints. (c) Orthogonal orthotrophy of a wood board.

Figure 1 – Structural model assumptions.

In this work, the orthotropic model is reduced to the transversal isotropic model with two main directions the longitudinal also named parallel to the main fibres direction $E_L = E_x$, and the perpendicular direction that includes the radial and tangential material direction $E_R = E_T = E_{yz}$. The basic stochastic properties proposed in this work are the longitudinal MOE and the mass density. In what follows, these stochastic properties will be represented by $\mathbb{E}_x$ and ρ. For a transversally isotropic material, the elastic and shear modulus are defined as $\mathbb{E}_{zy} = \mathbb{E}_x/15$, $\mathbb{G}_{xy} = \mathbb{G}_{xz} = \mathbb{E}_x/16$ (UNE-EN 408, 1995). Meanwhile, the Poisson coefficients for hardwood are $\nu_{RT} = 0.67, \nu_{LT} = 0.46$ and $\nu_{LR} = 0.39$ (Argüelles Álvarez, Arriaga Martitegui and Martínez Calleja, 2000). For a transversally isotropic formulation $\nu_{zy} = \nu_{RT} = 0.67$ and $\nu_{xzy} = (\nu_{LT} + \nu_{LR})/2 = 0.425$ and $\mathbb{G}_{zy} = \mathbb{E}_{zy}/2(1 + \nu_{zy})$.

PDF of the MOE and mass density

In order to determine the marginal PDF of the MOE and the mass density, the Principle of Maximum Entropy (PME) (Shannon, 1948) for a continuous random variable is applied. The measure of uncertainties of the continuous random variable X is defined by the following expression:

$$S(f_X) = -\int_D f_X(x) ln(f_X(x)) dx \tag{2}$$

in which f_X stands for the PDF of the random variable X and D is its domain. It is possible to demonstrate that the application of the principle under the constraints of positiveness and bounded second moment, leads to a gamma PDF. The PME conduces to this PDF due to the fact that the domain of the MOE and the mass density is real and positive.

To find the parameters of the marginal PDF of the MOE and mass density, experimental data presented by Piter (2003) are employed. These values were obtained by means of two point load bending tests, performed with 349 sawn beams of Argentinean $Eucalyptus\ grandis$ with structural dimensions and density measurements. Bending tests and density measurements were carried out according to the standard UNE-EN 408 (1995).

The parameters of the gamma marginal PDF of the MOE are estimated with the help of the maximum likelihood method (MLM). Then, the Kolmogorov-Smirnov (K-S) test of fit is used, ($e.g.$ Benjamin and Cornell, 1970). The PDF that best fits with the experimental values is the gamma, in agreement with the PME. The test of fit also was carried out with the lognormal and normal PDFs, the first one since Köhler, Sørensen and Faber (2007) proposed it to model the MOE and the second PDF as is often employed to represent mechanical properties. The normal PDF fits the experimental data best. However, the use of this PDF in the model would occasionally lead to negative values of the MOE which are physically unacceptable. Thus, the gamma and lognormal PDF are more suitable.

Köhler, Sørensen and Faber (2007) represent the variability of the mass density among structural timber beams with a normal PDF for European Softwood species. The parameters of the PDF of the mass density are estimated with the help of the MLM. Finally, the K-S test of fit is used to choose the PDF that fits best. As before, assuming α=0.05, the critical value for the K-S test of fit is equal to c=1.36. The results of the K-S test are the following: lognormal 0.65, gamma 0.69 and normal 0.75. As can be seen, the three PDFs fulfill the critical value, but the lognormal and gamma fit best with the experimental values. Here, following the PME and since the difference found among the lognormal and gamma is small, we adopted the last PDF in order to introduce in the stochastic model the mass density uncertainty. The gamma marginal PDF and CDF of the MOE and mass density are:

$$f(x \mid a, b) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{-\frac{x}{b}} \qquad F(x \mid a, b) = \frac{1}{b^a \Gamma(a)} \int_0^x t^{a-1} e^{-\frac{t}{b}} dt \tag{3}$$

where a and b denote the shape and scale parameters, respectively. For the MOE, a = 34.582 and b= 0.402 with a mean value of the MOE equal to 13.902 GPa and a standard deviation of 1.498 GPa. In the case of the mass density, a = 72.179 and b = 7.659, the mean value of the mass density equal to 552.819 kg/m^3 and a standard deviation of 65.069 kg/m^3 .

PLATE AND LAMINATED BEAM FORMULATION

The First Order Shear Deformation Theory (FSDT) accounts for the transverse shear strains. The displacement field is the following:

$$u(x,y,z) = z\phi_x, \qquad v(x,y,z) = -z\phi_y \qquad \text{and} \quad w(x,y,z) = w_0 \tag{4}$$

where ϕ_x and ϕ_y denote rotations about the y and x axes, respectively (Reddy, 2004). For the bending in plates, we obtain:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = z \begin{Bmatrix} \frac{\partial \phi_y}{\partial x} \\ \frac{\partial \phi_x}{\partial y} \\ \frac{\partial \phi_x}{\partial x} - \frac{\partial \phi_y}{\partial y} \end{Bmatrix} \qquad \text{and} \qquad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \phi_x + \frac{\partial w_0}{\partial x} \\ \phi_y + \frac{\partial w_0}{\partial y} \end{Bmatrix} \tag{5}$$

Then, the displacement field of each layer (k) in the laminated beam is given by

$$u(x,y,z) = z\phi_x + \theta\psi_k \qquad v(x,y,z) = -\theta xz \qquad \text{and} \qquad w(x,y,z) = w_0 + \theta xy \tag{6}$$

where θ is the angle of twist per unit length and ψ_k is the warping function for the kth layer. The torsion effects in the laminated beams is considered as reported by Swanson (1998) in the second term of u and w and in v. The strain-displacement relations are employed:

$$\gamma_{xy} = \theta\left(\frac{\partial \psi_k}{\partial y} - z\right) \qquad \text{and} \qquad \gamma_{xz} = \theta\left(\frac{\partial \psi_k}{\partial z} - y\right) \tag{7}$$

In Swanson (1998), the warping function ψ_k for laminated orthotropic rods is obtained in order to evaluate the effective torsional stiffness:

$$\begin{aligned} GJ_{eff} \quad = \quad & \frac{32(b)^3(h)}{\pi^4} \sum_k G_{xy_k} \sum_{m=1,3,5\ldots} \frac{1}{m^4} \\ & \left[\frac{h_k - h_{k-1}}{h} - \frac{A_{km}}{h\beta_{km}}\left(\sinh \beta_{km} h_k - \sinh \beta_{km} h_{k-1}\right) - \frac{B_{km}}{h\beta_{km}}\left(\cosh \beta_{km} h_k - \cosh \beta_{km} h_{k-1}\right)\right] \end{aligned} \tag{8}$$

in which b and h are the width and height of the beam, h_k the interlayer location, G_{xy_k} the shear modulus of the kth layer, β_{km} is a coefficient which accounts for the relation $\frac{G_{xz_k}}{G_{xy_k}}$ and finally A_{km} and B_{km} coefficients of the warping function in each layer.

LOAD MODELS

Two load models that represent the effect of an individual walking are considered in this work. In both models, the load position is changed according to the individual location along the footbridge. The time functions have a space and time description in both cases.

The first walking load model is composed of the static load of the individual's weight and a combination of harmonic forces represented by a Fourier series:

$$F(t) = P\left[1 + \sum \alpha_i \cos\left(2\pi i f_s t + \Phi_i\right)\right] \tag{9}$$

where P is the individual's weight, considered here equal to 700 N, α_i is the dynamic coefficient of the harmonic force, i is the harmonic order, f_s is the step frequency and Φ_i the harmonic phase angle as listed in Table 1. This load model is frequently employed in the structural design standards for the verification of serviceability requirements (Živanović, Pavic and Reynolds, 2005).

The second load model is more realistic. It incorporates the human heel impact effect over the floor (Figueiredo et al.,

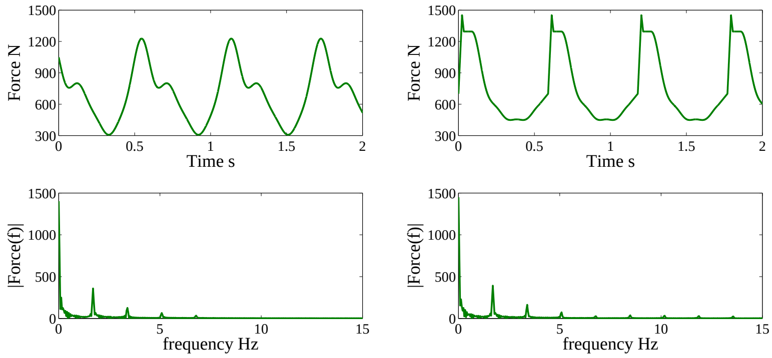


Figure 2 – Walking force models, time and frequency representations. First load model (left plots) and second load model (right plots).

2008). The formulation of this model is presented in the following equation:

$$F(t)=\begin{cases}\left(\frac{f_{mi}F_m-P}{0.04T_p}\right)t+P & \text{if} \quad 0\le t<0.04T_p\\ f_{mi}F_m\left[\frac{C_1(t-0.04T_p)}{0.02T_p}+1\right] & \text{if} \quad 0.04T_p\le t<0.06T_p\\ F_m & \text{if} \quad 0.06T_p\le t<0.15T_p\\ P\left\{1+\sum_{i=1}^{nh}\alpha_i\sin\left[2\pi if_s\left(t+0.1T_p\right)-\Phi_i\right]\right\} & \text{if} \quad 0.15T_p\le t<0.90T_p\\ 10\left(P-C_2\right)\left(\frac{t}{T_p}-1\right)+P & \text{if} \quad 0.90T_p\le t<T_p\end{cases} \tag{10}$$

$$F_m=P\left(1+\sum_{i=1}^{nh}\alpha_i\right) \qquad C_1=\left(\frac{1}{f_{mi}}-1\right) \qquad \text{and} \qquad C_2=\begin{cases}P\left(1-\alpha_2\right) & \text{if} \quad nh=3\\ P\left(1-\alpha_2+\alpha_4\right) & \text{if} \quad nh=4\end{cases} \tag{11}$$

where F_m is the maximum value of the Fourier series, f_{mi} is the heel impact factor (here equal to 1.12), T_p is the step period, C_1 and C_2 are force coefficients. This model has the same parameters than the first model, but changes the harmonic phase angle Φ_i (Table 1). The velocity of walking, the step distance and frequency considered in this work are depicted in Table 2. Time and frequency representations of both load models are shown in Fig. 2.

Table 1 – Forcing frequencies f_s, dynamic coefficients α_i and harmonic phase angle Φ_i. Load models 1 and 2.

Harmonic i	if_s Hz	α_i	Φ_i	
			Load model 1	Load model 2
1	1.6-2.2	0.5	0	0
2	3.2-4.4	0.2	$\pi/2$	$\pi/2$
3	4.8-6.6	0.1	$\pi/2$	π
4	6.4-8.8	0.05	$\pi/2$	$3\pi/2$

Table 2 – Human walking characteristics.

Activity	Step distance (m)	Step frequency (Hz)
Slow walking	0.6	1.60-1.85
Normal walking	0.75	1.85-2.15
Fast walking	0.9	2.15-2.30

The excitation frequency of the loads is adopted in such a way that if_s, second column of Table 1, be equal to the first natural frequency of the footbridge (F_1). Hence, the step frequency f_s becomes a random variable related with the values of F_1. Depending on the value that this variable can adopt, third column Table 2, the activity and the step distance of the force model applied to the footbridge are defined.

FINITE ELEMENT METHOD

The application of the Hamilton's Principle leads directly to the equation of motion of the system:

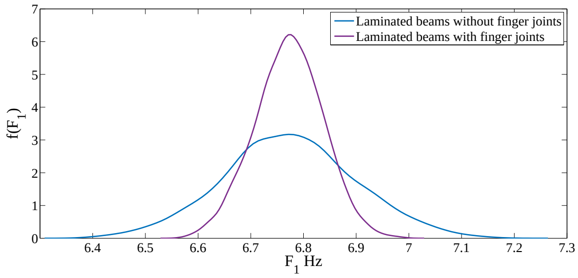


Figure 3 – PDFs of the first natural frequencies. Laminated beams with and without finger joints.

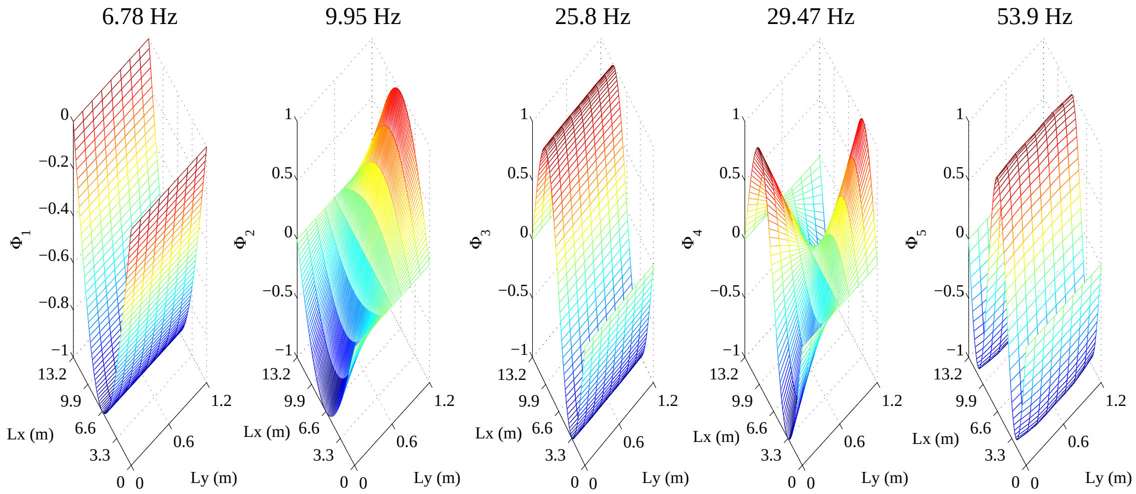


Figure 4 – Mode shapes and natural frequencies of the mean model.

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{12}$$

The equation of motion is discretized for the laminated beams with Timoshenko beam elements with two nodes and three degrees of freedoms per node. Cubic and quadratic shape functions for the translational and rotational degrees of freedom and lineal shape function for the torsional one are employed respectively. Rotational, translational and torsional inertial contributions are taken into account. The desk of the footbridge is modeled with rectangular bilinear plate elements with four nodes and three degrees of freedom per node (Reddy, 1993). The damping matrix in Eq. (12) is considered proportional to the stiffness matrix, considering damping ratios equal to 5 $\%$ and 7 $\%$ in the first natural vibration mode. These values have been suggested by Chopra (1995) for wood structures with nailed or bolted joints. Experimental studies carried out in real footbridges exhibit similar values (Segundinho, 2010).

NUMERICAL RESULTS

A convergence study of the first natural frequency was carried out due to the fact that the step harmonic, the step frequency and the step distance of the load model are considered in this work as functions of the first natural frequency of the footbridge. A number of 3000 independent Monte Carlo Simulations (MCS) was adopted for the stochastic study. In Fig. 3, the PDF of F_1 ($f(F_1)$) is shown for laminated beams with and without finger joints union. As can be observed, the mean value remains equal in both cases, $\mu(F_1) = 6.78$ Hz, while the standard deviation decreases in the first case, $\sigma(F_1) = 0.06$ Hz and $\sigma(F_1) = 0.12$ Hz respectively. This effect is based in the variation of the effective stiffness and mass properties along the beams, introduced by the stochastic model with finger joints.

In Fig. 4, the first five mode shapes and natural vibration frequencies of the mean model are shown. The study of the modes shapes and the frequency content of the forces allows to determine the number of modes employed in the mode superposition method for the resolution of the equation of motion. For the load models applied at the centre of the desk, the flexural vibration modes have the higher contribution in the response.

The comparison between the displacements at the midspan point of the footbridge along the time, obtained for the two considered loads models and a damping ratio of 7 $\%$ is shown in Fig. 5. As can be observed, the shape of the time

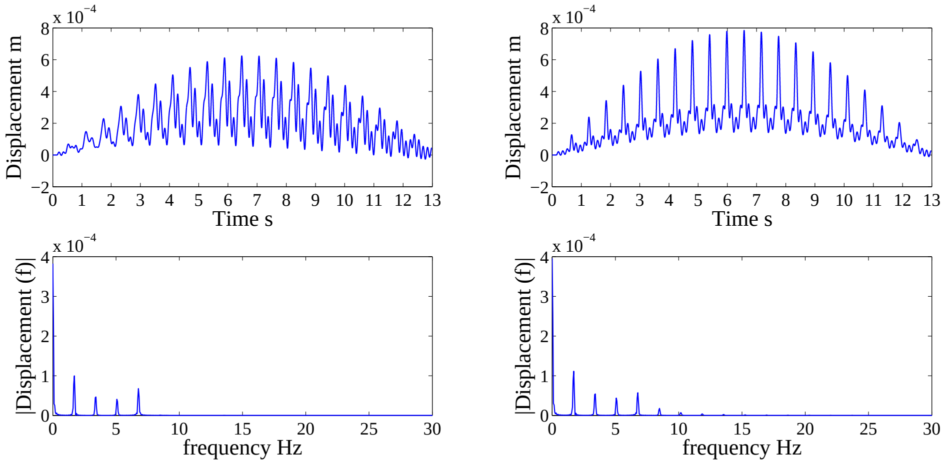


Figure 5 – Displacement functions at the midspan point of the footbridge. First load model (left plots) and second load model (right plots).

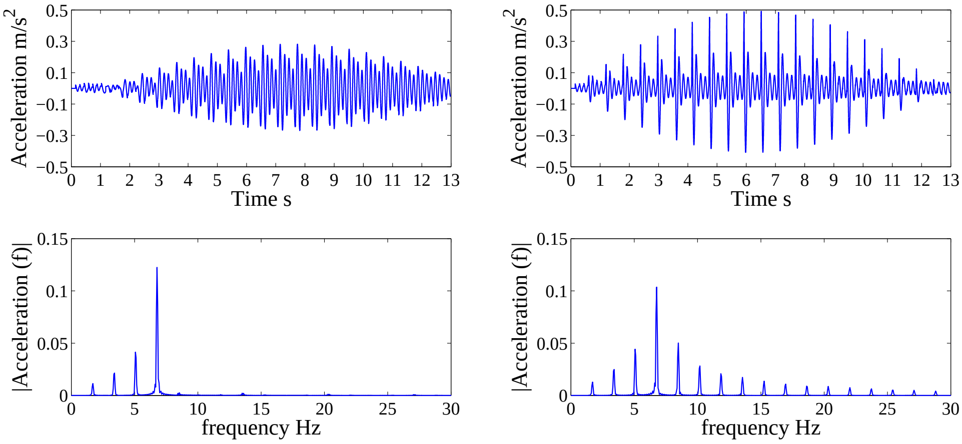
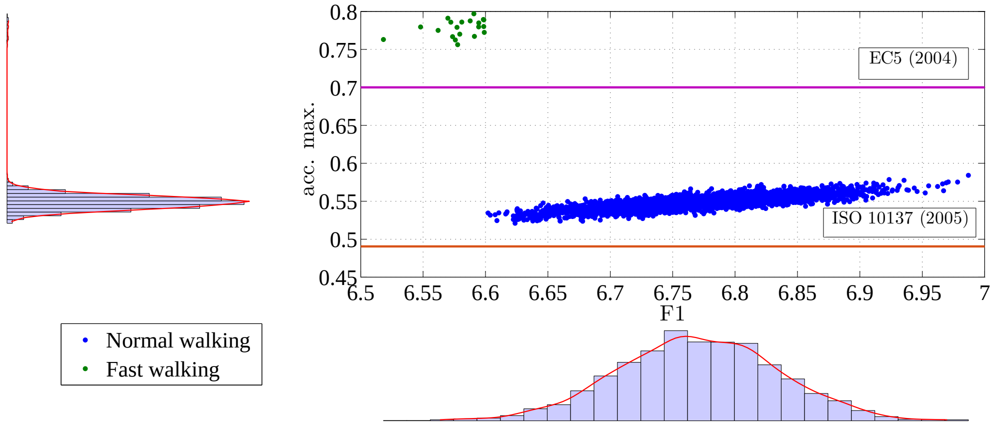


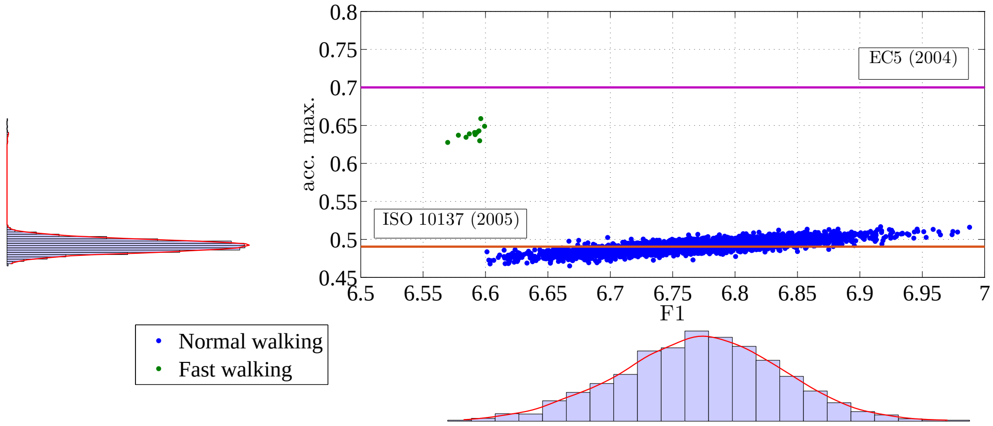
Figure 6 – Acceleration functions at the midspan point of the footbridge. First load model (left plots) and second load model (right plots).

functions and the frequency representations change with the applied load model. The heel impact effect in the second load model is clearly observed in the peaks of the displacement function. Similar behaviour is obtained for the acceleration results, Fig. 6. The second load model produces structural responses in terms of displacements and accelerations of the midspan point of the footbridge with more frequency components and higher values of these parameters are observed in comparison with the results obtained when the first load model is applied. In comparison with footbridge structures with similar natural frequencies and different materials (Da Silva et al., 2007 and Figueiredo et al., 2008), the accelerations are important and the displacements small. This can be explained due to the high relation stiffness/weight of the timber material. In what follows, for the stochastic study of the serviceability performance of the structure, the second load model that conduces to higher accelerations than the first one will be employed.

According to the Argentinian standard CIRSOC 601 (2013) when the first natural frequency of a timber structure is lower than 8 Hz, a dynamical study is needed to assure the serviceability behavior. Hence, the serviceability performance of this type of structures must be evaluated through the peak accelerations and the first natural frequencies, according to several standards and studies (Živanović, Pavic and Reynolds, 2005). In Fig. 7, the serviceability performance of the results obtained through a stochastic study carried out with the second load model is presented. Two limit values of peak accelerations according to the standards EUROCODE 5 (2004) and ISO 10137 (2005) for outdoor footbridges were considered. The third and fourth harmonic of the force model are coincident with the first natural frequencies F_1. This result is due to the frequency range of this random variable, Fig. 3. As can be observed, for a 5 $\%$ of damping ratio, the peak accelerations in some samples are higher than the EC5 limit (0.67 $\%$). This occurs when the structure is subjected to a fast walking with the third harmonic component of the forces coincident with the fist natural mode of the footbridge. In this cases, the dynamic coefficient α_i is higher than for the fourth harmonic, Table 1. When the fourth harmonic of the walking force is coincident with the first natural frequency the footbridge, the requirement of the EC5 is fulfilled but not



(a) 5% damping ratio.



(b) 7% damping ratio.

Figure 7 – Serviceability study of the footbridge with the second load model.

the requirements of the standard ISO 10137 for a normal walking (99.33 %). When the damping ratio is changed to 7 %, the requirement of the EC5 are satisfied for all the samples and some results that fulfill the two normative requirements are obtained (44.63 %). The marginal histograms of the maximum accelerations and the first natural frequencies are also presented in the figure. As can be observed, the range of variation of the natural frequencies is higher than the amplitude of variation of the peak accelerations. Hence, the traditional analysis with a mean model would be inadequate in the frequency zone in which the footbridge could be excited for a fast walking with the third harmonic or for a normal walking with the fourth harmonic of the force coincident with the first natural frequency of the footbridge. The variation of the material properties could lead to an unacceptable serviceability performance of the structure.

CONCLUSIONS

A stochastic study of the structural behaviour of a timber footbridge was presented. The material properties and the assumptions of the model were presented. The stochastic analysis allows us to extend the range of the response starting from a model with mean properties. The analysis of the results obtained from the mean model allows us to understand and compare the effect in the structure of the two load models considered in this work. The first load model is frequently employed in the design practice. The second load model introduces the heel impact effect and a more realistic description of the human walking. This approach conduces to higher values of displacements and accelerations than the first model. In comparison with footbridges structures with similar natural frequencies and different materials, the timber footbridge presented here exhibits similar levels of accelerations while the displacements are smaller. This effect can be explained with the high relation stiffness/weight of the timber material.

The influence of the finger joints distances in the natural frequencies was presented. From a visual survey carried out in structural size laminated beams the PMF of the distance between finger joints was constructed. The variation in the effective stiffness and mass along the elements provoked by the union of boards with different properties produces a lower standard deviation in the natural frequencies than when the finger joints union are not considered. The mean values of the natural frequency remain equal and are not influenced by the laminated beam model.

The serviceability performance of the structure proposed in this work was also analyzed. Two limit values of peak accelerations according to the standards EUROCODE 5 (2004) and ISO 10137 (2005) for outdoor footbridges were considered. For a 5 % of damping ratio, the peak accelerations in some samples are higher than the EC5 limit. This occurs when the structure is subjected to a fast walking with the third harmonic component of the forces coincident with the fist natural mode of the footbridge. When the fourth harmonic of the walking force was coincident with the first natural frequency the footbridge, the requirement of the EC5 is fulfilled but not the requirements of the standard ISO 10137 for a normal walking. When the damping ratio was changed to 7 %, the requirement of the EC5 is satisfied and some results that fulfill the two normative requirements are obtained.

The marginal histograms of the maximum accelerations and the first natural frequencies are also presented. The range of variation of the natural frequencies is higher than the amplitude of variation of the peak accelerations. But for certain range of the first natural frequencies, the excitation condition can change and some results with higher values of acceleration appear. Hence, the traditional analysis with a mean model would be inadequate in the frequency zone in which the footbridge could be excited for a fast walking with the third harmonic or for a normal walking when the fourth harmonic of the force is close to the first natural frequency of the footbridge. The variation of the material properties can lead to an unacceptable serviceability performance of the structure in some cases.

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