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Nonstationary Gaussian process as surrogate for uncertainty and sensitivity analysis in hydraulic fracture problem

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Abstract: Complex computer models are widely used to predict the behavior of complex physical phenomena. However, this computer models become too expensive when taking into account variabilities present in the physical properties. To circumvent this problem, the expensive computer model is replaced by the inexpensive mathematical function, called surrogates. In this work, we use the nonstationary multivariate Gaussian process (MGP) as surrogate to predict the evolution of hydraulic fracture (HF) . To access uncertainty and sensitivity in the fracture evolution, we use the statistical approaches such as the uncertainty quantification and the global sensitivity analysis. The mathematical model of the HF relies on complex non-linear and free boundary problems simulated here, through an implicit level set algorithm. The numerical examples are presented to show the efficiency of the MGP to predict the evolution of the HF in the rock which presents discontinuities and uncertainty in the rock properties. The MGP results are compared to the Monte Carlo method(MC) and the stochastic collocation method (SCM).

Keywords: Nonstationary, Gaussian Process, Hydraulic Fracture , Uncertainty

1 INTRODUCTION

Hydraulic fracture generally takes place in complex geological formation and characterized by a set of parameters that may not be exactly known or measure with uncertainties. Therefore, assess uncertainty and sensibility of the geological rock medium can help to understanding the evolution of the hydraulic fracture. However when the number of parameters is large, and the rare events are need to be predict, the computational cost may become prohibitive. To avoid the problem of huge calculation time in uncertainty and sensitivity analysis, we replace the complex computer code by a mathematical approximation, called surrogate model or metamodel. The literature presents several models built using: the Gaussian process, polynomials function, and neutral networks.

In this work we follow the approach developed by [1] . This approach proposes the use of the nonstationary Multivariate Gaussian process(MGP) as surrogate to approximate the complex computer codes. The non-stationary Gaussian process is obtained by dividing adaptively the input space into disjoint subspace and fitting separate Multivariate Gaussian process models within each subspace. The Multivariate Gaussian process is obtained by considering that all output has the same covariance function. The surrogate obtained is used to facilitate the application of Uncertainty Quantification (UQ) and Global sensitivity analysis (GSA).

The UQ approach is used to know the variabilities or the uncertainties on the model outputs and to reduce uncertainties on the model output prediction. There are several methods used in literature for UQ analysis. Methods such as Monte Carlo (MC), general Polynomial Chaos(gPC) [2], Multi-element generalized Polynomial Chaos(ME-gPC)[3], stochastic collocation, Adaptive stochastic collocation (ASCM) [4] present some difficulties related to computational demand, convergence, and the calculation of some statistical information. The GSA approach is different to the UQ, this approach quantifies the influence of uncertain input variable or group of input variables on the variability of model output values. In literature [5-6], the GSA approach is used to understanding the model behavior and identify parameters that significantly affect the model output. It can be used also to model calibration. Here we present two methods such as the Sobol'/Saltelli method and the Morris One-At-Time (OAT) sensitivity method. Using the Morris and Sobol'/Saltelli sensitivity method we will provide additional physical information to better understand the evolution of HF. The GSA explore the parameter space so that they provide robust sensitivity measures compare to the local sensitivity analysis. One of the limitation of the use of these important statistical methods is the computer time. Herein the Gaussian surrogate constructs is use to replace the computer code which decreases the computer cost and facilitate the application o these statistical methods.

The work is divided in 5 sections: In section 1, we present the mathematical model used to describe the evolution of HF in the rocky medium. In section 3 we describe the construction of the Nonstationary Gaussian process as surrogate. The results are presented in section 4 and the conclusion in section 5.

2 HYDRAULIC FRACTURE

The fracture, after its initiation, is propagated by the injection, through an well, of a highly pressurized incompressible Newtonian fluid with dynamic viscosity μ. The direction of propagation is represented by the coordinate χ and the domain occupied by the fracture along the time τ is described by the interval $\ell(t) = [\ell_{low}(\tau), \ell_{up}(\tau)]$, what is schematically depicted in Fig 1. Moreover, we assume linear elastic fracture mechanics, that the fracture grows in a limit equilibrium regime. The whole process is controlled by the injection rate Q_0 and the multiple physical process at the fracture tip and geological conditions imposed by the rock formation, namely: the confining stress σ, perpendicular to the fracture aperture direction, and the four material parameters E', μ', K', and C' defined as: $E' = \frac{E}{1-\nu^2}$; $\mu' = 12\mu$; $K' = 4(\frac{2}{\pi})^{\frac{1}{2}} K_{ic}$; $C' = 2C_l$. Here, E' is the plane strain modulus, μ' the alternate viscosity, C' the leak-off coefficient and K' the rock toughness. The governing equations are presented below:

Elasticity Equation

$$\Pi = \Pi_f(\chi,\tau,\varpi) - \Sigma_0(\varpi)\Phi(\chi,\varpi) = \frac{-G_E}{4\pi}\int_{-\ell_{low}(\tau,\varpi)}^{\ell_{up}(\tau,\varpi)} \frac{\Omega(\chi',\tau,\varpi)}{(\chi-\chi')} d\chi', \tag{1}$$

where Π stands for the nondimensional net pressure.

Lubrication Equation:

$$\frac{\partial\Omega(\chi,\tau,\varpi)}{\partial\tau} + G_c\frac{H(\tau-\tau 0(\chi))}{\sqrt{(\tau-\tau_0(\chi))}} = G_m\frac{\partial}{\partial\chi}[\Omega(\chi,\tau,\varpi)^3\frac{\partial\Pi_f(\chi,\tau,\varpi)}{\partial\chi}] + G_v\Psi(\tau,\varpi)\delta_0(\chi), \tag{2}$$

where ϖ represents the stochastic dimension.

Boundary and Propagation Conditions:

$$\begin{aligned}\lim_{s\to 0}\Omega^3(\chi,\tau,\varpi)\frac{\partial\Pi_f(\chi,\tau,\varpi)}{\partial s} &= 0,\\ \Omega(\ell_{low,up},\tau,\varpi) &= 0,\end{aligned} \tag{3}$$

$$\lim_{s\to 0}\frac{\Omega}{s^{1/2}} = G_k, \tag{4}$$

where $s = \ell_{low,up}(\tau) - \chi$ is a local coordinate representing the distance from a fracture interior point to the fracture tip.
Dimensionless quantities G_j resulting from the above definitions are: $G_E = \frac{p_*\ell_*}{E'w_*}$, $G_m = \frac{\mu' Q_0}{w_*^3 p_*}$, $G_v = \frac{Q_0 t_*}{w_*\ell_*^2}$, $G_c = \frac{C'\ell_*^2}{Q_0 t_*^{1/2}}$, $G_k = \frac{K'\ell_*^{1/2}}{E'w_*}$.

3 NONSTATIONARY GAUSSIAN PROCESSES AS SURROGATES TO COMPUTATIONAL MODELS

The nonstationary GPs are important to capture the response of the function in some parts of input space in which the function may vary quickly. These approaches are important in the physical problems which take place in the heterogeneous area, like the evolution of fracture in the so-called "stress-jump" in which the fracture trespass a boundary between layers with different confining stresses. Here the nonstationary GP is obtained using several locally stationary MGPs obtained adaptively by the decomposition of the input space into I disjoint subspaces X^i. The input space is represented by the set of reservoir rock properties like (Elasticity modulus, confining stresses,..)

The statistics of any Gaussian process $f(.)$ is completely specified through its mean and covariance functions. Considering that the simulator is represented by the Gaussian process and as a mapping, connecting inputs x (material, initial conditions,..) and outputs y (fracture lengths and aperture) are denoted as $f(x)$ with a K-dimensional input $\mathbf{x} \in X \subset \mathbf{R}^K$, i.e. $X = \times_{k=1}^K [a_k, b_k]$, $-\infty \leq a_k < b_k \leq \infty$. To reflect uncertainties in the input parameters, $\mathbf{x}$ is considered a random vector with probability density $p(\mathbf{x})$ defined as $\prod_{k=1}^K p_k(x_k)$, assuming independency between the individual inputs. Indeed, here we are considering only uncertain inputs. The rest is considered known with certainty and keep fixed along the analysis. The multiple outputs of the simulator are organized in a vector form: $\mathbf{y} \in Y \subset \mathbf{R}^M$, where M represents the number of outputs. At this point, it is important to realize that the components of this vector might have different physical meanings (pressure, velocity or length,..), which requires normalization before the construction of the simulator surrogate. The simulator is run at selected training points $X = (\mathbf{x}^1, ..., \mathbf{x}^N)$, leading to the training outputs $Y = (\mathbf{y}_r^1 = \mathbf{f}(\mathbf{x^1}), ..., \mathbf{y}_r^N = \mathbf{f}(\mathbf{x}^N))$ where $r = 1, .., M$. We assume that we have observed a fixed number $N \geq 1$ and training set $\mathbf{D} = \{(X, Y)\}$. The training outputs Y are used to compute mean and standard deviation of the observations, such that the observed mean is:

$$\mu_{obs,r} = \frac{1}{N}\sum_{n=1}^N \mathbf{y}_r^n, \tag{5}$$

and the observed variance is:

$$\sigma_{obs,r}^2 = \frac{1}{N}\sum_{n=1}^N (\mathbf{y}_r^n - \mu_{obs,r})^2, \tag{6}$$

the scaled version z_r of the observation is used to put all output in the same signal strength, defined by:

$$z_r^n = \frac{y_r^n - \mu_{obs,r}}{\sigma_{obs,r}}. \tag{7}$$

From a Bayesian perspective, we regard $\mathbf{f}(.)$ as an unknown function and consider that the prior information about $\mathbf{f}(.)$ is a Gaussian Process (GP) conditioned on hyper-parameters. The posterior distribution of $\mathbf{f}(.)$ is then regarded as the surrogate noted by:

$$\mathbf{f}(.) \sim \mathbf{GP}(\mathbf{m}(\mathbf{x}), C(\mathbf{x}, \mathbf{x'}; \Theta)), \tag{8}$$

where $\mathbf{m}(\mathbf{x})$ and $C(\mathbf{x}, \mathbf{x'}; \Theta)$ are the mean and the covariance function of the GP and Θ unknown hyper-parameters.

The simulator is run at selected training points $X = (\mathbf{x}^1, ..., \mathbf{x}^N)$, leading to the training outputs $Y = (\mathbf{y}_r^1 = \mathbf{f}(\mathbf{x^1}), ..., \mathbf{y}_r^N = \mathbf{f}(\mathbf{x}^N))$ where $r = 1, .., M$. We assume that we have observed a fixed number $N \geq 1$ and training set $\mathbf{D} = \{(X, Y)\}$. The training outputs Y are used to compute mean and standard deviation of the observations, such that the observed mean is:

$$\mu_{obs,r} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{y}_r^n, \tag{9}$$

and the observed variance is:

$$\sigma_{obs,r}^2 = \frac{1}{N} \sum_{n=1}^{N} (\mathbf{y}_r^n - \mu_{obs,r})^2, \tag{10}$$

the scaled version z_r of the observation is used to put all output in the same signal strength, defined by:

$$z_r^n = \frac{y_r^n - \mu_{obs,r}}{\sigma_{obs,r}}. \tag{11}$$

The squared exponential covariance is used as the covariance function which is expressed as:

$$C(\mathbf{x}_n, \mathbf{x'}_m; \Theta, \mathbf{g}) = C_{SE}(\mathbf{x}_n, \mathbf{x'}_m; \Theta) + \mathbf{g}\delta_{n,m}, \tag{12}$$

where $\delta_{n,m}$ is the Kronocker delta which permits adding **g** in the diagonal element of the matrix C_{SE}, herein, we consider that $\mathbf{g} = 10^{-6}$ [1].

The hyperparameters $\Theta = (\mathbf{s_f}, \ell_\mathbf{1}, ..., \ell_\mathbf{k})$ are obtained by maximizing the logarithm of the marginal likelihood. Since the scaled responses are conditionally independent given Θ, the marginal likelihood is simply the sum of the marginal likelihood of each output and the GPs becomes the multivariate Gaussian process MGPs:

$$\ell(\Theta) = -\frac{1}{2} \sum_{r=1}^{M} z_r^T C^{-1} z_r - \frac{M}{2} \log |C| - \frac{NM}{2} \log 2\pi, \tag{13}$$

where $C = C_{nm} = C(\mathbf{x}_n, \mathbf{x'}_m; \Theta)$, $\{n, m\} = 1, .., N$, and $|C|$ its determinant.
Thus Θ is obtained by maximize $\ell(\Theta)$:

$$\Theta^* = \arg \max_{\Theta} \ell(\Theta), \tag{14}$$

The maximum is obtained by a Conjugate Gradient method(CG). Knowing the hyperparameters Θ^*, we can calculate the predictive statistics of outputs such as mean, error bar, and the probability distribution(PDF) at any test point $\mathbf{x*} \in X \subset \mathbf{R}^K$. The predictive statistics of order q are obtained by considering that the power q of the response is also a MGPs conditional by the hyper -parameters Θ^q. Let us write the predictive distribution for the q power of the response at **x** as

The predictive distribution:

$$m_r^q | D, \theta^q \sim N(\mu_{m_r^q}; \sigma_{m_r^q}^2), \tag{15}$$

predictive mean:

$$\mu_{m_r^q} = \int_{\mathbf{X}} \mu_{\mathbf{f_r^q}}(\mathbf{x}; \Theta^q) p(\mathbf{x}) d\mathbf{x}, \tag{16}$$

predictive error bar:

$$\sigma_{m_r^q}^2 = \int_{\mathbf{X}} \sigma^2{}_{\mathbf{f_r^q}}(\mathbf{x}; \Theta^q) p(\mathbf{x}) d\mathbf{x}. \tag{17}$$

Those integrals can be computed with the aid of Monte Carlo schemes. It can be also calculated also using analytical solutions when the random value **x** is uniformly distributed. The error is used to construct adaptively nonstationary MGPs.

4 GLOBAL SENSITIVITY ANALYSIS

Global sensitivity analysis (GSA) explores the input parameters space and provides robust sensitivity measures in the presence of nonlinearity and interactions between input parameters. Typically, GSA is used to have a better understanding of the complex physical model and to simplify the model by eliminating the input parameters which have insignificant contributions to the outputs variability. Herein, we use the GSA to better understand the dynamics of evolution of a hydraulic fracture in a rocky medium formed by three layers with different values of confining stresses depicted in Fig 2.

The literature presents different GSA methods[7]. Here, we use a Variance-Based method ,namely, the Sobol'/Satelli method [8] and a difference-based method, namely, the Morris method[9]. Both methods provide the sensitivity of the parameters using different strategies. The Sobol method provides the quantitative measure of sensibility, it can give an additional important information coupling with UQ method. The Morris method provides the effect of the parameters in the evolution of the physical model. This method can help to understand the effect of each parameter in the complex physical model evolution.

The Sobol'/Saltelli method analyses the impact of the input parameters by calculating the first and total sensitivity index S_k and S_{tk}. The Morris one At time analyses the effect of each input parameter by computing the mean μ_{EE} and the standard deviation σ_{EE}.

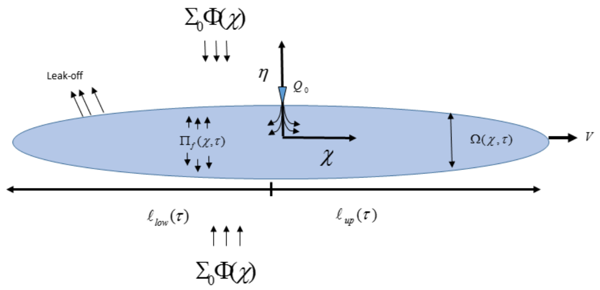

Figure 1 – Schematic view of the fracture in rocky medium

5 RESULTS

In order to assess the performance of Gaussian surrogate and the application of this surrogate to UQ and GSA in the problem of hydraulic fracture. We study a challenging HF problem in which the fracture propagates vertically within a three layered medium, as described schematically in see Fig 2. In this complex geological formation, the fracture growth tends to experiement a non-symmetric evolution due to abrupt changes in the confining stresses which assumes different value at each layer (Σ_0^1 for the layer 1, Σ_0^2 layer 2 and Σ_0^3 for layer 3). The layers are separated by interfaces χ_1 and χ_2, that the fracture will pass through at unknown injection times. In such a scenario, numerical simulations of the fracture propagation might help the operation in the field, trying to reduce the risks or improving the stimulation.

In this particular example, we elected the Elasticity Modulus and the confining stresses on each layer as uncertain parameters. We analyse the efficiency of MGPs compared to Monte Carlo method and stochastic collocation. We consider that the solution of MC obtained at 30,000 samples as reference solution. We construct the surrogate considering the fracture length obtained at upper and lower side of the fracture at each injection time τ, $\ell_{up}(\tau)$, $\ell_{low}(\tau)$.

In Figs 4-5 we observe that the MGPs surrogate provide the lower absolute error using few call of the computer code compare to the stochastic collocation method. The Gaussian surrogate construct is used to facilitate the GSA analysis.

To deeper understand the evolution of a fracture facing stress-jump as it evolves through the interfaces χ_1 and χ_2, we consider as outputs, the normal velocities of fracture propagation at both side of fracture tips: V_{lower} and V_{upper} obtained at each injection time τ. In the Fig 6, we present the evolution of V_{lower} and V_{upper}, this figure presents the velocities evolution obtained using the mean value of the input parameters $E[G_E], E[\Sigma_0^1], E[\Sigma_0^2], E[\Sigma_0^3]$ presented in the previous section. We observe that at $\tau = \tau_{p1} \simeq 128$, the front velocities V_{lower} and V_{upper} change abruptly, at this time the velocities diverge. The V_{lower} is decreasing and the V_{upper} keeps constant until to present an abrupt change at the time $\tau = \tau_{p2} \simeq 228$ and takes a constant value greater than the last values. These abrupt changes indicate that the fracture reached the interfaces located at the positions χ_1 and χ_2. The interface χ_2 closer to the point of injection $\chi = 0$ and between the layer 2 and 3 was reached at time τ_{p1} and the interface χ_1 between the layer 1 and 2 was reached by the fracture at τ_{p2}.

To analysis the sensitivity of the uncertainties parameters listed bellow we construct the surrogate of the fracture front velocities $V_{lower}(\tau)$ and $V_{upper}(\tau)$ The surrogate is constructed assuming $\delta = 10^{-7}$ and M=170 i.e τ is discretized at 70 time steps defined by $d\tau = 3.9$, $\tau = 50.72 : d\tau : 319.82$. The surrogate is obtained after 8 divisions of the input space and with the total number of computer code call Nt=88.

In Fig 7, we present Morris sensitivity coefficients μ_{EE} related to inputs $G_E, \Sigma_0^1, \Sigma_0^2, \Sigma_0^3$ on V_{low}. We observe that before that the fracture reaches the interface χ_2 at the time τ_{p1}, the input parameter G_E is the parameter which has the greatest contribution in the fracture evolution on the lower side. When the fracture trespasses interface χ_2, the confining stresses Σ_0^2 and Σ_0^3 become the inputs which presents the main influence of the evolution of the fracture on the lower side. The confining stress Σ_0^1 and G_E have the less significative effect on the fracture velocity V_{lower} when the fracture reaches the interfaces.

In Fig 8 we observe that until time $\tau < 150$, only the elasticity modulus has a significant effect on V_{upper}. At times $\tau > 150$, we observe that all the input parameters have an influence on the fracture evolution of the upper side. the confining stress Σ_0^2 resists to the fracture evolution in the time interval [150, 250] i.e that V_{upper} will decrease if the confining stress Σ_0^2 increases in this time interval. At the approximate times $\tau > 250$, the confining stress Σ_0^2 contributes positively to the fracture evolution and decreases at the time $\tau = \tau_f$. The confining stress Σ_0^1 resists also to the fracture evolution but at time $\tau > 250$ this resistance decreases. The confining stress Σ_0^3 has a positive sign of μ_{EE}. The contribution of the G_E increases when the fracture reaches the interface χ_2 and decreases at the time $\tau > 250$.

In Figs 9 - 10, we show the evolution of the total Sobol sensitivity indices S_{tk} on the outputs $V_{lower}(\tau)$ and $V_{upper}(\tau)$. We observe that S_{tk} presents the same behavior on the contribution of each input parameters on the outputs V_{lower} and V_{upper}. However, the Sobol method presents the contribution of each input parameters on the uncertainty of the model outputs.

In Fig 9, we observe that when the fracture does not reach the interface χ_2, the elasticity modulus is the input parameter which controls the variance on the output V_{lower}. When the fracture reaches the interface χ_2, the contribution of the elasticity modulus G_E on the total variance of V_{lower} decreases and at $t > 148$, the contribution of the inputs Σ_0^2 and Σ_0^3 on the total variance of V_{lower} increases, respectively, from 10% to 72% for Σ_0^2 and from 10% to 20% for Σ_0^3. The contribution of the inputs G_E and Σ_0^1 on the total variance of V_{lower} decreases from 10% to 3% for Σ_0^1 and from 70% to 5% for G_E.

In Fig 10 we can observe that the elasticity modulus G_E, the confining stresses Σ_0^2 and Σ_0^1 contribute highly on the variance of the output V_{upper} in the different time intervals. At the time τ between the interval $[50, 155]$, the elasticity modulus has approximately 65% of contribution to the variance of V_{upper}. In the interval [155, 250], the confining stress Σ_0^2 has the greatest contribution to the variance of the output V_{upper} and takes the maximum value at the time $\tau = 200$ which is 67%. In the time interval [250, 339], the confining stress Σ_0^1 controls the variance of V_{upper} with 50% in contribution of variance of V_{upper}.

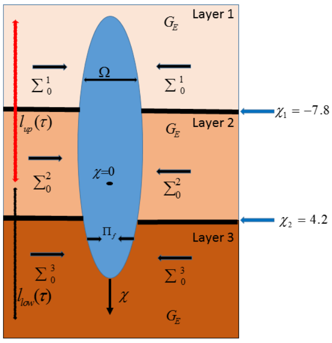

Figure 2 – Schematic view of the layered medium

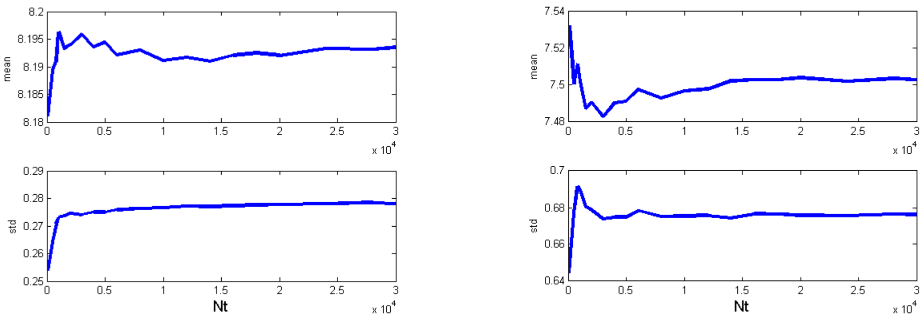

Figure 3 – Convergence in mean and standard deviation of the fracture lower and upper length ℓ_{low} and ℓ_{up} at $\tau = 240$ using MC

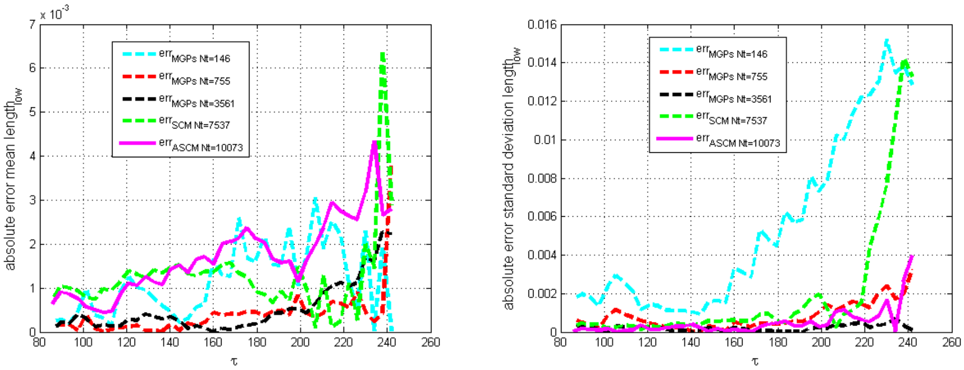

Figure 4 – Evolution of the absolute error calculate using the mean and standard deviation of $\ell_{low}(\tau)$ obtained by the sampling of MGPs and the standard deviation of reference solution obtained by using MC at Nt=30,000

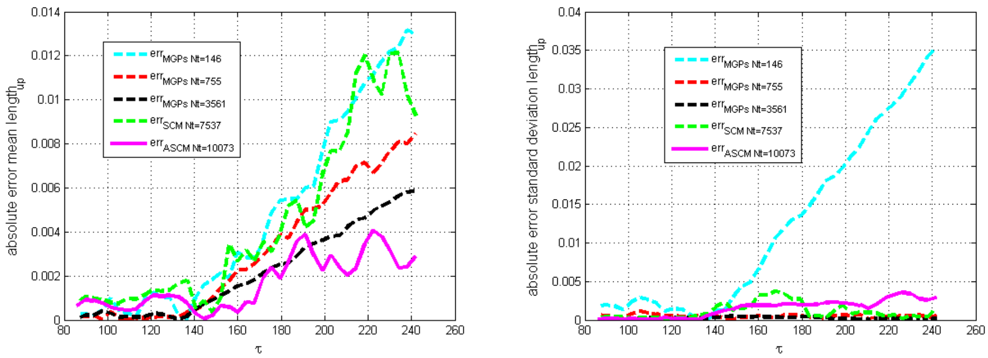

Figure 5 – Evolution of the absolute error calculate using the mean and standard deviation of $\ell_{up}(\tau)$ obtained by the sampling of MGPs and the standard deviation of reference solution obtained by using MC at Nt=30,000

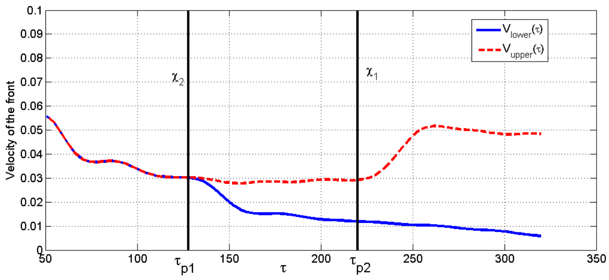

Figure 6 – The normal velocity of the front on upper and lower sides of the fracture.

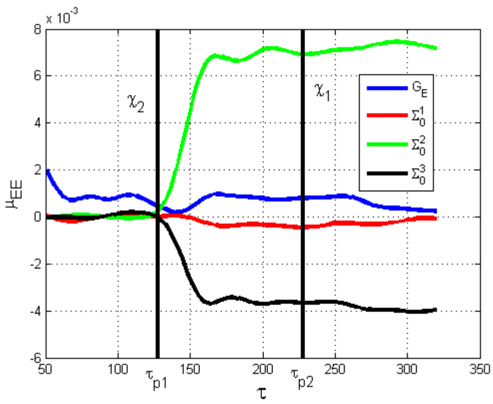

Figure 7 – Time evolution of the Morris μ_{EE} with respect to the normal velocity of the front on lower part of the fracture: V_{low}.

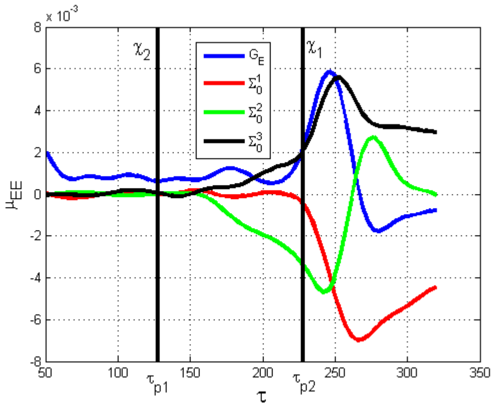

Figure 8 – Time evolution of the Morris μ_{EE} with respect to the normal velocity of the front on lower part of the fracture: V_{lup}.

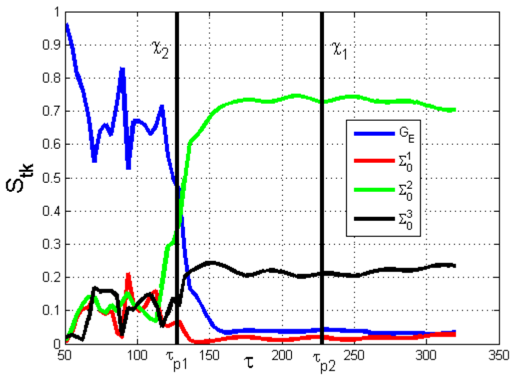

Figure 9 – Time evolution of the Sobol index S_{tk} with respect to the normal velocity of the front on lower part of the fracture: V_{low}.

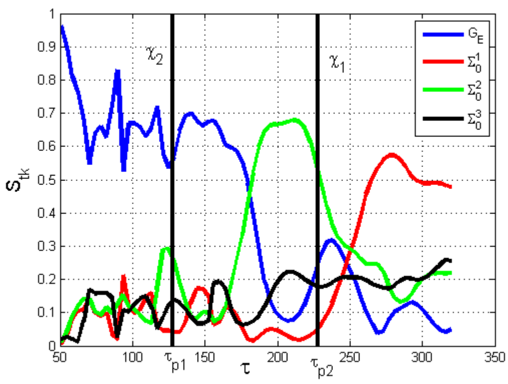

Figure 10 – Time evolution of the Sobol index S_{tk} with respect to the normal velocity of the front on lower part of the fracture: V_{lup}.

6 UQ AND GSA

In this section we present the statistics on the fracture lengths and the contribution of each input parameter on the variance of fracture lengths. The statistics such as mean and variance of the fracture lengths $\ell_{low}(\tau)$ (μ_{low} and var_{low})and $\ell_{upper}(\tau)$ (μ_{lup} and var_{lup}) are calculated using UQ. To know the contribution of each stochastic input $G_E, \Sigma_0^1, \Sigma_0^2, \Sigma_0^3$ on the calculation of the variance of $\ell_{low}(\tau)$, $\ell_{upper}(\tau)$, we calculate the total sensitivity coefficient S_{tk} of each input parameter $S_{tG_E}, S_{t\Sigma_0^1}, S_{t\Sigma_0^2}, S_{t\Sigma_0^3}$ using Sobol's method.

The surrogate is constructed by considering as outputs, the lengths of fracture $\ell_{low}(\tau)$ and $\ell_{upper}(\tau)$. We consider that the fracture begins at time $\tau_0 = 11.72$ and stops at time $\tau_f = 319.82$, with a time step $d\tau = 3.9$. Therefore, the surrogate is build with M=160 outputs, we choose $\delta = 10^{-5}$. We obtain the surrogate after 28 divisions of stochastic space and Nt=316.

In Table 9 - 10, we present the statistical results of the fracture lengths and the contribution of each input parameter on the calculation of the variance. These results are important for the construction of the fracture design project.

ℓ_{low}	UQ (surrogate MGPs)	GSA (Total Sobol sensitivity coefficient %)
$\tau = 70.22$	μ_{low}=3.6, var_{low}=0.001	$S_{tG_E} = 98.88, S_{t\Sigma_0^1} = 0.15, S_{t\Sigma_0^2} = 0.37, S_{t\Sigma_0^3} = 0.57,$
$\tau = 124.82$	μ_{low}=5.22, var_{low}=0.004	$S_{tG_E} = 91.40, S_{t\Sigma_0^1} = 0.12, S_{t\Sigma_0^2} = 6.72, S_{t\Sigma_0^3} = 1.74,$
$\tau = 202.82$	μ_{low}=6.85, var_{low}=0.2	$S_{tG_E} = 12.14, S_{t\Sigma_0^1} = 0.39, S_{t\Sigma_0^2} = 65.95, S_{t\Sigma_0^3} = 21.52,$
$\tau = 319.82$	μ_{low}=8.70, var_{low}=1.37	$S_{tG_E} = 9, S_{t\Sigma_0^1} = 0.39, S_{t\Sigma_0^2} = 67.90, S_{t\Sigma_0^3} = 22.69,$

Table 1 – Total sensitivity $S_{tk}(\%)$ and UQ results of the fracture length $\ell_{low}(\tau)$.

ℓ_{up}	UQ (surrogate MGPs)	GSA (Total Sobol sensitivity coefficient %)
$\tau = 70.22$	μ_{lup}=3.6, var_{lup}=0.001	$S_{tG_E} = 98.88, S_{t\Sigma_0^1} = 0.15, S_{t\Sigma_0^2} = 0.37, S_{t\Sigma_0^3} = 0.57,$
$\tau = 124.82$	μ_{lup}=5.22, var_{lup}=0.005	$S_{tG_E} = 98.56, S_{t\Sigma_0^1} = 0.10, S_{t\Sigma_0^2} = 0.74, S_{t\Sigma_0^3} = 0.57,$
$\tau = 202.82$	μ_{lup}=7.22, var_{lup}=0.014	$S_{tG_E} = 54.80, S_{t\Sigma_0^1} = 0.40, S_{t\Sigma_0^2} = 34.58, S_{t\Sigma_0^3} = 10.22,$
$\tau = 319.82$	μ_{lup}=11.65, var_{lup}=0.31	$S_{tG_E} = 12.28, S_{t\Sigma_0^1} = 8.56, S_{t\Sigma_0^2} = 31.04, S_{t\Sigma_0^3} = 48.10,$

Table 2 – Total sensitivity $S_{tk}(\%)$ and UQ results of the fracture length $\ell_{up}(\tau)$.

7 CONCLUSION

In this work, the we show the efficiency of MGPs to calculate with low computer cost the statistics of the HF in the complex rocky medium with the presence of a discontinuity. The Gaussian surrogate constructs here, is used to calculate the sensitivity index with the low cost and helps to understand the importance of each parameter in hydraulic fracture evolution between complex reservoir rock. In the future work, we will use the MGPs to the Optimization under Uncertainty in the HF design.

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