Proceedings of the 3rd International Symposium on Uncertainty Quantification and Stochastic Modeling
D.A. Rade; A. M. G. Lima (Editors), ABCM, Maresias, SP, Brazil, February 15-19, 2016

Interval Kinematic and Dynamic Analysis of a Planar Parallel Kinematic Manipulator

R.F. Bastos [1], J.V.C. Fontes [1], and M. M. daSilva [1]

[1] Department of Mechanical Engineering, São Carlos School of Engineering, University of São Paulo, Av. Trabalhador Sancarlense, 400. CEP: 13566-590, São Carlos SP, Brazil

Abstract: Interval analysis can be exploited to determine upper and lower bounds of any function considering scattering parameters. In robotics, this method can be employed to evaluate clearance effects, to treat calibration and to identify accuracy issues. This approach can support the design of collision-free trajectory planning among several others possible applications. In this manuscript, this technique is employed to evaluate the impact of some scattering parameters on the kinematic and dynamic performance of a planar parallel kinematic manipulator, the 3RRR. Parallel kinematic machines may suffer from singularities in the workspace, calibration and accuracy issues. Scattering geometric parameters and/or dynamic properties may have a huge impact on the kinematic and dynamic performance of this kind of manipulators. This impact is going to be assessed by employing interval analysis to evaluate the kinematic and dynamic performance of the manipulator using direct and inverse kinematic and dynamic models. This analysis is performed using INTLAB, an internal analysis toolbox in MATLAB. Using this technique, one can infer which design parameters should be close monitored during their manufacturing since any scattering may cause important performance variations.

Keywords: Parallel kinematic manipulators, interval analysis, kinematic and dynamic analysis.

NOMENCLATURE

A = active joint position
B = passive joint position
C = end-effector joint position
J = Jacobian matrix
M = inertia matrix
V = Coriolis matrix
l_1 = link length, m
l_2 = link length, m
m_1 = link mass, kg
m_2 = link mass, kg
m_{ee} = end-effector mass, kg
h = distance between vertices and barycenter, m

Greek Symbols

β_i = angle joint B_i, rad.
θ_i = angle joint A_i, rad.
α = angle end-effector, rad.
γ_i = complementary angle α to *h*, rad.
τ = torque, N.m

Subscripts

i = relative of the links *i*.
j = relative of the chain *j*.

INTRODUCTION

Parallel kinematic machines (PKM) have attracted the attention of academic and industrial communities due to their advantages over serial architectures. Among these advantages, it can be mentioned the lightness, the high speed/acceleration, the rigidity and the load capacity (Merlet, 1996). The most promising industrial application for these alternative architectures is the pick-and-place operation required in the food, pharmaceutical and electronic industries (da Silva & Gonçalves, 2013; da Silva *et al.*, 2010, among others).

Regarding the performance of PKMs, the scattering of some parameters may have a huge impact. In this way, these parameters should be clearly identified. The most relevant parameters should be carefully selected and designed in order to avoid unwanted issues on accuracy and performance regarding kinematics and dynamics of the understudy robotic system. For this reason, it is important to investigate the mechanism performance sensitivity regarding its geometric and dynamic properties (Caro *et al.*, 2009). For instance, Binaud *et al.* (2011) and Caro *et al* (2009) have compared the performance and its sensitivity regarding uncertain geometry of a 3-RPR manipulator. Muller (2010) have addressed the effects of geometric imperfections on the control performance of redundantly actuated parallel kinematic manipulators. Recently, Tannous *et al.* (2014) have evaluated the sensitivity of 3RRR to variations on geometric parameters using interval linearization method.

In this paper, the 3RRR planar parallel manipulator is evaluated regarding some scatter geometric and dynamic parameters (see Fig. 1). The former is evaluated by imposing some variability on the length of the manipulator's links, while the latter is evaluated by considering some variability on the mass and inertia of the manipulator's links. The end-effector's position and the required torque to perform a pre-defined trajectory are evaluated regarding these scatter properties. The 3RRR is composed by three kinematic chains containing three revolute joints each (RRR). The underlined letter highlights the active joint. The manipulator's end-effector presents three degrees of freedom.

Interval Analysis has been employed to study the effects of such variabilities on the kinematic and dynamic performance of the understudied manipulator. The purpose of this technique is to determine guaranteed bounds for the minimum and maximum of a given function over ranges of unknowns (Merlet, 2011). This determination is called interval evaluation. This technique has been employed in the field of robotics with different purposes: the evaluation of clearance effect on robotic manipulators (Wu and Rao, 2004; Chen *et al.*, 2013), the assessment of scatter geometric parameters on the localization and navigation (Ashokaj *et al.*, 2004), the impact of variabilities on trajectory planning (Piazzi and Visioli, 2000), calibration issues regarding uncertainties (Daney *et al.*, 2006), among others.

The numerical methodology employed to model the 3RRR manipulator is briefly described in Section 2. Some concepts regarding Interval Analysis are treated in Section 3. Numerical results are presented in Section 4. Finally, in Section 5, conclusions based on these numerical results are drawn.

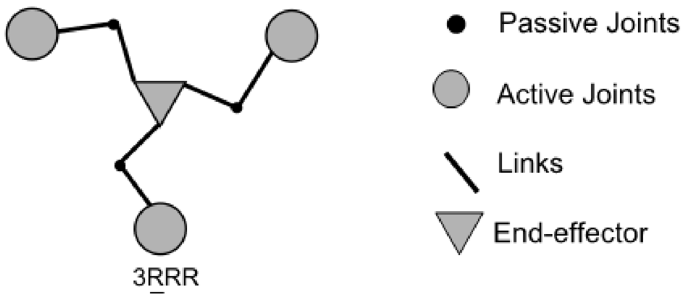

Figure 1 – Planar parallel manipulator 3RRR.

MODELING METHODOLOGY

In this section, the 3RRR kinematic and dynamic models are briefly introduced. Figure 2 shows the manipulator's geometrical characteristics. The position of the end-effector is defined by x, y and α, where x and y are linear and α angular positions. The active joint angle position is defined by θ_i, and β_i is the rotation angle of the passive joint, where *i* represents the chain *1, 2* or *3.* The distance of the vertex C_i , to the end-effector's centroid is represented by h.

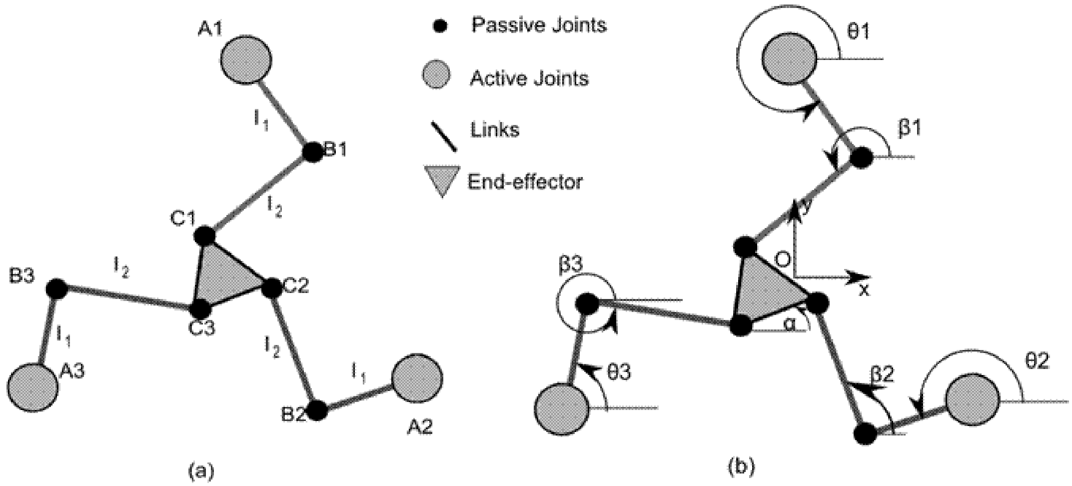

Figure 2 – 3RRR geometrical characteristics: (a) length of the links and (b) joint angles.

Forward Geometry

The forward geometry is used to describe the manipulator geometrically. In order to do that, the geometrical relationships for each chain are described in order to find the position of the end-effector centroid, x_u and y_u, the active joint position, x_{0_i} and y_{0_i}, and the angles θ_i,β_i, α and γ_i. The angle $\alpha + \gamma_i$ is the angle between the horizontal to the joint C_i tangent to the imaginary line that connects this vertex to the centroid. For each chain i, the geometrical relations can be given by:

$$\begin{bmatrix} x_u \\ y_u \end{bmatrix} = \begin{bmatrix} x_{0_i} \\ y_{0_i} \end{bmatrix} + l_1 \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix} + l_2 \begin{bmatrix} \cos(\beta_i) \\ \sin(\beta_i) \end{bmatrix} + h \begin{bmatrix} \cos(\alpha+\gamma_i) \\ \sin(\alpha+\gamma_i) \end{bmatrix} \tag{2.1}$$

Inverse Kinematics

The inverse kinematics is evaluated in order to calculate the angles θ_i and β_i for a given position of the end-effector. This can be done by evaluating some geometrical relationships related to the links l_1 and l_2 of the manipulator and the position of the joints and the centroid of the end-effector (Fig. 2).

The position of C_i can be written by:

$$r_{C_i} = r_{B_i} + l_2 \begin{bmatrix} \cos(\beta_i) \\ \sin(\beta_i) \end{bmatrix} \tag{2.2}$$

where $r_{B_i} = (x_{B_i}, y_{B_i})$ and $r_{C_i} = (x_{C_i}, y_{C_i})$. The equation (2.1) can be described by:

$$\left\| r_{C_i} - r_{B_i} \right\| = l_2 \tag{2.3}$$

Using the definition of μ_i and ρ_i (Eq. (2.4)), Eq. (2.3) can be rewritten as Eq. (2.4):

$$\begin{bmatrix} \mu_i \\ \rho_i \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + h \begin{bmatrix} \cos(\alpha+\gamma_i) \\ \sin(\alpha+\gamma_i) \end{bmatrix} + \begin{bmatrix} x_{0_i} \\ y_{0_i} \end{bmatrix} \tag{2.4}$$

$$l_2 = \left\| \begin{bmatrix} \mu_i - l_1 \cos(\theta_i) \\ \rho_i - l_1 \sin(\theta_i) \end{bmatrix} \right\| \tag{2.5}$$

Eq. (2.4) describes the following geometrical constraint:

$$-2\mu_i l_1 \cos(\theta_i) - 2\rho_i l_1 \sin(\theta_i) + {\mu_i}^2 + {\rho_i}^2 + {l_1}^2 - {l_2}^2 = 0 \tag{2.6}$$

In order to describe the angles θ_i, one can define:

$$u_{i1} = -2\rho_i l_1 \tag{2.7}$$

$$u_{i2} = -2\mu_i l_1 \tag{2.8}$$

$$u_{i3} = {\mu_i}^2 + {\rho_i}^2 + {l_1}^2 - {l_2}^2 \tag{2.9}$$

Using these definitions, the angles θ_i can be calculated:

$$\theta_i = 2tan^{-1}\left(\frac{-u_{i1} \pm \sqrt{{u_{i1}}^2 + {u_{i2}}^2 - {u_{i3}}^2}}{u_{i3} - u_{i2}} \right) \tag{2.10}$$

Based on the angles θ_i, one can calculate the angle β_i through the Eq. (2.3):

$$l_2 \begin{bmatrix} \cos(\beta_i) \\ \sin(\beta_i) \end{bmatrix} = \begin{bmatrix} \mu_i - l_1 \cos(\theta_i) \\ \rho_i - l_1 \sin(\theta_i) \end{bmatrix} \tag{2.11}$$

$$\beta_i = tan^{-1}\left(\frac{\rho_i - l_1 \sin(\theta_i)}{\mu_i - l_1 \cos(\theta_i)} \right) \tag{2.12}$$

Dynamic

The inverse dynamics is exploited in order to evaluate the required actuator's torque to perform a pre-defined trajectory (end-effector's position, velocity and acceleration). To this end, the Jacobian is firstly defined. This matrix described the relation between the actuators' angular velocities $\dot{\Theta}$ and end-effector's velocities $\dot{X}$. This relation is defined by:

$$\dot{X} = J\dot{\Theta} \tag{2.13}$$

with $X = [x \quad y \quad \alpha]^T$ and $\Theta = [\theta_1 \quad \theta_2 \quad \theta_3]^T$.

The link A_iB_i is rotating around the joint A_i, while the link B_iC_i presents rotational and translational movements. Considering these aspects, the Principle of Virtual Work is exploited to evaluate the required torque for a given set of position, velocities and accelerations. Details can be found in Fontes *et al.* (2014). In order to summarize the technique, the vectors p_{ij} and p_{ee} can be defined. They are composed by forces (F) and moments (M) applied to the body j of the chain i:

$$p_{ij} = \begin{bmatrix} F_{ij} \\ M_{ij} \end{bmatrix} \tag{2.14}$$

where $j=1$ is the link A_iB_i and $j=2$ is the link B_iC_i, and to the end-effector:

$$p_{ee} = \begin{bmatrix} F_{ee} \\ M_{ee} \end{bmatrix} \tag{2.15}$$

Both p_{ij} and p_{ee} can be rewritten according to the variation of the linear position, $r_{x_{ij}}$ and $r_{y_{ij}}$, and its angular position $\phi_{ij} = (\theta_i, \beta_i, \alpha)$:

$$p_{ij} = Z_{ij} \begin{bmatrix} \ddot{r}_{x_{ij}} \\ \ddot{r}_{y_{ij}} \\ \ddot{\phi}_{ij} \end{bmatrix} + N_{ij} \begin{bmatrix} \dot{r}_{x_{ij}} \\ \dot{r}_{y_{ij}} \\ \dot{\phi}_{ij} \end{bmatrix} \tag{2.16}$$

$$p_{ee} = Z_{ee} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\alpha} \end{bmatrix} \tag{2.17}$$

where:

$$Z_{ij} = \begin{bmatrix} m_j & 0 & \dfrac{-m_j l_{ij} \sin\phi_{ij}}{2} \\ 0 & m_j & \dfrac{m_j l_{ij} \cos\phi_{ij}}{2} \\ \dfrac{-m_j l_{ij} \sin\phi_{ij}}{2} & \dfrac{m_j l_{ij} \cos\phi_{ij}}{2} & I_j \end{bmatrix} \tag{2.18}$$

$$N_{ij} = \begin{bmatrix} 0 & 0 & \dfrac{-m_j l_{ij} \cos\phi_{ij}}{2} \\ 0 & 0 & \dfrac{-m_j l_{ij} \sin\phi_{ij}}{2} \\ 0 & 0 & 0 \end{bmatrix} \tag{2.19}$$

and I_j is the inertia of the body j, and is m_1, m_2 and m_{ee} are the masses of each link A_iB_i, B_iC_i and the end-effector, respectively.

The required torque τ to perform a predefined trajectory is:

$$\tau = M\ddot{\Theta} + V\dot{\Theta} \tag{2.20}$$

where:

$$M = J^t Z_n J + \sum_{i=1}^{3} \sum_{j=1}^{3} K_{ij}^t Z_{ij} K_{ij} \tag{2.21}$$

$$V = J^t Z_n \dot{J} + \sum_{i=1}^{3} \sum_{j-1}^{3} K_{ij}^t Z_{ij} \dot{K}_{ij} + \sum_{i=1}^{3} \sum_{j=1}^{3} K_{ij}^t N_{ij} K_{ij} \tag{2.22}$$

The matrices K are defined in Fontes *et al.* (2014).

INTERVAL ANALYSIS

In this manuscript, interval analysis is exploited to evaluate the impact of some scattering parameters on the kinematic and dynamic performance of the manipulator. This performance is evaluated using the abovementioned kinematic and dynamic models. For sake of completeness, a briefly explanation on interval analysis is described hereafter.

Considering the interval set of real numbers F =$[\underline{F}, \overline{F}]$, we can define the function $F(X)$ as $\{F \colon \underline{F} \leq f(X) \leq \overline{F}\}$ for a unknown parameters X. These interval sets presents some interesting characteristics and/or properties (Moore et al., 2009):

- Two intervals are equal only if they have the same extreme, for example: $X = Y$ if $\underline{X} = \underline{Y}$ e $\overline{X} = \overline{Y}$.
- The width of an interval F is defined and denoted by $\overline{F} - \underline{F}$;
- The absolute value of F, denoted $|F|$, is the maximum of the absolute values of its endpoints:

  $$|F| = \max\{|\overline{F}|, |\underline{F}|\};$$
- The midpoint of F is given by $(\overline{F} + \underline{F})/2$.

Classical mathematical operators can be employed when dealing with interval numbers. These operations are defined and implemented so that all possible values of the corresponding interval operation contain all possible results of the corresponding real operation for real arguments inside the interval arguments (Tannous *et al.*, 2014). For example, adding two interval numbers $x = [\underline{x}, \overline{x}]$ e $y = [\underline{y}, \overline{y}]$ is given by $x + y = [\underline{x} + \underline{y}, \overline{x} + \overline{y}]$.

However, interval operations may depend on how the equations are treated. For instance, one may find different results when evaluating $f = x^2 - 4x$ for x belonging to the interval [1, 2]. One can consider that if x is between [1, 2], x^2 is between [1, 4] and $4x$ is between [4, 8], yielding $f = [1,4] + [-8,-4] = [-7,0]$. In a different way, if x is between [1, 2], the value of $(x-2)$ is between [-1, 0] and consequently $(x-2)^2$ is between [0, 1]. Thus, the function $f = [0,1] - 4$ is between [-4, -3]. This shows that the function f with $x = [1,2]$, is between [-4, -3] in both cases, and this implies that the numerical range can be overestimated the maximum and minimum values of the function.

In order to assess the influence of some scattering parameters on the kinematic and dynamic performance of the planar parallel manipulator, an interval analysis have been carried out. In order to do so, some parameters of the 3<u>R</u>RR are considered to have interval values: $l_1 = [\underline{l_1}, \overline{l_1}]$, $l_2 = [\underline{l_2}, \overline{l_2}]$, $m_1 = [\underline{m_1}, \overline{m_1}]$, $m_2 = [\underline{m_2}, \overline{m_2}]$ and $m_{ee} = [\underline{m_{ee}}, \overline{m_{ee}}]$ (see Fig. 2). Consequently, as the masses are varying, the inertias also vary, yielding $I_1 = [\underline{I_1}, \overline{I_1}]$, $I_2 = [\underline{I_2}, \overline{I_2}]$ and $I_{ee} = [\underline{I_{ee}}, \overline{I_{ee}}]$.

Because of the variation on the geometrical and/or inertial properties, the required torque to perform a pre-defined trajectory, τ (Eq. 2.20), can also be expressed according to interval values:

$$torque = \tau = [\underline{\tau}, \overline{\tau}] \tag{2.23}$$

The variation on the dynamic performance of the manipulator is assessed by evaluating the impact of the geometrical and/or inertial properties' variation on this interval (Eq. 2.23).

The end-effector has three degrees-of-freedom: two translational and one rotational degrees-of-freedom. The translational ones are described by the position of the centroid of end-effector, x and y. Due to the scatter geometrical parameters, this position can be represented by interval values, $x = [\underline{x}, \overline{x}]$ and $y = [\underline{y}, \overline{y}]$. These interval values can be treated as a rectangular uncertain area. The area of such region can be calculated by:

$$area = (\overline{x} - \underline{x})(\overline{y} - \underline{y}) \quad (2.24)$$

The variation on the manipulator's accuracy is assessed by evaluating the impact of the geometrical variation on this interval (Eq. 2.24).

INTLAB®, a MATLAB® toolbox, was employed to perform the interval analysis of this work. This toolbox takes into considerations the abovementioned issues.

RESULTS

The 3<u>R</u>RR manipulator is illustrated in Fig. 3. The positions of the actuators 1, 2 and 3 (regarding θ_1,θ_2 and θ_3) are (-0.17,-0.10), (-0.17,0.10) and (0.00,0.20), respectively. The distance of the vertex C_i to the end-effector's centroid is h=50mm. The nominal values of the links' lengths and masses are described in Table 1.

The impact of the variations on the dynamic performance and accuracy of the planar 3<u>R</u>RR manipulator has been evaluated considering a pre-defined trajectory. In this trajectory, the centroid of the end-effector of the manipulator moves from the initial point with coordinate $(-0.04, -0.04{,}0)$ to the final point $(0.04, 0.04, 0)$ in a straight line, with no variation in angle α, as illustrated in Fig. 3. In this trajectory, the initial and final velocities and accelerations are equal to zero.

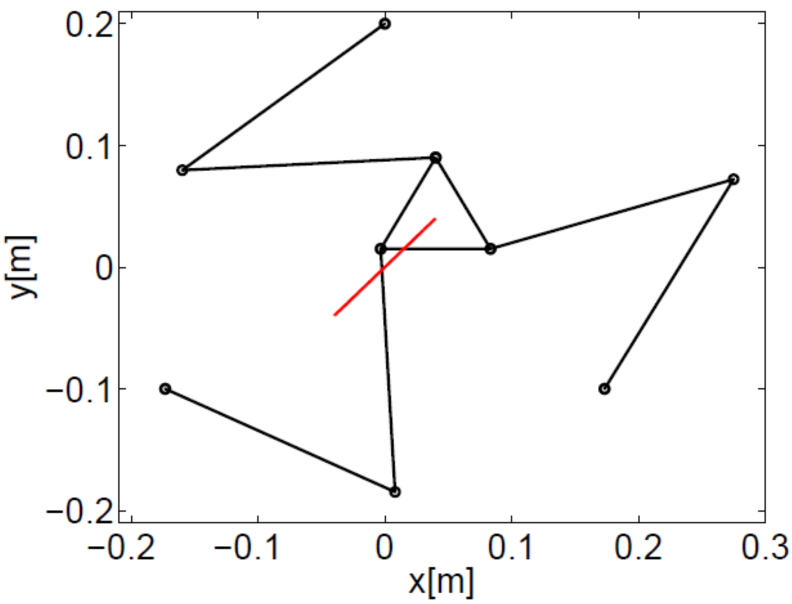

Figure 3 – 3<u>R</u>RR manipulator and trajectory executed.

Table 1 show the nominal and interval values of the masses and length of the 3<u>R</u>RR components.

Table 1 – Mass and length of the 3<u>R</u>RR components, the nominal values and their respective interval values

Parameters	Nominal Value	Interval Value
m_1 and m_2 (kg)	0.2	[0.18,0.22]
m_{ee} (Kg)	0.4	[0.36,0.44]
l_1 and l_2 (m)	0.2	[0.199,0.201]

In Table 1, m_1 and m_2 are the masses of the first and second links of each chain i, respectively, and m_{ee} is the mass of the end-effector. A variation of 10% is allowed for each mass parameter and consequently the inertia varies also 10%. The geometrical parameters, the links' lengths l_1 and l_2 are allowed to vary 0.5%. Higher values would lead to a non-realistic analysis due to singularities in the workspace.

Regarding the computational time, it could be assessed that the geometrical variations leads to more expensive computational evaluation than inertial variation. This may due to the fact that there are more subroutines in the program depending on the geometrical parameters.

Three case studies are considered hereafter.

Case I: The influence of the masses' variation, $\mathrm{m_1}$, $\mathrm{m_2}$ e $\mathrm{m_{ee}}$, on the dynamic performance of the manipulator

As aforementioned, in order to assess the influence of the masses' variation on the dynamic performance of the manipulator, the required torques to perform the selected predefined trajectory are compared (Eq. 2.23). The columns in Fig. 4 depict the required torque for each actuator denoted as 1, 2 and 3 according to the degrees of freedom $\Theta = [\theta_1 \quad \theta_2 \quad \theta_3]^T$. The lines (a), (b) and (c) depict the required torque for each interval values of the evaluated masses $\mathrm{m_1}$, $\mathrm{m_2}$ e $\mathrm{m_{ee}}$, respectively.

According to Figs. 4 (1a, 2a e 3a), one can conclude that variations on the mass m_1 have little impact on the required torque. This is not the case for the analysis considering variations on the masses m_2 and m_{ee}. In these way, it can be concluded that these uncertainties should be closely reduced by using better design and prototyping approaches or should be considered during the design of the manipulator's control system.

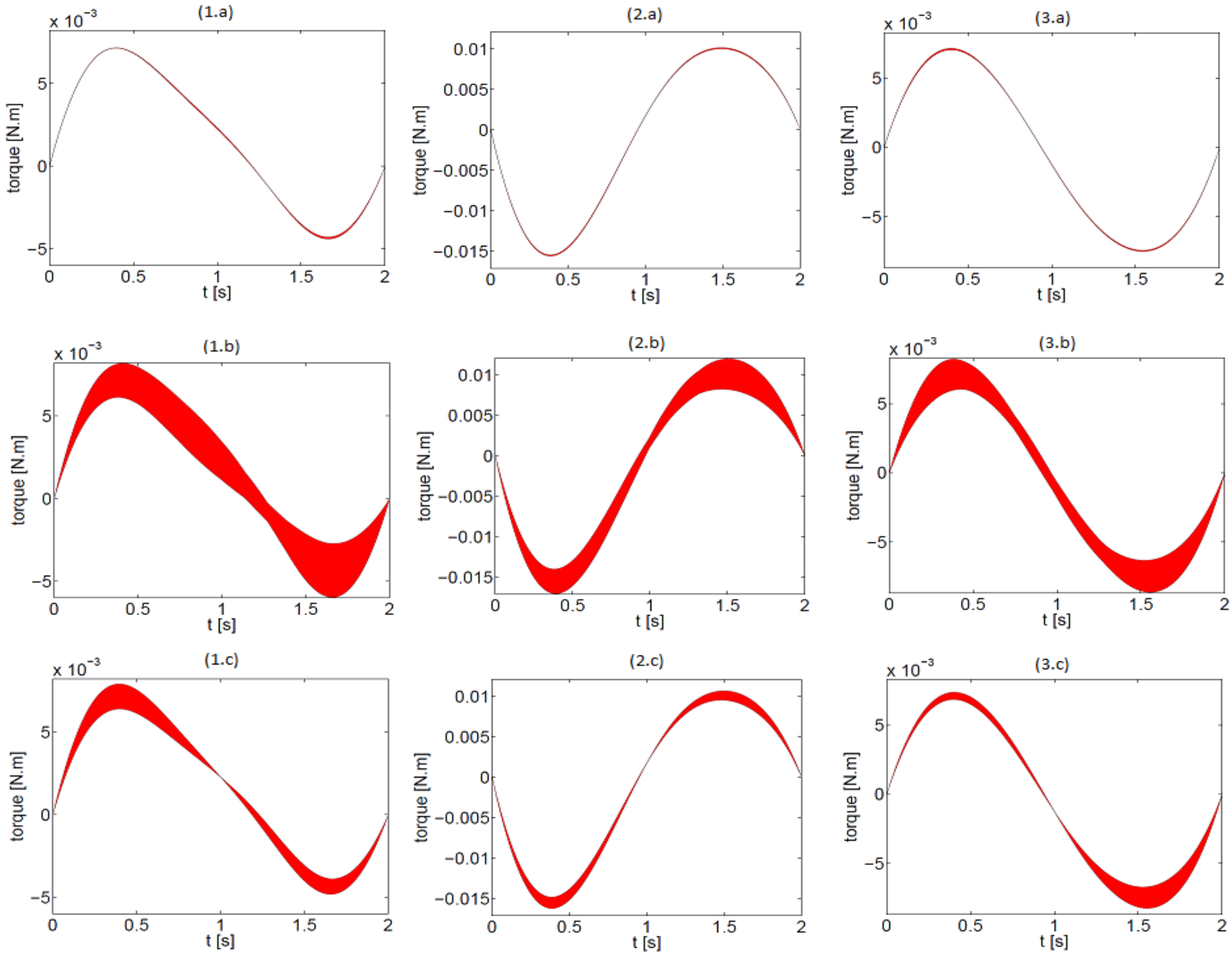

Figure 4 – Case I: Interval values of the required torque to perform the pre-defined trajectory regarding the manipulators' actuators (columns 1, 2 and 3): (a) $\{\underline{m_1}, \overline{m_1}\}$, (b) $\{\underline{m_2}, \overline{m_2}\}$ and (c) $\{\underline{m_{ee}}, \overline{m_{ee}}\}$

Case II: The influence of the variation on the length of the links, l_1 and l_2, on the dynamic performance of the manipulator.

In Case II, the required torques to perform the trajectory are compared considering geometrical variations (Eq. 2.23). The columns in Fig. 5 depict the required torque for each actuator denoted as 1, 2 and 3 according to the degrees of freedom $\Theta = [\theta_1 \quad \theta_2 \quad \theta_3]^T$. The lines (a) and (b) depict the required torque for each interval values of the evaluated lengths l_1 and l_2 respectively.

According to Figs. 5 (1a, 2a e 3a), one can conclude that variations on the length l_1 have more influence on the dynamic performance of the manipulator than variations on the length l_2. This is not an intuitive result. Moreover, the impact of geometrical variation on the required torque has been considerably larger than the impact of inertia properties. Possibly, the reason for that is the fact the geometrical variations influence not only the position of the end-effector but also the singular regions. Clearly, more investigation on this issue is necessary before conclusions are drawn.

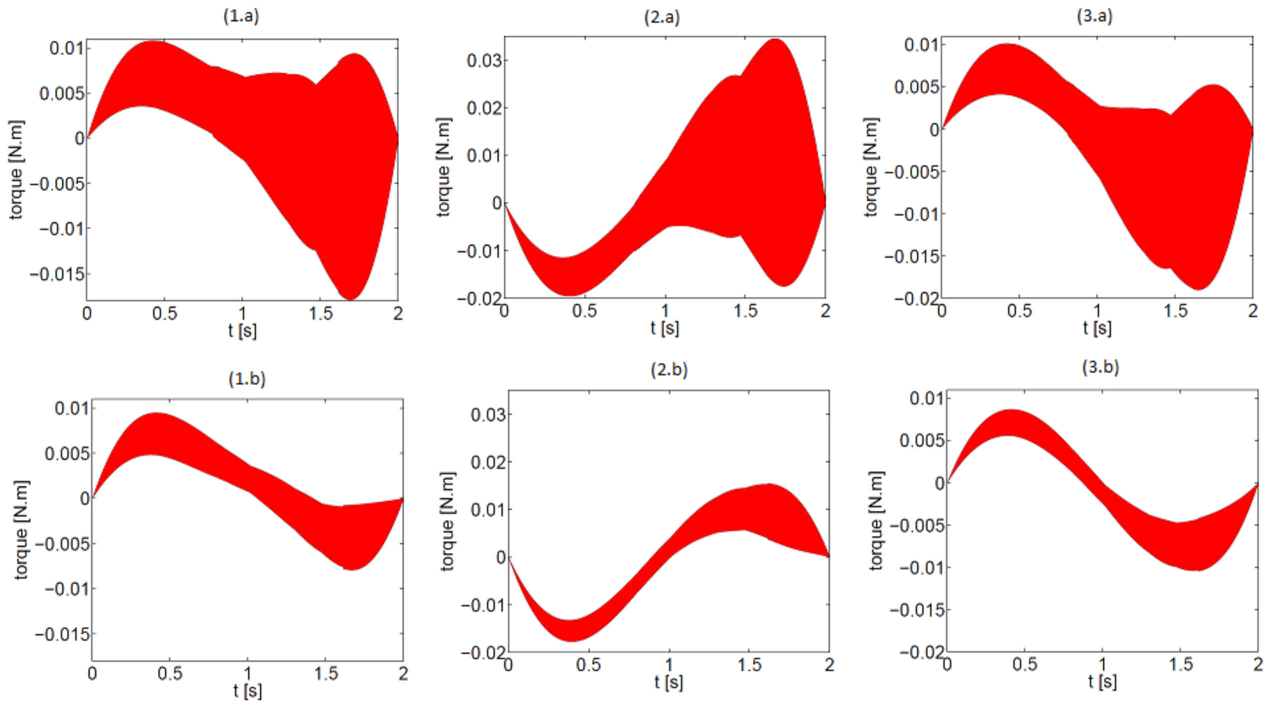

Figure 5 – Case II: Interval values of the required torque to perform the pre-defined trajectory regarding the manipulators' actuators (columns 1, 2 and 3): (a) $l_1 = \{\underline{l_1}, \overline{l_1}\}$ and (b) $l_2 = \{\underline{l_2}, \overline{l_2}\}$

Case III: The influence of the variation on the length of the links, l_1 and l_2, on the accuracy of end-effector's position

The impact on the accuracy of end-effector's position considering geometrical variations is evaluated by comparing the area of the interval end-effector's position values (Eq. 2.24). Figure 6 depicts the values of the areas for (ΔAB) the variation on the length of the link 1, (ΔBC) the variation on the length of the link 2 and (ΔAC) variations on the lengths of both links 1 and 2. The columns 1, 2 and 3 depict the chains 1, 2 and 3 respectively.

According to Figs. 6 (1, 2 e 3), one can conclude that the variations on both lengths l_1 and l_2 have increased the influence on the error area on end-effector's position than the variations on the lengths l_1 and l_2 separately.

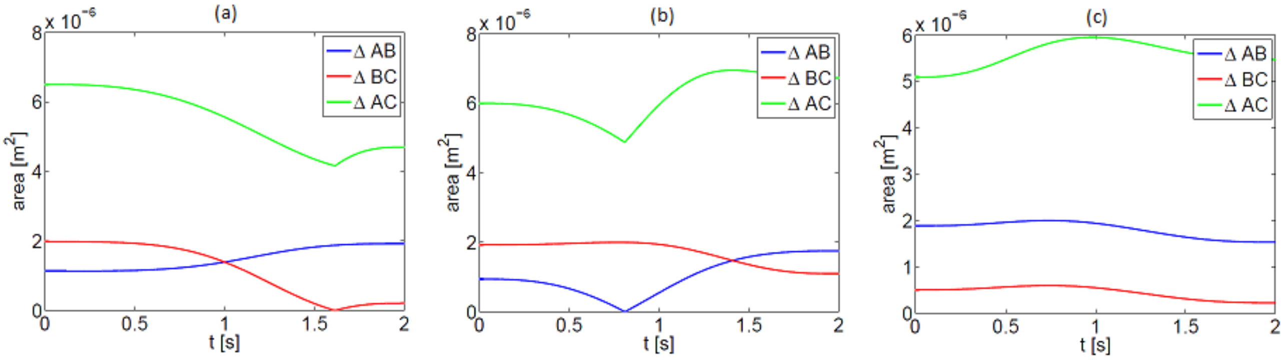

Figure 6 – Case III: size of the possible area on the end-effectors' movement to perform the pre-defined trajectory regarding the manipulators' actuators (columns 1, 2 and 3): ∆AB=variation on l_1 , ∆BC=variation on l_2 and ∆AC= variation on both l_1 and l_2

CONCLUSIONS

Based on the numerical results previously presented, one can conclude that:

- Variations on the mass of the link A_iB_i have little impact on the required torque to perform the selected trajectory;
- Variations on the masses of the end-effector and link B_iC_i have more influence on the dynamic performance of the manipulator than variations on the mass of the link A_iB_i;
- Variations on the length l_1 have more influence on the dynamic performance than variations on the length l_2;
- No large impact on the accuracy has been identified due the length's variations. Studies on their flexibility may be important to evaluate this issue in a more precise way.

The impact of some uncertainties could be realized by using interval analysis and dynamic parametric models of the manipulator. This is a straightforward methodology that can aid the designer to infer about these quests.

Future works

- Study the impact of these variations considering the redundancy in the actuation of the manipulator;
- Improve the algorithm for better filtering the results.

REFERENCES

Ashokaraj, I., Tsourdos, A., Silson, P., & White, B. (2004, July). Sensor based robot localisation and navigation: Using interval analysis and extended Kalman filter. In Control Conference, 2004. 5th Asian (Vol. 2, pp. 1086-1093). IEEE.

Binaud, N., Caro, S., & Wenger, P. (2011). Comparison of 3-RPR planar parallel manipulators with regard to their kinetostatic performance and sensitivity to geometric uncertainties. Meccanica, 46(1), 75-88.

Caro, S., Binaud, N., & Wenger, P. (2009). Sensitivity analysis of 3-RPR planar parallel manipulators. Journal of mechanical design, 131(12), 121005.

Chen, G., Wang, H., & Lin, Z. (2013). A unified approach to the accuracy analysis of planar parallel manipulators both with input uncertainties and joint clearance. Mechanism and Machine Theory, 64, 1-17.

Daney, D., Andreff, N., Chabert, G., & Papegay, Y. (2006). Interval method for calibration of parallel robots: Vision-based experiments. Mechanism and Machine Theory, 41(8), 929-944.

Fontes, J. V. C., Santos, J. C., & Da Silva, M. M. (2014, October). Optimization Strategies for Actuators of Kinematically Redundant Manipulators to Achieve High Dynamic Performance. In Robotics: SBR-LARS Robotics Symposium and Robocontrol (SBR LARS Robocontrol), 2014 Joint Conference on (pp. 31-36). IEEE.

Merlet, J. P. (1996). Redundant parallel manipulators. Laboratory Robotics and Automation, 8(1), 17-24.

Merlet, J. P. (2009). Interval analysis and reliability in robotics. International Journal of Reliability and Safety, 3(1-3), 104-130.

Merlet, J. P. (2011). Interval analysis and robotics. In Robotics Research (pp. 147-156). Springer Berlin Heidelberg.

Moore, R. E., Kearfott, R. B., & Cloud, M. J. (2009). Introduction to interval analysis. Siam.

Muller, A. (2010). Consequences of geometric imperfections for the control of redundantly actuated parallel manipulators. Robotics, IEEE Transactions on, 26(1), 21-31.

Piazzi, A., & Visioli, A. (2000). Global minimum-jerk trajectory planning of robot manipulators. Industrial Electronics, IEEE Transactions on, 47(1), 140-149.

Rocha, D. M., & da Silva, M. M. (2013, February). Workspace and singularity analysis of redundantly actuated planar parallel kinematic machines. In Proc. of the XV International Symposium on Dynamic Problems of Mechanics, Búzios, RJ, Brazil (pp. 1-10).

Rump, S. M. (1999). INTLAB—interval laboratory (pp. 77-104). Springer Netherlands.

da Silva, M. M., De Oliveira, L. P., Brüls, O., Michelin, M., Baradat, C., Tempier, O., De Caigny J., Swevers J., Desmet W., Van Brussel, H. (2010). Integrating structural and input design of a 2-DOF high-speed parallel manipulator: A flexible model-based approach. Mechanism and Machine Theory, 45(11), 1509-1519.

da Silva, M. M., & Goncalves, L. A. (2013). Mechatronic design concept and its application to pick-and-place robotic systems. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 35(1), 31-40.

Tannous, M., Caro, S., & Goldsztejn, A. (2014). Sensitivity analysis of parallel manipulators using an interval linearization method. Mechanism and Machine Theory, 71, 93-114.

Wu, W., & Rao, S. S. (2004). Interval approach for the modeling of tolerances and clearances in mechanism analysis. Journal of Mechanical Design, 126(4), 581-592.

Wu, W., & Rao, S. S. (2007). Uncertainty analysis and allocation of joint tolerances in robot manipulators based on interval analysis. Reliability Engineering & System Safety, 92(1), 54-64.

RESPONSIBILITY NOTICE

The authors are the only responsible for the material included in this paper.