Proceedings of the 3rd International Symposium on Uncertainty Quantification and Stochastic Modeling
D.A. Rade; A. M. G. Lima (Editors), ABCM, Maresias, SP, Brazil, February 15-19, 2016

Variability in the dynamic response of connected structures – A mobility approach for point connections

M. R. Souza[1], N. S. Ferguson[1]

[1] ISVR, University of Southampton, Highfield Campus, SO17 1BJ, Southampton, UK

Abstract: This paper applies a mobility method to simple and idealised structures comprising one-dimensional wave bearing systems, such as cable bundles or hydraulic pipes, connected to isotropic a plate in order to explore potential methods for coupling together uniform structures. Considering point attachments of a beam to a plate, a mobility approach has been developed and implemented to quantify and understand the significance of the dynamic properties of the attachments and the variability that might be introduced by random spacing of the attachments points, variability in the stiffness of the attachments or in the properties of the attached beam. The advantage of this mobility method lies on its analytical solution from classical Euler-Bernoulli beam and thin plate solutions. Examples of a stiff or flexible beam attached to a plate through a set of elastic springs is explored. Results show how the variability in the different parameters have effects on the subsequent structural dynamic variability in different frequency ranges of the response spectrum.

Keywords: Connections, Mobility, Analytical, Variability

NOMENCLATURE

a = amplitude of the wave, m
B = plate bending stiffness
E = Young's modulus, Pa
f = applied transverse force, N/m
I = second moment of area, m^4
k = flexural wavenumber, m^{-1}
m = mass per unit area, kg/m^2
p = transverse force, N/m^2
r = distance between points, m
S = area of beam cross-section, m^2
$\dot{w}$ = velocity vector, m/s
W = transverse displacement, m
Y = mobility, m/Ns
$\boldsymbol{Y}$ = mobility matrix

Greek Symbols

α, β relative to points α and β
Δ = spacing between connections, m
κ = spring stiffness, N/m
$\boldsymbol{\kappa}$ = spring stiffness matrix
ρ = density, kg/m^3
ω = angular frequency, rad/s

Subscripts

n near-field waves
z to the z-axis
$+$ for positive going waves
$-$ for negative going waves
b beam
p plate
K spring

Superscripts

∞ relative to the infinite structure

INTRODUCTION

Widely found in engineering are one-dimensional vibration waveguides, such as hydraulic pipes or cable bundles, attached to host-structures, like thin plates. Design engineers frequently consider these attachments as simply lumped masses at the attachment points, which is not enough to predict all of the structural dynamic interaction of cables or determining when this interaction is relevant (Coombs *et al.*, 2011). Moreover, manufacturing processes produce variability in the nominal properties of the materials and deviations from the design geometries (Fabro, 2014). This paper presents simple structural models using a mobility method to couple different beams to a thin plate through a set of point connections. The effects of variability can be introduced by having different beam to plate bending wavenumber ratios (stiff beam or stiff plate and some random distribution around these values); a different number of connections, randomly spaced connections, flexible connections (in the form of elastic springs) and all of these are explored. These models presented provide results for the point mobilities of coupled infinite structures to capture and best understand the underlying physics of these connections. The behaviour of finite structures can be approximated to an infinite structure at higher frequencies (White, 1986) and (Skudrzyk, 1958). Moreover, infinite structures have simpler expressions and are related to the frequency-averaged response of finite structures (Skudrzyk, 1968). Since there are analytical formulations in the frequency domain for both the mobility of an infinite Euler-Bernoulli beam and an infinite plate, the modelling is numerically efficient and there is no need to use, for instance, FEA. In previous experimental work (Mayr and Gibbs, 2011) measurements of point-connected ribbed plates were compared to infinite structures results, but the effects of variability in the spacing of the attachments or in the wavenumber ratio were not investigated.

MODELLING

Mobility is a particular form of a scaled dynamic frequency response of a mechanical system, which is defined as the ratio of velocity to force as a function of frequency. In a three-dimensional system, one can measure (or estimate) the translational velocity in the three coordinate directions, X, Y, and Z, and one can also measure the rotational response.

In addition, mobility is in general a complex function in the frequency domain. One can find the driving point, or point, mobility, when the velocity and force are in the same direction and at the same point (White, 1986). Mobility (or impedance) methods are a very useful and straightforward tool to analyse linear mechanical systems under periodic, transient or random loads. It can be used for simple lumped parameters or more elaborate systems, in the form of a mobility matrix (Gardonio and Brennan, 2004). This kind of approach was developed in order to be able to consider the dynamic behaviour, in the frequency domain, of both a source and receiver of vibration and then predict a coupled system performance in a manner analogous to what electrical engineers use for circuit analysis (White, 1986).

Using mobilities to couple an infinite beam to an infinite plate

For an infinite and uniform Euler-Bernoulli beam, the equation of motion for flexural vibration, $w(x,t)$, due to a transverse distributed force per unit length is given by (Gardonio and Brennan, 2004) and (Bishop and Johnson, 1960):

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho S\frac{\partial^2 w(x,t)}{\partial t^2} = f_z(x,t) \tag{1}$$

For a harmonic response, Eq. (1) has a solution in the form:

$$W(x) = a_{+n}e^{-k_b x} + a_+ e^{-jk_b x} + a_{-\mathrm{n}}e^{k_b x} + a_- e^{jk_b x} \tag{2}$$

Applying the appropriate boundary conditions, differentiating $W(x)$ with respect to time, and then dividing it by the applied force, the transfer mobility of the infinite beam between points $x = \alpha$, excitation point, and $x = \beta$, response point, is as follow:

$$Y_b^{\infty}(\omega,\alpha,\beta) = \frac{\dot{W}(x)}{F} = \frac{-\omega}{4EIk_b^3}\left(je^{-k_b r_{\alpha\beta}} - e^{-jk_b r_{\alpha\beta}}\right) \tag{3}$$

where $k_b = \sqrt[4]{\frac{\rho S}{EI}}\sqrt{\omega}$ is the flexural wave number in the beam and $r_{\alpha\beta}$ is the distance between the points.

From the classical plate theory for thin plates (Cremer, Heckl and Ungar, 1988), the equation of motion for the transverse displacement, $w(x,y,t)$, subject to a distributed transverse force per unit area $p_z(x,y,t)$ is:

$$B\left(\frac{\partial^4 w(x,y,t)}{\partial x^4} + 2\frac{\partial^4 w(x,y,t)}{\partial x^2\partial y^2} + \frac{\partial^4 w(x,y,t)}{\partial y^4}\right) + m\frac{\partial^2 w(x,y,t)}{\partial t^2} = p_z(x,y,t) \tag{4}$$

where B and m are the plate bending stiffness and mass per unit area, respectively.

After some algebraic manipulation, the mobility of the plate is given by (Gardonio and Brennan, 2004):

$$Y_p^{\infty}(\omega,\alpha,\beta) = \begin{cases} \frac{\omega}{8Bk_p^2}\left[H_0^2(k_p r_{\alpha\beta}) - \frac{2j}{\pi}K_0(k_p r_{\alpha\beta})\right], & \alpha \neq \beta \\ \frac{1}{8\sqrt{Bm}}, & \alpha = \beta \end{cases} \tag{5}$$

where H_i^2 is an i^{th} order Hankel function of the second kind, K_i is an i^{th} order modified Bessel function of the second kind, $k_p = \sqrt[4]{\frac{m}{B}}\sqrt{\omega}$ is the flexural wave number in the plate.

Infinite beam attached to an infinite plate – Rigid links

For simplicity, initially, an external harmonic point force is applied to the plate at one of the attachments. In addition, the numbers of points of interest in the plate and in the beam are the same and are the attachment points. A certain proportion of this applied force is transmitted from the plate to the beam through the rigid link connecting them and, as a reaction, the beam applies the same force on the plate. The present analysis also neglects the offset of the beam neutral axis from the plate. Since the structures are connected through rigid links, velocity continuity implies at the attachment points:

$$\dot{\boldsymbol{w}}_{\boldsymbol{p}} = \dot{\boldsymbol{w}}_{\boldsymbol{b}} \tag{6}$$

The velocities can be calculated using:

$$\dot{\boldsymbol{w}}_{\boldsymbol{p}} = \boldsymbol{Y}_{\boldsymbol{p}}^{\infty}(\boldsymbol{f} - \boldsymbol{f}') \tag{7}$$

$$\dot{\boldsymbol{w}}_{\boldsymbol{b}} = \boldsymbol{Y}_{\boldsymbol{b}}^{\infty}\boldsymbol{f}' \tag{8}$$

where $\boldsymbol{f}$ is the vector of external forces applied to the plate and $\boldsymbol{f}'$ is the vector of transmitted forces.

From the previous set of equations, it is possible to determine the vector of transmitted forces, as follows:

$$\boldsymbol{f}' = \left(\boldsymbol{Y}_{\boldsymbol{p}}^{\infty} + \boldsymbol{Y}_{\boldsymbol{b}}^{\infty}\right)^{-\mathbf{1}}\boldsymbol{Y}_{\boldsymbol{p}}^{\infty}\boldsymbol{f} \tag{9}$$

The previous equations consider that the number of points of interest are the attachment points connecting the beam to the plate. If one is interested, for instance, in the response of points on the plate that are not connected to the beam, some transformation matrices are required to match the matrices dimensions and make the relationships agree with the required transfer mobilities.

Infinite beam attached to an infinite plate – Flexible connections

In order to get a more realistic model of the attachments, it is fair to consider some flexibility in the connections. It is possible to achieve that using elastic springs to connect the beam and plate. At each point attachment one will find a force that is produced by the spring on the connected beam and plate. These forces, at end of the spring, are equal in magnitude, but opposite in direction. The force on the beam, for example, is by Hooke's Law equal to the spring stiffness, k, multiplied by the extension of the spring, which is given by the difference in the displacement of its ends. This extension is the difference in the displacement of the beam and the displacement of the plate at the respective attachment points. Assuming a harmonic point force at an arbitrary Point α on the plate and a single spring connecting the plate to the beam at another point, Point β. The transverse velocity of Point β in the plate is given by:

$$\dot{W}_p(\beta) = Y_p^{\infty}(\omega,\alpha,\beta)F + Y_p^{\infty}(\omega,\beta,\beta)F_K \tag{10}$$

where $\dot{W}_p$ is the velocity of Point β on the plate, F is the external force applied to the plate at Point α, and F_K is the force due to the spring.

In a similar manner, the response of the beam is given by:

$$\dot{W}_b(\beta) = -Y_b^{\infty}(\omega,\beta,\beta)F_K \tag{11}$$

Assuming a harmonic response for the system under examination, the force due to the spring can be written as a function of the velocities at its ends as:

$$F_K = \frac{\kappa}{j\omega}\left(\dot{W}_p(\beta) - \dot{W}_b(\beta)\right) \tag{12}$$

After some manipulation, it is possible to write in a matrix form:

$$\begin{Bmatrix} \dot{W}_p(\beta) \\ \dot{W}_b(\beta) \end{Bmatrix} = \begin{bmatrix} \dfrac{1-\frac{\kappa}{j\omega}Y_p^{\infty}(\omega,\beta,\beta)}{Y_p^{\infty}(\omega,\alpha,\beta)} & \dfrac{\kappa Y_p^{\infty}(\omega,\beta,\beta)}{j\omega Y_p^{\infty}(\omega,\alpha,\beta)} \\ \dfrac{\kappa}{j\omega}Y_b^{\infty}(\omega,\beta,\beta) & \left(1-\dfrac{\kappa}{j\omega}Y_b^{\infty}(\omega,\beta,\beta)\right) \end{bmatrix}^{-1} \begin{Bmatrix} F \\ 0 \end{Bmatrix} \tag{13}$$

If there are an arbitrary number of elastic point connections, Fig. 1, Eq. (10) and Eq. (11) can be generalized in a matrix form, shown in Eq. (14) and Eq. (15).

$$\dot{\boldsymbol{w}}_{\boldsymbol{p}} = \boldsymbol{Y}_{\boldsymbol{p}}^{\infty}\boldsymbol{f} + \boldsymbol{Y}_{\boldsymbol{p}}^{\infty}\boldsymbol{f}_{\boldsymbol{K}} \tag{14}$$

$$\dot{\boldsymbol{w}}_{\boldsymbol{b}} = -\boldsymbol{Y}_{\boldsymbol{b}}^{\infty}\boldsymbol{f}_{\boldsymbol{K}} \tag{15}$$

where $\boldsymbol{f}$ is the vector of external forces applied to the plate and $\boldsymbol{f}_K$ is the vector of forces due to the springs.

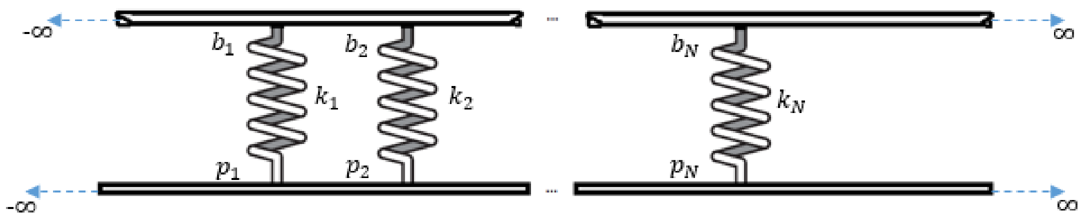

Figure 1 – Infinite beam attached to an infinite plate through an arbitrary number N of elastic spring attachments

The vector of forces due to the springs can be written as:

$$\boldsymbol{f}_{\boldsymbol{K}} = -\boldsymbol{Y}_{\boldsymbol{b}}^{\infty^{-1}}\dot{\boldsymbol{w}}_{\boldsymbol{b}} \tag{16}$$

$$\boldsymbol{f_K} = \frac{1}{j\omega}\boldsymbol{\kappa}(\dot{\boldsymbol{w}}_{\boldsymbol{p}} - \dot{\boldsymbol{w}}_{\boldsymbol{b}}) \tag{17}$$

where $\boldsymbol{\kappa}$ is a diagonal matrix containing the stiffness of the connecting springs.

Using Eq. (16) to rewrite Eq. (14) and Eq. (17):

$$\dot{\boldsymbol{w}}_{\boldsymbol{p}} = \boldsymbol{Y}_{\boldsymbol{p}}^{\infty}\boldsymbol{f} - \boldsymbol{Y}_{\boldsymbol{p}}^{\infty}\boldsymbol{Y}_{\boldsymbol{b}}^{\infty-1}\dot{\boldsymbol{w}}_{\boldsymbol{b}} \tag{18}$$

$$-\boldsymbol{Y}_{\boldsymbol{b}}^{\infty-1}\dot{\boldsymbol{w}}_{\boldsymbol{b}} = \frac{1}{j\omega}\boldsymbol{\kappa}(\dot{\boldsymbol{w}}_{\boldsymbol{p}} - \dot{\boldsymbol{w}}_{\boldsymbol{b}}) \tag{19}$$

Rearranging:

$$\dot{\boldsymbol{w}}_{\boldsymbol{p}} + \boldsymbol{Y}_{\boldsymbol{p}}^{\infty}\boldsymbol{Y}_{\boldsymbol{b}}^{\infty-1}\dot{\boldsymbol{w}}_{\boldsymbol{b}} = \boldsymbol{Y}_{\boldsymbol{p}}^{\infty}\boldsymbol{f} \tag{20}$$

$$\frac{1}{j\omega}\boldsymbol{\kappa}\dot{\boldsymbol{w}}_{\boldsymbol{p}} + \left(\boldsymbol{Y}_{\boldsymbol{b}}^{\infty-1} - \frac{1}{j\omega}\boldsymbol{\kappa}\right)\dot{\boldsymbol{w}}_{\boldsymbol{b}} = \boldsymbol{0} \tag{21}$$

In a matrix form:

$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{Y}_{\boldsymbol{p}}^{\infty}\boldsymbol{Y}_{\boldsymbol{b}}^{\infty-1} \\ \frac{1}{j\omega}\boldsymbol{\kappa} & \boldsymbol{Y}_{\boldsymbol{b}}^{\infty-1} - \frac{1}{j\omega}\boldsymbol{\kappa} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{w}}_{\boldsymbol{p}} \\ \dot{\boldsymbol{w}}_{\boldsymbol{b}} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{Y}_{\boldsymbol{p}}^{\infty}\boldsymbol{f} \\ \boldsymbol{0} \end{Bmatrix} \tag{22}$$

$$\begin{Bmatrix} \dot{\boldsymbol{w}}_{\boldsymbol{p}} \\ \dot{\boldsymbol{w}}_{\boldsymbol{b}} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{Y}_{\boldsymbol{p}}^{\infty}\boldsymbol{Y}_{\boldsymbol{b}}^{\infty-1} \\ \frac{1}{j\omega}\boldsymbol{\kappa} & \boldsymbol{Y}_{\boldsymbol{b}}^{\infty-1} - \frac{1}{j\omega}\boldsymbol{\kappa} \end{bmatrix}^{-1} \begin{Bmatrix} \boldsymbol{Y}_{\boldsymbol{p}}^{\infty}\boldsymbol{f} \\ \boldsymbol{0} \end{Bmatrix} \tag{23}$$

where $\boldsymbol{I}$ is the identity matrix.

NUMERICAL SIMULATIONS AND DISCUSSIONS

The general properties of the infinite plate and the infinite beams are given in Tab. 1. When different wavenumber ratios are considered, the Young's modulus of elasticity of the beam is adjusted to match the proposed ratio. The cross-section of the beam is square. In all cases, the external load was applied at the central attachment point on the infinite plate. In the cases where there are some randomness around a nominal value, uniform probability density functions were considered.

Table 1 – General properties of the plate and the beams

Properties	Value
Density (kg/m^3)	7850
Plate thickness (m)	0.002
Plate Young's modulus (GPa)	200
Beam cross-sectional side (m)	0.02
Poisson's ratio	0.30
Loss factor	0.001

Figure 2 shows a set of six different beams connected to a plate. In Fig. 2(a), they are connected through a single perfectly rigid link; whilst in Fig. 2(b) a single spring is used as connection. Each of the beams has a different wavenumber ratio to the plate. The smaller this wavenumber ratio, the stiffer the beam. The stiffness of the springs is 4000 kN/m.

At lower frequencies, in both cases, the beam and plate behave in a similar way when rigidly or elastically connected. The presence of the elastic spring though decouples the structural components above a certain frequency. Therefore, in this latter case, independently of the beam, the response of the plate is given at higher frequencies solely by the combination of the mobilities of the spring and the plate.

In Fig. 3, the effects of having a different number of attachments is presented. The wavenumber ratio is fixed at 0.35 and the stiffness of the springs is identical and equal to 4000 kN/m. The spacing between the connection points, Δ, was kept constant and equal to 0.1875 m.

When the beam and the plate are connected through multiple attachments, the structure can respond in a similar way of having a standing wave between the attachments, increasing its response at a frequency where this phenomenon can occur. The different number of attachments has some influence on the frequency when this happens, from around 370

Hz when there are 3 or 5 attachments to 313 Hz when there are 7, 9, 11 or 13 attachments. Although, this effect is more sensitive to the spacing between the attachments, as shown in Fig. (4) ($k_b/k_p = 0.35$), and the wavenumber ratio.

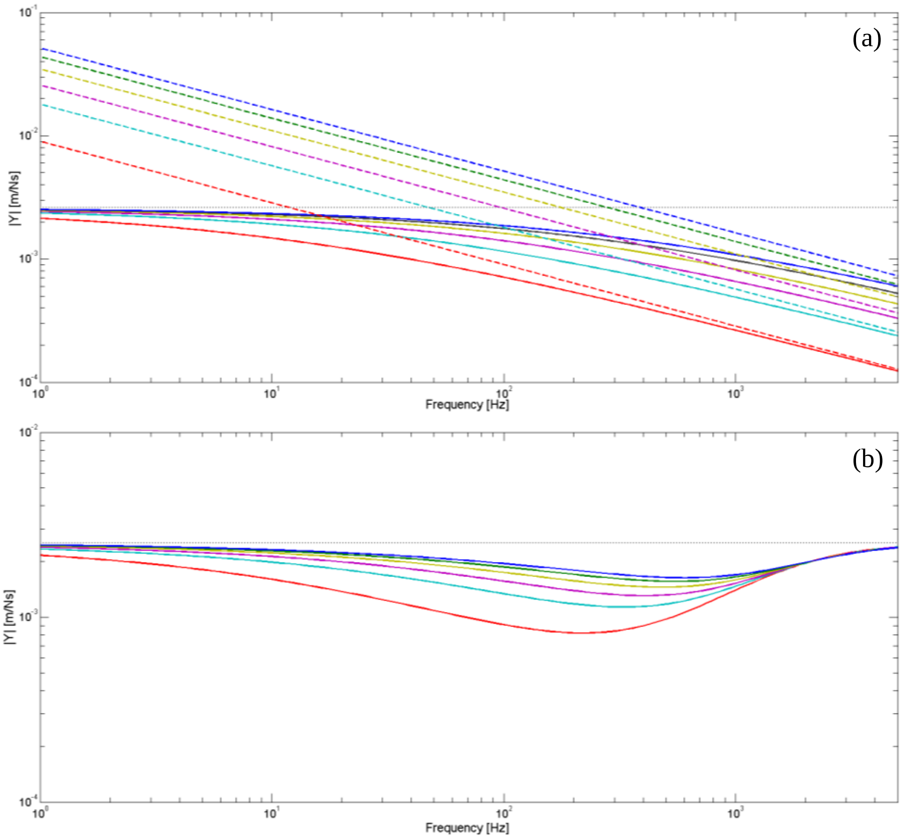

Figure 2 – Point mobilities with a single attachment

In both cases of Figure 2, the coupled system mobilities are plotted for the following free wave beam to plate bending wavenumber ratios: $\frac{k_b}{k_p} = 0.35$, $\frac{k_b}{k_p} = 0.70$ $\frac{k_b}{k_p} = 1.00$, $\frac{k_b}{k_p} = 1.35$, $\frac{k_b}{k_p} = 1.70$, $\frac{k_b}{k_p} = 2.00$ and is the infinite uncoupled plate. The dashed lines in (a) are the mobility of the matching infinite beam, the solid lines in the same colour when the said beam is rigidly connected to the plate at one point. Figure 2 (b) corresponds to the same cases when the beam is connected to the elastic plate by an elastic spring ($\kappa = 4000\ kN/m$) at one point. In (a), the attachment is a rigid link, whereas in (b), the connection is made through an elastic spring.

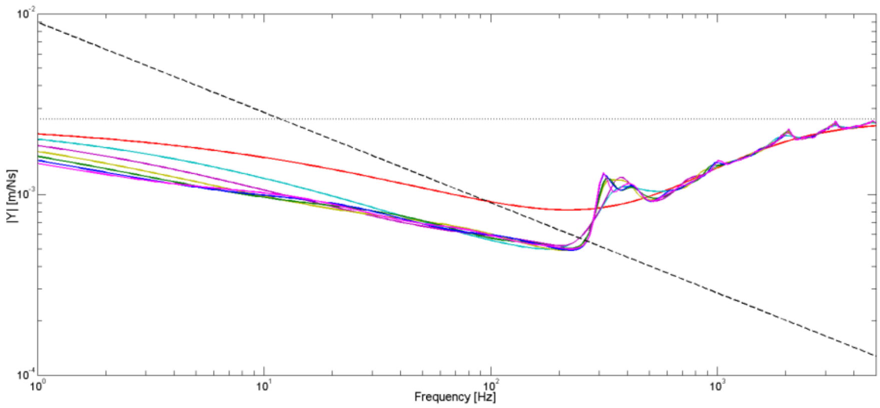

Figure 3 – Point mobilities with different number of spring attachments

In Figure (3), the curves shown these different number of spring connections: 1, 3, 5, 7, 9, 11, 13. is the uncoupled plate and is the uncoupled beam.

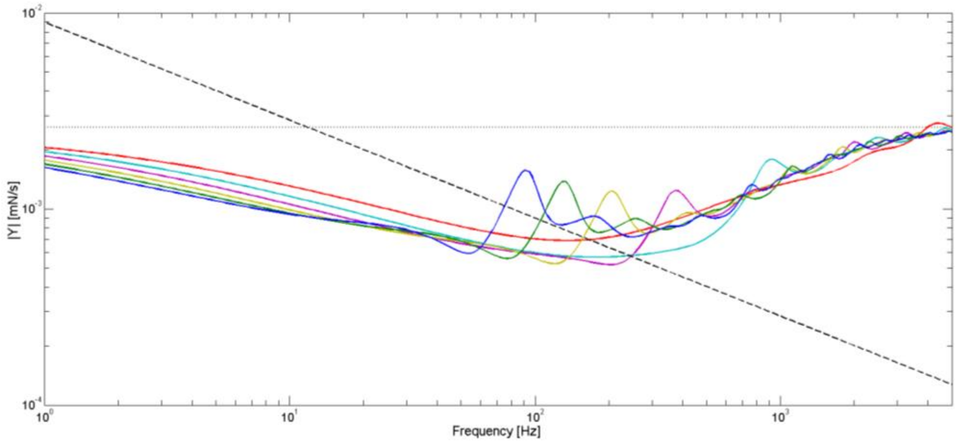

Figure 4 – Point mobilities with different spacing between the spring attachments

The spacing between each of the five attachments used to calculate Figure (4) are: $\mathit{\Delta} = 0.0625\ m$, $\mathit{\Delta} = 0.125\ m$, $\mathit{\Delta} = 0.1875\ m$, $\mathit{\Delta} = 0.25\ m$, $\mathit{\Delta} = 0.3125\ m$, $\mathit{\Delta} = 0.375\ m$ is the uncoupled plate and is the uncoupled beam.

When these mobilities are plotted versus the dimensionless quantity of the wavenumber times spacing, $k_b\Delta$, it is possible to see that the maxima occur in the same region, as shown in Fig. 5(a), which presents the mobilities for different evenly spaced connections and it indicates the coupled structures are accommodating the same kind of wave. Figure 5(c) is contour plot of the real part of the plate velocity at 371 Hz when it is connected to a beam through 5 attachments $\frac{k_b}{k_p} = 0.35$ and Fig. 5(d) is the transverse response of the attached section at 371 Hz. The solid blue line is the real part of the plate velocity and black line is the real part of the beam velocity. They both, Fig. 5(c) and Fig. 5(d) show the wave behaviour and response when there are 5 attachments. In Fig. 5(b), the mobility of the randomly spaced connections. The yellow solid line is the average of the response and the dashed red line is the response of the evenly spaced connections. In this case, a hundred different scenarios were numerically simulated when the spacing between the springs were allowed to randomly change up to 15% from their nominal position, breaking the periodicity of the structure. Even for this case, the response still exhibits this standing-like wave. In this latter case, the mobilities were plotted considering the spacing $\mathit{\Delta}$ for the evenly spaced case.

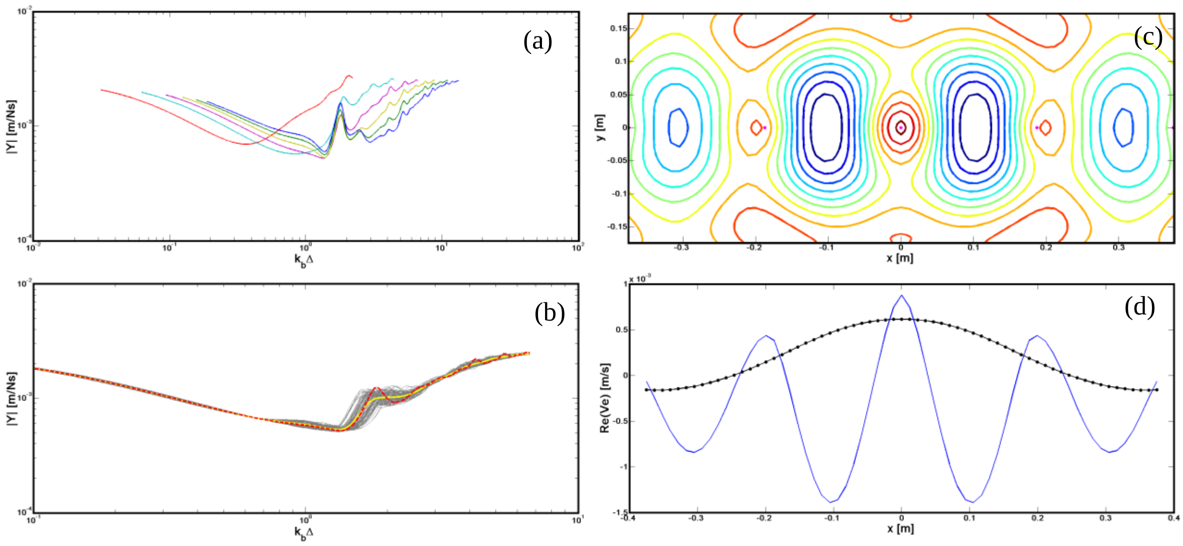

Figure 5 – Standing-like wave behaviour

The effects of having some variability in the beam properties, namely its Young's modulus and some variability in the springs' stiffness were also considered. For each of these cases, a hundred cases were calculated. The wavenumber ratio was allowed to vary up to 15% around its nominal value of $\frac{k_b}{k_p} = 0.35$. The stiffness of the springs were also allowed to vary up to 15% around their nominal value of 4000 kN/m. The results are shown in Fig. 6. For the case of the aforementioned random spacing, a logarithmic dB plot was produced considering:

$$Y = 20\log\left|Y/Y_{ref}\right| \tag{24}$$

where Y_{ref} is the mobility of the evenly spaced case ($\Delta = 0.1875\ m$), $\frac{k_b}{k_p} = 0.35$ and $\kappa = 4000\ kN/m$.

It is clear that variability in the beam affects the response of the system at lower frequencies, whereas variability in stiffness of the attachments has an effect at higher frequencies, when the combination of the mobility of the spring and the plate play a major part in governing the response of the system. The variability of the spacing between the attachments is relevant for the frequency at which the amplification of the response occurs. On a dB scale, this variation can be up to 6 dB for the response for the random spacing of the nominally identical spacing attachments. For the other cases, random properties or random stiffness of the springs, the variation is less than 2 dB.

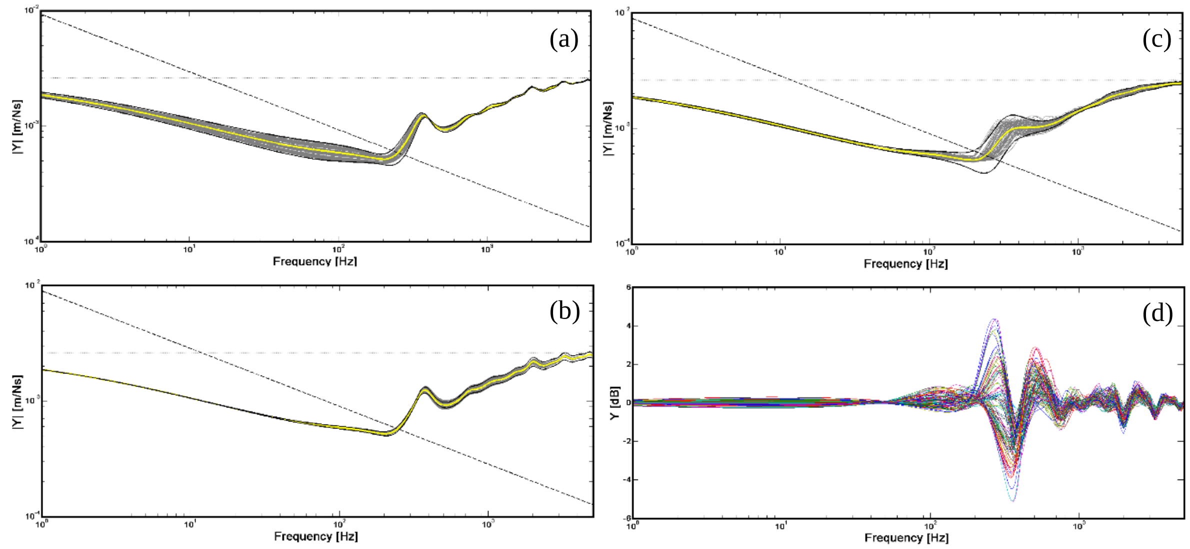

Figure 6 – Effects of variability

Figure 6(a) shows the effect of varying the wavenumber up to 15% of its nominal value (0.35). Figure 6(b) shows the response when the stiffness of the spring is allowed to vary up to 15% of its nominal value (4000 kN/m). Figure 6(c) shows the effect of randomly spaced connections. In these three cases: the grey lines are the response of a hundred different cases, the yellow line is the average response, the black solid lines are the two standard deviation interval, the black dotted line is the response of the uncoupled plate and the black dashed line is the response of the uncoupled beam. Figure 6(d) shows the result of a dB scaling of the response of the randomly spaced connections.

All of the previous cases considered a stiff infinite beam attached to an infinite plate. This could be the case of a hydraulic pipe system or when the beam has an important role as a structural load-bearing element. There are cases though when the attached beam does not have a structural role, such as cable bundles that carry power or data from different parts of a structure. In this scenario, the beam is more flexible than the host structure. To illustrate this scenario, examples for a wavenumber ratio $k_b/k_p = 2.00$ were calculated. In all of the cases, 5 elastic springs were considered as the connections between the infinite beam and the infinite plate.

In Fig. 7(a), the effects of variability in the properties of the attached beam were considered. Again, the wavenumber ratio was allowed to vary up to 15% around its nominal value of 2.00. Figure 7(b) has the same data in a dB scale. For this case, the variations can reach up to 10 dB, much larger than the previous case (stiff beam). Figure 7(c) shows the result of when the spacing between the spring attachments can again vary up to 15% around its nominal value. Figure 7(d) shows the data in a dB scale and, again, the variation can reach up to 10 dB. Also, larger variation in the plate response is predicted compared to the case of the stiff beam, which had a maximum variation of 6 dB. In the case of a flexible attachment the response is typically governed by the stiffer plate and the variation in the coupled structure is about this plate mobility line. For these cases, Y_{ref} is the mobility of the evenly spaced case: $\Delta = 0.1875\ m,\ k_b/k_p = 2.00$ and $\kappa = 4000\ kN/m$

In Fig. 8, the response of the coupled structures were plotted *versus* the dimensionless quantity of the plate bending wavenumber k_p, times the spacing of the evenly spaced case, $\Delta = 0.1875\ m$. In both, $\kappa = 4000\ kN/m$ and the grey lines are a hundred different randomly spaced cases, the solid yellow line is the average response and the red dashed line is the response of the evenly spaced case. Figure 8(a) the beam is stiffer than the plate, $k_b/k_p = 0.35$. The results of the flexible beam, $k_b/k_p = 2.00$, is shown in Fig. 8(b). In the flexible beam case, it is clear that there are more peaks, which indicates where the coupled structures present a similar behaviour of having standing waves between the attachments.

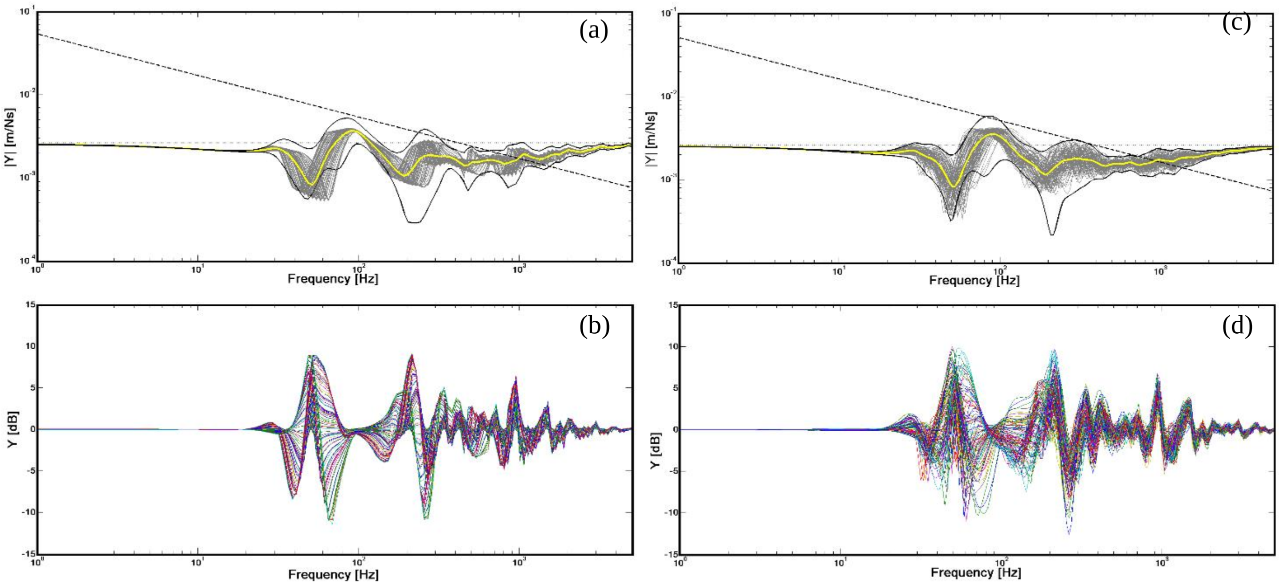

Figure 7 – Variability in the presence of a flexible beam (nominal values: $\Delta = 0.1875$ m, $\frac{k_b}{k_p} = 2.00$ and $\kappa = 4000\ kN/m$)

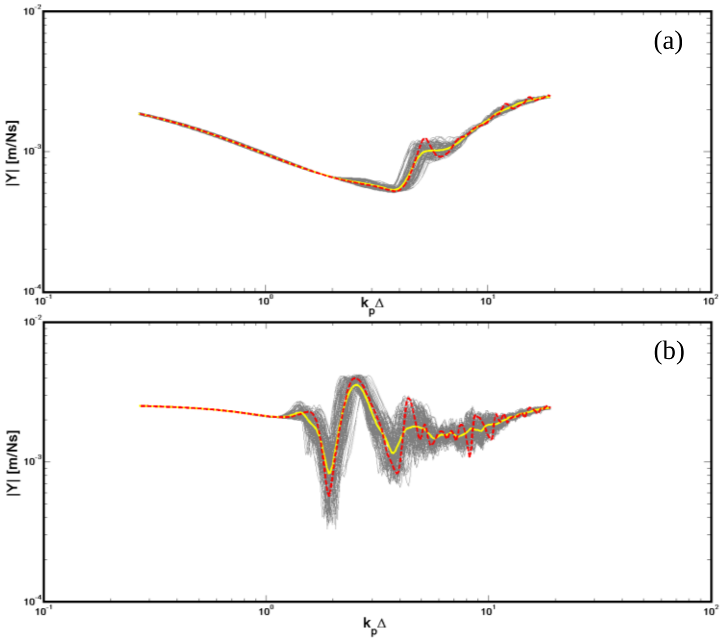

Figure 8 – Stiff beam *versus* Flexible beam - Standing-like wave behaviour in the point mobility for elastically connected infinite beam to infinite plate

A flexible beam allows the plate to accommodate more of these standing-like waves. Therefore, they occur at lower frequencies, Fig 9(a) to Fig. 9(d), and can also occur between the frequencies that a stiff beam would produce. At 371 Hz, the plate exhibits the same kind of wave in both cases, stiff of flexible beam, but the waves in the beam are much different, as Fig. 9(e) and Fig. 9(f) show. In all cases, the support spacing is $\Delta = 0.1875\ m$, $\kappa = 4000\ kN/m$ and $k_b/k_p = 2.00$. Figure 9(a) is contour plot of the real part of the plate velocity at 88 Hz. Figure 9(b) is the transverse response of the attached section at 88 Hz. The solid blue line is the real part of the plate velocity and black line is the real part of the beam velocity. Figure 9(c) is contour plot of the real part of the plate velocity at 263 Hz. Figure 9(d) is the transverse response of the attached section at 263 Hz. The solid blue line is the real part of the plate velocity and black line is the real part of the beam velocity. Figure 9(e) is contour plot of the real part of the plate velocity at 371 Hz. Figure 9(f) is the transverse response of the attached section at 371 Hz. The solid blue line is the real part of the plate velocity and black line is the real part of the beam velocity.

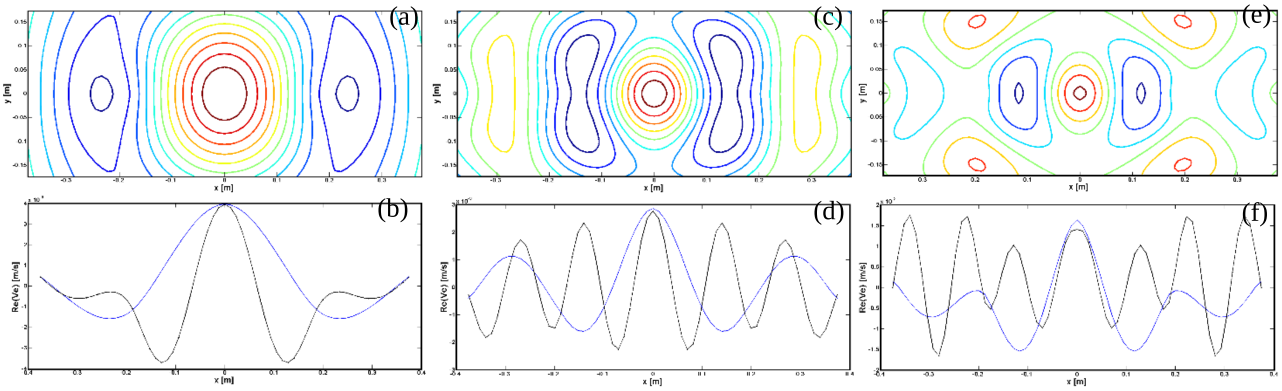

Figure 9 – Standing-like waves behaviour in the presence of a flexible beam

CONCLUDING REMARKS

The effects of variability in commonly found simple structures connected together through points connections were investigated. A mobility method that has analytical solutions was chosen as a modelling approach to predict the coupled system dynamic response.

Examples of a stiff beam connected to a thin plate through spring connections allows one to get some insight into the general behaviour, such as how variability in the attached beam properties is more relevant at lower frequencies in the response of the connected structures. Whilst variability in the stiffness of the attachment is responsible for affecting the response at higher frequencies. This could be the case one is interested of the effects of a hydraulic pipe system attached to a host structure. If one is interested in how cable bundles, for example, could affect the host structure, the case of a flexible infinite beam attached to an infite plate was also shown. For the flexible beam, the effects of variability in the beam properties can extend to higher frequencies and are more relevant than when there is a stiff beam attached. The effects of having variations in the attachment positions is also more relevant in the flexible beam case. When multiple attachments are considered, the infinite structures can behave similiarly to having standing waves between the attachments at some frequencies. These frequencies are given by the spacing between the attachments and the wavenumber ratio considered. Even when the attachments are allowed to have some small variation around their nominal positions, this behaviour persists. When attached to a flexible beam, the plate can accommodate more of these waves and they can start to occur at lower frequencies.

Further steps in the study will be to include and consider the rotational degrees-of-freedom in the mobility modelling and slow varying properties of the beam. The findings can then be adopted for numerically modelling and incorporation and estimation of the potential variability which might arise in finite coupled systems.

ACKNOWLEDMENTS

The authors gratefully acknowledge the financial support of the Brazilian National Council for Scientific and Technological Development, CNPq. Process number 231744/2013-7.

REFERENCES

Bishop, R.E.D., Johnson, D., 1960, “The Mechanics of Vibration”, Cambridge University Press.

Coombs, D., Gooding, J.C., Babuška, V., Ardelean, E.V., Robertson, L.M., Lane, S.A., 2011, “Dynamic Modeling and Experimental Validation of a Cable-Loaded Panel”, Journal of Spacecraft and Rockets, Vol. 48(6).

Cremer, L., Heckl, M., Ungar, E.E., 1988, “Structure-Borne Sound”, Springer-Verlag.

Fabro, A.T., 2014, “Propagation in Waveguides With Slowly Changing Variability”, Ph.D. Thesis, University of Southampton.

Gardonio, P., Brennan, M. J., 2004, “Chapter 9 – Mobility and impedance methods in structural dynamics In Advanced Applications in Acoustic, Noise and Vibration”, F. J. Fahy and J. G. Walker (Eds), Spon Press.

Mayr, A., Gibbs, B., 2011, “Point and transfer mobility of point-connected ribbed plates”, Journal of Sound and Vibration, Vol. 330, pp. 4798-4812.

Skudrzyk, E., 1958, “Vibrations of systems with finite or infinite number of resonances”, Journal of the Acoustical Society of America, Vol. 30, pp. 1140-1152.

Skudrzyk, E., 1968, "Simple and Complex Vibratory Systems", London, The Pennsylvania State University Press.
White, R.G., 1986, "Chapter 26 – Vibration control (II) In Noise and Vibration", R.G. White and J.G. Walker (Eds), Ellis Horwood Publishers.

RESPONSIBILITY NOTICE

The authors are the only responsible of the material included in this paper.