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Why Critical State Soil Mechanics should be taught to all civil engineers

David Airey
University of Sydney, Sydney, Australia, david.airey@sydney.edu.au

Guien Miao
University of Sydney, Sydney, Australia, guien.miao@sydney.edu.au

SUMMARY: Recent discussion on the teaching of soil mechanics has discussed what topics should be kept and which discarded. One of the issues raised was whether critical state soil mechanics (CSSM) should have a place in an undergraduate education. In this paper it is argued that CSSM, or at least the basics of it, are essential to the understanding of soil mechanics and should be a non-negotiable part of introductory soil mechanics. The paper will also discuss how to introduce the topic and also touches on what topics should be included in courses introducing soil mechanics. This to some extent will depend on the structure of the program. The paper is based on an assumption that the course is part of a program for civil engineers and that there is only a single course in soil mechanics. The majority of the students will not be proceeding to majors in soil mechanics, but need to understand the fundamental mechanics that underlie geotechnical engineering, many of which are relevant to (civil) engineering more generally. Students find the concept of the critical state line, initially, difficult to grasp and it thus provides an excellent opportunity for getting students to think, an important part of an undergraduate education. The paper presents some simple exercises that can be used to encourage students to grapple with the concept of the critical state line. Critical state soil mechanics is also the starting point for any course on soil behaviour, and it provides a framework that enables practitioners to understand soil mechanics and the link between consolidation, stress path and strength.

KEYWORDS: Soil Mechanics, Teaching, Critical State Soil Mechanics

1 INTRODUCTION

The question of what to teach in geotechnical engineering has recently been the subject of some discussion (Santamarina, 2015, Wesley, 2015), and this also spawned some online comments on the Géotechnique Letters Linkedin site. This paper is concerned with one of the main points of disagreement between the two principal contributors, that is, on the role of critical state soil mechanics (CSSM). Wesley (2015) argues strongly that critical state soil mechanics has no place in an undergraduate education in geotechnical engineering. His reasons are as follows: the idea of a uniform critical state is not relevant to clays because of progressive failure which obscures its observation; sands and clays are inherently different so that CSSM can't possibly work; critical state was developed for remoulded/artificial clays and does not account for the structure of natural soils; studying remoulded soils is a waste of money; the use of critical state parameters ϕ'_{cv} and c′ (=0) gives absurd results for slope stability, embankments and therefore is of no practical value; critical state soil mechanics has had only minimal (and sometimes negative) impact on the geotechnical profession; bending the concepts to make natural soils fit the model is mentally lazy; the use of ϕ'_{cv} and c′ (=0) in design codes is not sensible because they cannot be measured; failure is governed by peak not critical state values; and residual soils are incompatible with critical state concepts. The aim of this paper is not to respond directly to the criticisms raised by Wesley (2015), which mainly relate to the practical application of CSSM and are discussed in detail by Schofield and Wroth (1968) and Schofield (2005), but to put the case for the introduction of CSSM as a fundamental conceptual model that helps students make sense of soil behaviour.

Wesley (2015) proposes that what is taught in undergraduate soil mechanics courses should be relevant to practice and should be correct and relevant to geotechnical engineering, and in his view critical state soil mechanics does not satisfy these requirements. However, soil mechanics is not taught in isolation, it is generally taught as part of a program for civil engineers. The balance between the wider educational objectives and the depth of technical training is a challenge for all programs and for individual courses, such as soil mechanics within them (Burland, 2008). While there is widespread agreement that the teaching of soil mechanics should focus on core principles and that these should be relevant to practice, the extent to which training in design methods should be included is more contested.

As critical state soil mechanics, at least as considered here, captures the key characteristics of soil behaviour it can be considered as a core principle. Wesley (2015) argues that CSSM does not capture the response of natural and residual soils. This may be contrasted with the views of Burland (1990) that the intrinsic (remoulded and reconstituted) properties of a natural clay, which are described by critical state soil mechanics, provide a robust frame of reference against which to assess the in-situ state of the soil, its structure and the measured mechanical properties of undisturbed samples. Burland (1990) provided examples from a variety of natural soils with fabric and bonding, which included residual soils. More recently Leroueil and Hight (2003) have argued that critical state soil mechanics (defined by them as the integration of shear stress, mean effective stress, void ratio and shear strain) is the foundation of modern soil mechanics. Features of natural soils such as anisotropy, microstructure, viscosity, partial saturation can be understood through extensions to the basic critical state soil mechanics framework (Leroueil and Hight, 2003).

2 TEACHING SOIL MECHANICS

Many students find soil mechanics and geotechnical engineering to be challenging. While this is held as a universal truth there has been little research into the reasons. Burland (2008) has suggested it is because of the number of aspects that have to be considered in tackling ground engineering problems. This is undoubtedly part of the problem, but we also need to remember that for many civil engineering students understanding the concepts of stress and strain and elastic behaviour is not simple. When we introduce a material which has properties that depend on stress and has strength that depends on whether soil is free to drain or otherwise and on top of that the material is inherently heterogeneous we cannot expect students to easily grasp the concepts of soil mechanics. It is necessary to introduce simplified models of soil behaviour to help. Without a simple conceptual model we would be required to teach different aspects of soil behaviour (compression, consolidation, shearing, strength) in isolation and then for each aspect look at the effect of soil type (clay, silt, sand, gravel, effects of grading, etc.) on this behaviour. For most civil engineering students, who only take core fundamental courses in soil mechanics, the aspects of soil behaviour are joined with engineering mechanics, elastic solutions for settlement, solutions to Laplace's equation for flow, analytical and numerical solutions to the consolidation equation, and limit analyses. These are usually complemented by some laboratory exercises to demonstrate the nature of soil and rock materials and to demonstrate the applicability of the engineering mechanics solutions. In any introductory course the nature of the problems analysed has to be simplified as do the models of soil behaviour. Part of the responsibility of the lecturer is to emphasise the nature of the simplifications and assumptions which underlie all the analyses. Soil mechanics is one of the few places in the civil engineering curriculum where students are made to consider uncertainty and question assumptions. This is an important part of the education of engineering students and needs to be explicitly taught and tested. The engineering mechanics can largely be conducted by a computer nowadays, but whether the analysis is sensible, the simplifications reasonable, and whether the fundamental theories and their assumptions have been understood are important issues that can be addressed more explicitly in soil mechanics than other areas of civil engineering.

3 WHY CSSM?

In other branches of mechanics one of the first topics to be covered is the material behaviour. However, in most soil mechanics courses this is introduced in a disjointed way, firstly the nature of soils (clay, silt, sand) and some properties such as the Atterberg limits and grading curves are introduced, primarily as a means of understanding the variability of soil materials and emphasizing their particulate nature. This is followed by the principle of effective stress, and flow of water to understand the role of pore water pressures. There are then a range of topics involving the mechanical behaviour of the soil: 1-D compression and settlement; elasticity and settlement; consolidation; soil strength; and soil stability (usually retaining walls and foundations). Critical state soil mechanics offers a simple framework that can help to explain many of the important aspects of soil behaviour and provide a link between the properties required in these different analyses. However, it is often presented, if at all, as an afterthought making it difficult for students to see its relevance, and compounding students' difficulties with making sense of soil mechanics.

The important aspects that can be explained and integrated by CSSM include: soil compressibility; friction and volume change due to shearing (critical state line); the importance of stress path (drained/undrained strengths); and apparent cohesion. Note that elasto-plastic Cam Clay models are not in this list as they are considered to be beyond the scope of introductory soil mechanics courses. One of the most critical aspects of soil behaviour is the volume changes that accompany compression and shearing. Increasing effective stress whether one-dimensionally or isotropically leads to a reduction in void ratio. For clays the relation between e and σ′ is idealized by a unique normal compression line in a semi-logarithmic space and a family of parallel swelling lines depending on the yield stress. For real soils departures from this idealization can be expected and these will depend on soil type (sand, clay, residual soil) and state. However, caution is needed in discussing real soils as this is likely to lead to confusion and a failure to understand the CSSM framework. Volume changes also accompany shearing and diminish as soil heads towards a critical state where the behaviour is governed by friction and void ratio. The path followed as the soil heads towards the critical state is controlled by the applied stresses and drainage conditions. When the basic concepts underlying CSSM are understood the basis of apparent cohesion can be explained.

Knowledge of the critical state line also allows discussion of the consequences for the engineering performance of structures in soils of different relative densities or OCRs.

A further important aspect of the critical state line lies in its ability to predict apparent cohesions in particulate frictional materials. In undrained tests this is a consequence of the strength associated with a particular void ratio (moisture content), while in drained tests it arises from the tendency of soils to expand during shear. Importantly the expansive nature is limited.

4. HOW TO TEACH?

Critical State is sometimes seen as too difficult for students to grasp in introductory soil mechanics courses. Most texts that introduce critical state make use of deviator stress, q and mean effective stress, p' and refer to triaxial conditions. This is a step too far for most students and requires considerable time being given to explaining the triaxial test, and stress conditions therein. This detracts from an understanding of the basics of soil behaviour and is unnecessary for a conceptual understanding.

The approach recommended here is to simplify as much as possible and to start with the behaviour observed in shear box tests on sand. The typical soil responses in these tests can be found in all introductory soil mechanics texts and in our course are reinforced by laboratory classes carried out by all the students. Thus the starting point is the familiar responses shown in Figure 1, which indicate how the behaviour depends on relative density for a given normal stress. It is useful to emphasise that the behaviour varies with relative density and a range of intermediate responses are possible between the ideal "dense" and "loose" behaviours shown in Figure 1.

Then by referring to an ideal simple shear mode of deformation a relationship between height changes and volume strain can be obtained, and combined with the fundamental relation between volumetric strain, ε_v and change in void ratio, e:

$$\varepsilon_v = -\frac{\Delta V}{V} = \frac{-\Delta V_v}{V_v + V_s} = \frac{-\Delta e}{1+e} \quad (1)$$

where V is total volume, V_v is volume of the voids and V_s is the volume of the solids.

Hence the void ratio changes associated with the typical shear box behaviours can be sketched, as shown in Figure 2. From Figures 1 and 2 the following observations can be made. All samples approach the same ultimate conditions of shear stress and void ratio, irrespective of the initial density. Initially dense samples attain higher peak angles of friction, $\varphi' = \tan^{-1}(\tau/\sigma')$. Initially dense soils expand (dilate) when sheared, and initially loose soils compress.

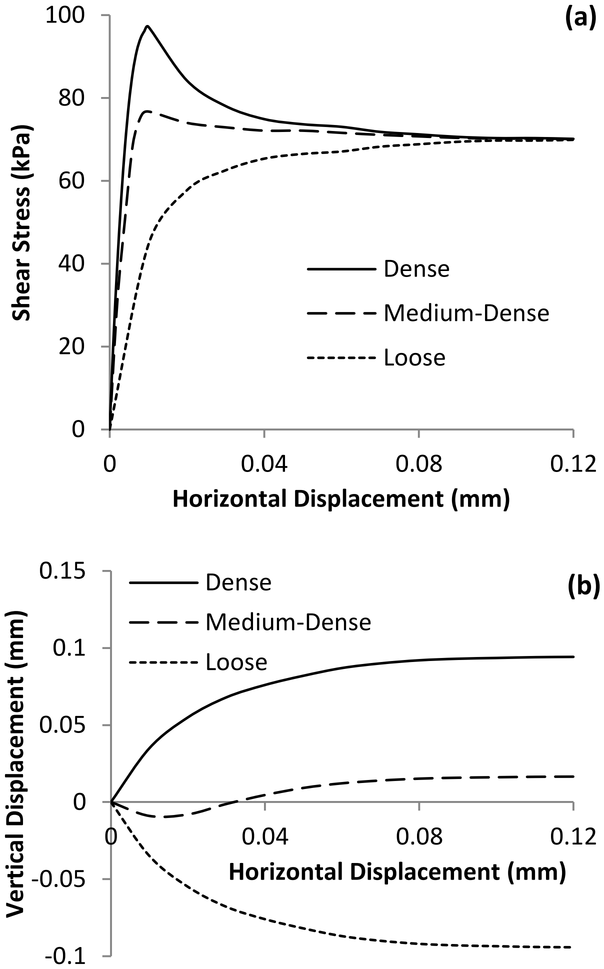

Figure 1. a) Shear stress and b) vertical displacement vs horizontal displacement behaviour for sands of differing initial densities as observed in shear box tests.

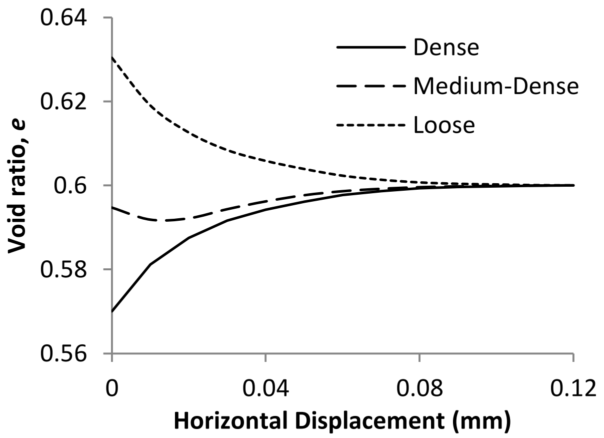

Figure 2. Void ratio vs horizontal displacement behaviour for sands of differing initial densities as observed in shear box tests.

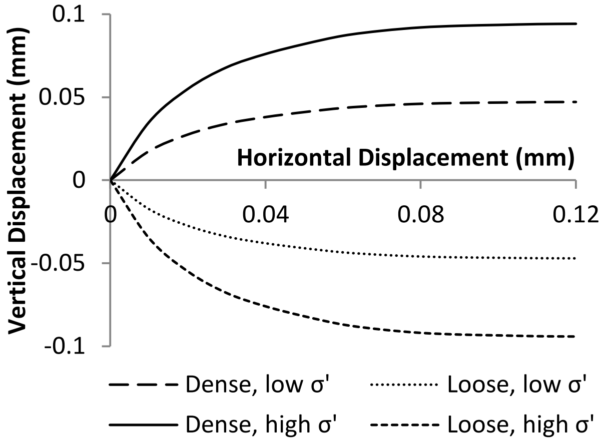

Figure 3. Vertical displacement vs horizontal displacement behaviour for sands of differing initial densities and under different confining stresses (σ'_v) as observed in shear box tests.

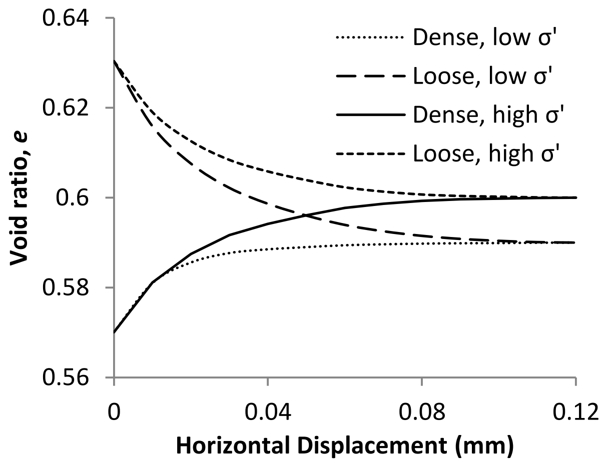

Figure 4. Void ratio vs horizontal displacement behaviour for sands of differing initial densities and under different confining stresses (σ'_v) as observed in shear box tests.

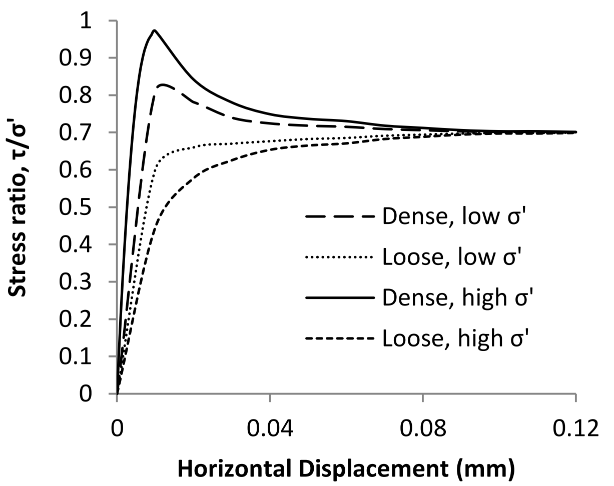

Figure 5. Stress ratio (τ/σ') vs horizontal displacement behaviour for sands of differing initial densities and under different confining stresses (σ'_v) as observed in shear box tests.

The effect of normal stress on the response can also be assessed by considering the results of shear box tests. Again reference can be made to experimental data from shear box tests conducted by the students. Idealised results from these tests are shown in Figure 3. It is assumed for simplicity that both dense tests have the same initial void ratio. Following the same approach as before we can determine how void ratio varies, shown in Figure 4, noting that the final void ratio depends on the normal stress. It is also helpful to introduce the stress ratio (τ/σ') and this is shown in Figure 5.

The following observations can be made. The ultimate values of shear stress and void ratio depend on the stress level, but the ultimate angle of friction $(\varphi'_{ult} = \tan^{-1}(\tau/\sigma')_{ult})$ is independent of both density and stress level. Initially dense samples attain higher peak angles of friction $(\varphi' = \tan^{-1}(\tau/\sigma'))$, but the peak friction angle reduces as the stress level increases. Initially dense soils expand (dilate) when sheared, and initially loose soils compress. Increasing stress level causes less dilation or greater compression.

We can then state that all soil when sheared will eventually attain a unique stress ratio given by $\tau/\sigma' = \tan\varphi'_{ult}$ and reach a critical void ratio which is uniquely related to the normal stress. This ultimate state is referred to as a Critical State. The locus of these critical states defines a line known as the Critical State Line (CSL). This is represented in Figure 6.

To explain the influence of volume change, one of the key fundamental issues that students need to grasp, reference can be made to stress-dilatancy relationships. For example the simple inclined sliding block model which leads to:

$$\varphi' = \varphi'_{ult} + \alpha \tag{2}$$

where α represents the rate of volume change. This equation then enables a simple understanding of the impact of soil state on behaviour. For initially dense soils which expand on shearing, α is positive and the frictional resistance is greater than φ'_{ult}, whereas for initially loose soils which compress, α is negative and the frictional resistance is less than φ'_{ult}. As shearing continues volume changes diminish as soil approaches the critical state, where volume change ceases and the frictional resistance approaches φ'_{ult}.

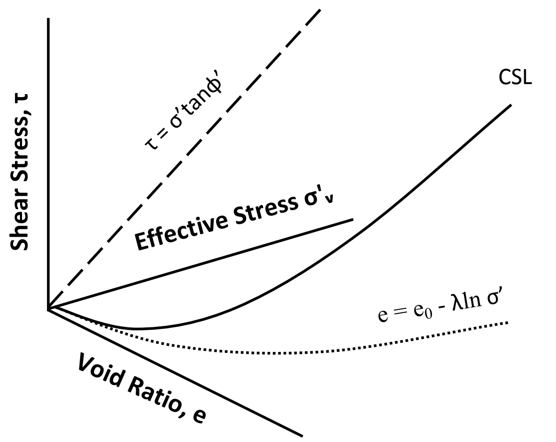

Figure 6. The Critical State Line (CSL) in *τ-σ'-e space*

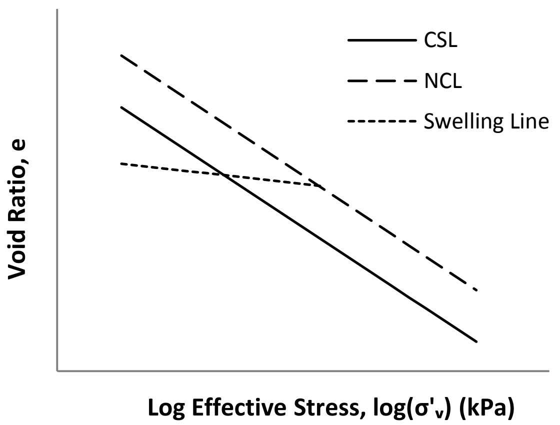

Figure 7. The Critical State Line (CSL) in relation to the Normal Consolidation Line (NCL)

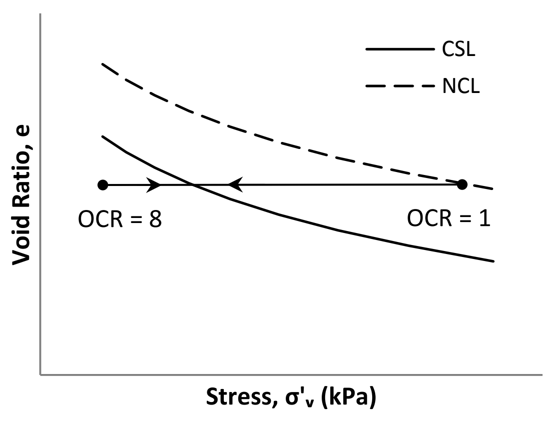

Figure 8. Paths towards the critical state in undrained tests on clays with the same initial void ratio, but differing OCRs.

The behaviour of clays is essentially identical to that of sands. The data however is usually presented in terms of the soils stress history (OCR) rather than relative density. We find that normally consolidated clays behave similarly to loose sands and heavily over-consolidated clays behave similarly to dense sands. As the OCR increases there is a gradual trend between these extremes.

To predict the behaviour of soils we need to combine the CSL with our knowledge of the compression behaviour of clays. Experience has shown that the CSL of reconstituted clays is parallel to the normal consolidation line and lies below it in a void ratio, effective stress plot. This is shown in Figure 7.

In an undrained test volume change is prevented and therefore the void ratio must remain constant. Because the soil always heads towards a critical state when sheared it is possible to show the path that will be followed in an e-σ' plot. This is shown in Figure 8 for normally consolidated (OCR=1) and heavily over-consolidated (OCR>8) samples having the same initial void ratio. Once the final states in this plot are known, so too are the final states in the τ- σ' plot. Also if the final total stresses are known then the excess pore pressures can be determined.

4.1 Typical questions

To assess whether students have understood the concept of the CSL and the importance of stress path a range of simple questions can be posed. Two examples are provided here.

a) Two specimens of a clayey soil have been sheared slowly (fully drained) to failure in a shearbox apparatus. At the ultimate failure state the following stresses and void ratios were measured.

	Normal effective stress, σ' (kPa)	**Shear stress, τ (kPa)**	**Void ratio, *e***
Sample 1	75	27.3	1.169
Sample 2	150	54.6	1.030

Determine the ultimate shear strength of this clay when it is tested undrained in a shearbox apparatus if the moisture content is 47.1% and G_s = 2.65. The Critical State Line for this clay is given by $\tau = \sigma' \tan \varphi$ and $e = e_0 - \lambda \ln \sigma'$.

b) Three clay specimens have been set up in shearbox apparatuses. Specimen 1 is sheared slowly (fully drained) to reach its ultimate failure state while maintaining a constant vertical stress of 52 kPa. At the ultimate state the shear stress is 30 kPa. Its initial void ratio is 0.9 and this increases during shearing to reach 1.0 at the ultimate state. Specimen 2 is sheared slowly (fully drained) to reach its ultimate failure state while maintaining a constant vertical stress of 100 kPa. Its initial void ratio is 1.0 and this reduces to 0.9 at the ultimate state. The third specimen is set up with a vertical effective stress of 100 kPa and void ratio of 1.0.

(i) Determine the ultimate shear stress and void ratio if it is sheared (fully drained) by increasing shear stress with $\Delta\tau/\Delta\sigma$ = - 0.625
(ii) Would you expect a peak in the shear stress, shear strain response? Explain briefly your answer.
(iii) If the third specimen had been sheared undrained what would the ultimate shear stress have been?

5. DISCUSSION

One of the key aspects of soils is that they are comprised of particles and their mechanical behaviours, which are controlled by particle interactions, are all fundamentally the same. This has been demonstrated by many studies of reconstituted clays and sands that have also demonstrated the fundamental validity of the critical state framework. The CSSM framework can be used to explain why in practice sand and clay are considered differently even though both are particulate materials and captured by the same framework. The difference arises because the yield stress, indicated in Figure 7, is much higher for sand than for clay. For a loose sand to become dense very high stresses must be applied so that in practice an initially loose sand will remain loose, and thus relative density provides a useful indication of sand behaviour. In contrast for normally and lightly over-consolidated clays the yield stress is low and significant changes in void ratio can occur.

One of the powerful features of the critical state framework is that it allows the role of stress path and drainage conditions to be considered. For example, it can be used to explain why staged loading is needed for embankments constructed on soft clay, or alternatively why excavations constructed in over-consolidated clay will initially stand up, but collapse over time.

Finally the combination of critical state and dilatancy allows a critical appreciation of the role of the apparent cohesion in the Mohr-Coulomb failure criterion. Students can easily appreciate the absurdity of tensile forces being generated between dry sand and retaining walls with the use of non-zero c′, and hence appreciate the limitations of apparent cohesion in particulate soil with no bonding between the particles. The fact that dilation is limited also leads to the possibility of progressive failures and lower than expected peak stresses. All these aspects can be simply explained by reference to the critical state framework.

The structure, fabric and bonding of natural soils cause departures from the simple critical state framework. However, because of the variability of natural soils, and residual soils in particular, it is not sensible to explain the details of the responses of "real" soils in an introductory soil mechanics course. These details detract from the simplicity of the CSSM framework and prevent students attaining mastery of the concept. Nevertheless, an understanding of the detailed soil response is essential for geotechnical specialists and should be included in advanced level courses.

6. CONCLUSIONS

It has been demonstrated that the critical state framework underlies our current understanding of soil behaviour. It therefore makes sense to teach the simple critical state framework to introduce students to the important aspects of soil behaviour. The model is able to explain key aspects of soil behaviour and allows a broad understanding of ground behaviour. At the same time it should be emphasised that this is an idealised model and that detailed geotechnical design will require more sophisticated approaches.

Part of the answer to the question posed by Santamarina (2015) and Wesley (2015), what to teach or not to teach, should be that we teach critical state soil mechanics. It is suggested that including the response of real soils with fabric and structure is not appropriate in introductory courses, but that the students should understand that the model is necessarily simple.

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