Iterative Combination of Wavelet Artificial Neural Networks for Short-Term Solar Radiation Forecasting

Julio C. Royer
IFPR
Foz do Iguaçu-PR, Brazil
julio.royer@ifpr.edu.br

Luiz Albino Teixeira Junior
ILATIT
UNILA
Foz do Iguaçu-PR, Brazil
luiz.junior@unila.edu.br

Edgar M. C. Franco
Electrical Engineering Department
UNIOESTE
Foz do Iguaçu-PR, Brazil
emfra@ieee.org

***Abstract*—An iterative Combination of Wavelet Artificial Neural Networks to produce short-term solar radiation time series forecasting is presented in this work. A decomposition of level *r*, defined in terms of a wavelet basis of a given solar radiation time series is performed, generating *r+1* Wavelet Components; these components are individually modeled by different Artificial Neural Networks and the best forecasts of each component are combined by means of another Artificial Neural Network; last, the combined forecasts are added, generating the forecasts of the underlying solar radiation data. An iterative algorithm is developed to search for the optimal values for the method parameters. Ten real solar radiation time series of the Brazilian system were modeled, achieving significantly better forecasting performances when compared with naïve predictor, a traditional Artificial Neural Network and another hybrid Wavelet-Artificial Neural Network method.**

***Index Terms*-- Artificial neural networks, Forecasting, Solar power generation, Wavelet transforms.**

I. INTRODUCTION

The conversion of solar energy into electrical energy is one of the most promising alternatives to generate electricity from clean and renewable way. It can be done through large solar farms connected directly to transmission systems or by means of small solar panels for isolated systems. The Sun provides annually to the Earth's atmosphere, approximately, 1.5×10^{18} kWh of energy, but only a fraction of this energy reaches the Earth's surface, due to the reflection and absorption of sunlight by the Earth's atmosphere [1]. One problem of renewable energy, for instance, wind and solar energies is the fact that the production of these sources depends on meteorological factors. In the case of solar energy, the alternation of day and night, seasons, passage of clouds and rainy periods cause great variability and discontinuities in the production of electricity. In addition, it is desirable to have access to energy storage during the day in order to make it available during the night such as battery banks or salt tank [2]. Thus, the safe economic integration of alternative sources in the operation of the electric system depends on accurate predictions of energy production so that operators may make decisions about the maintenance and dispatch of generating units that feed the system as a whole.

There are several techniques employed in solar radiation forecasting, for example Auto-Regressive Integrated with Moving Average (ARIMA) [3], Artificial Neural Networks (ANN)[4-8], Kalman Filter [9] and different ways of combining wavelet orthonormal basis and ANN [10-12]. The wavelet methods combined with several types of predictive models (as the ANNs) have been proposed, achieving remarkable accuracy gains. The wavelet methods consist of auxiliary pre-processing procedures of the data in question, which can be accomplished in two ways: by decomposition [13] or by noise shrinkage of the time series to be forecasted [14]. Several studies show the predictive gains achieved by combining wavelet decomposition and ANN approaches in different time series, as in river flow forecasting [15]; wind flows [16,17]; solar radiation [12]; and number of terrorist attacks in the world [18].

Due to the complexity to predict the solar radiation time series, two aspects should be considered: Firstly, although it is well-known that ANNs integrated with wavelet decompositions, referred to here as wavelet ANN, commonly lead to remarkable predictive gains, their best configuration are obtained in a manual way - and not iteratively by means of a computational algorithm. Secondly, the adoption of individual forecasting methods (as the ANNs and wavelet ANNs) in forecasting processes underestimates the structural risk exhibited mainly by non-stationary time series (as is the case of solar energy time series). However, as pointed out by [19], this problem is solved performing a combination of unbiased forecasts provided by individual forecasting methods.

This paper put forwards an interactive Combination of Wavelet Artificial Neural Networks (CWANN) which is aimed to produce short-term solar radiation forecasting. Summarily, the CWANN method can be split into three steps: in Step 1, a decomposition of level *r*, defined in terms of a wavelet basis, of a given solar radiation time series is

Sponsors: Araucaria Foundation - 44721, CNPq - 473679/2013-2, CAPES - 1419066

performed, generating *r+1* Wavelet Components (WC); secondly, these *r+1* WCs are individually modeled by *k* different ANNs, where *k>5*. The *5* best forecasts are combined by means of other ANNs, and the best combination is selected; and, thirdly, all the combined forecasts of the WCs are added, producing the solar radiation time series forecast. For this, an iterative algorithm is used to iteratively find the optimal CWANN parameters. To demonstrate the usefulness for this approach, ten time series of global horizontal solar radiation of different Brazilian locations were modeled and forecasted. All statistical results have shown that CWANN method has achieved better accuracy performances when compared with the ANN and wavelet ANN described, respectively, in Sections 2.1 and 2.2, and used in [12].

This paper is divided into five sections. In Section 2 some theoretical aspects about the wavelet decomposition and ANNs are introduced. Section 3 describes the CWANN method. The main statistical results are exposed and commented upon in Section 4 and some conclusions are presented on Section 5.

II. LITERATURE REVIEW

A. Wavelet Decomposition of Level r

Based on [20], an y_t in the sequence $y_t(t \in \mathbb{Z})$ can be orthogonally expanded as in Eq. 1.

$$y_t = y_{A_{m'},t} + \sum_{m=m'}^{+\infty} y_{D_m,t} \tag{1}$$

The expansion in Eq. 1 is usually referred to as "wavelet decomposition" of y_t. According to [12] any finite time series, denoted by $y_t(t = 1, \dots, T)$, can be decomposed as in Eq 2.

$$y_t = y_{A_{m'},t} + \sum_{m=m'}^{m'+(r-1)} y_{D_m,t} \tag{2}$$

The expansion in Eq. 2 is usually regarded as a wavelet decomposition of level r of the state y_t.

B. Artificial Neural Networks

According to [21], the feed-forward multi-layer perceptron Artificial Neural Network (ANN) are the most widely used prediction model for time series forecasting. Particularly, a single hidden layer ANN is characterized by a network composed by three layers of simple processing units numerically connected by acyclic links, as illustrated in Figure 1. The relationship between the output at instant *t*, denoted by y_t, and the p-lagged inputs, represented by the sequence $\mathrm{y_{t-k}}\ (\mathrm{k} = 1, \dots, \mathrm{p})$, has the following mathematical representation:

$$y_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \, g\left(\beta_{0j} + \sum_{i=1}^{p} \beta_{ij} \, y_{t-i}\right) + \varepsilon_t \tag{3}$$

where $\alpha_j\ (j = 0, 1, \dots, q)$ and $\beta_{ij}\ (i = 0,1,, \dots, p;\ j = 0, 1, \dots, q)$ are the (single hidden layer) ANN parameters, which are often called the connection weights; p is the number of input nodes; q is the number of hidden nodes; ε_t is the approximation error at time *t*; and $g(.)$ is the transfer function, for example, the logistic function, although it would be possible to adopt another transfer function (see [21] for more details). The logistic function is widely used as the hidden layer transfer function in forecasting processes and its mathematical representation is given by Eq 4.

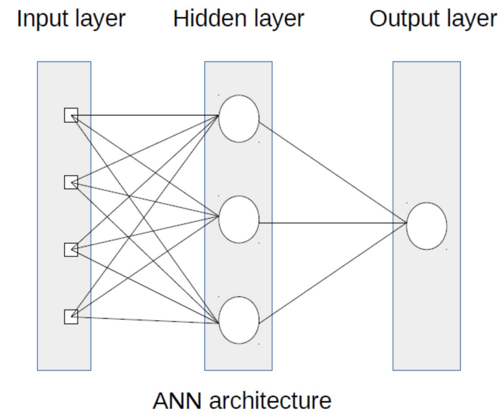

Figure 1. Single hidden layer ANN architecture

$$g(x_t) = \frac{1}{1+exp(-x_t)} \tag{4}$$

where $x_t := \beta_{0j} + \sum_{i=1}^{p} \beta_{ij} \, y_{t-i}$ and $exp(.)$ is the exponential function with Euler's basis (as in [21]). Due to $g(.)$ is a non-linear transfer function, the ANN model, in Eq. 3, in fact performs a non-linear mapping from the past observations $y_{t-k}\ (k = 1, \dots, p)$ to the future state y_t. Equivalently, the model in Eq. 3 can be rewritten, as follows:

$$y_t = f\left(y_{t-1}, y_{t-2}, \dots, y_{t-p}, w\right) + \varepsilon_t \tag{5}$$

where w denotes a vector of all ANN parameters and $f\left(y_{t-1}, y_{t-2}, \dots, y_{t-p}, w\right)$ is the model determined by the network structure and connection weights. Indeed, the neural network is equivalent to a non-linear auto-regressive model.

One of the main algorithms for neural training is the *backpropagation*. This algorithm does the synaptic weights numeric adjust using an optimizations process with two phases: *forward* and *backward*, depicted in Figure 2. In the *forward* phase, an input standard is presented and the ANN answer is calculated. In the *backward* phase, the error (difference) between the ANN answer and the desired answer is used in the synaptic weights (w) updating process.

In order to find the optimal solution $\widehat{w}$, accounting for some criteria, for the vector of ANN parameter w, some optimization algorithm must be employed. Although there are several methodologies in specialized literature, maybe the Levenberg-Marquardt's algorithm (as in [22]) might be considered the most frequently used for this assignment. The minimization in-sample squared error mean (i.e., $min_w \sum_{t=1}^{T} {\varepsilon_t}^2$) is often used as numerical criteria. Thus, it is desired that the solution $\widehat{w}$ of this optimization problem is the argument that minimizes the $\sum_{t=1}^{T} {\varepsilon_t}^2$. Once obtained $\widehat{w}$, it has

$$y_t = f\left(y_{t-1}, y_{t-2}, \dots, y_{t-p}, \widehat{w}\right) + \hat{\varepsilon}_t \tag{6}$$

where $f\left(y_{t-1}, y_{t-2}, \dots, y_{t-p}, \widehat{w}\right)$ is the final ANN output at instant t which consists of forecast, denoted by $\hat{y}_t$, of the state y_t, and $\hat{\varepsilon}_t$ is its forecasting error.

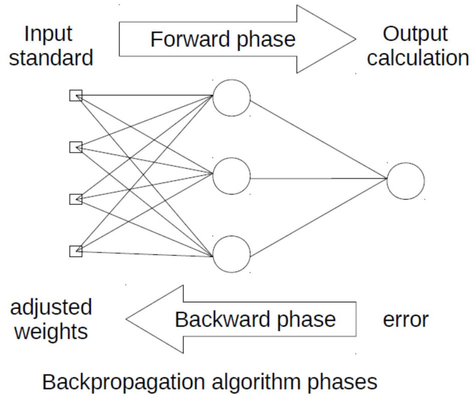

Figure 2. Backpropagation algorithm phases

III. PROPOSED METHODOLOGY

Let y_t $(t = 1, \dots, T)$ denotes a solar radiation time series that exhibits auto-dependence structure, and assume it is required to produce its out-of-sample forecasts. With this purpose, the proposed iterative methodology, referred to as a Combination of Wavelet Artificial Neural Networks (or simply CWANN), can be carried out according to the following four steps.

- Step 1: a wavelet decomposition of level r (as in Section 2.A), defined in terms of an orthogonal wavelet basis (e.g., the Haar, Daubechies, Minimum-Bandwidth, Fejér-Korovkin, Battle-Lemarie, and Symlet families [14,23,24], of y_t $(t = 1, \dots, T)$ is performed, producing 1 WC of approximation at level m', denoted by $y_{A_{m'},t}$ $(t = 1, \dots, T)$, where $\mathrm{m}' \in \mathbb{Z}$, and r WCs of detail at levels m', $m' + 1$, …, $m' + (r - 1)$, denoted by $y_{D_m,t}$ $(t = 1, \dots, T)$, respectively, where $r \in \mathbb{Z}$.
- Step 2: Each WC from Step 1 is individually modeled by k different ANNs (as in Section 2.B), where $k > 5$. So, the following sequences of forecasts are generated: $\hat{y}_{A_{m'},t,i}$, $\hat{y}_{D_{m'},t,i}$, …, $\hat{y}_{D_{m'+(r-1)},t,i}$ $(t = t' + 1, \dots, T + h)$, where $i = 1, \dots, k$ and t' represents the degree of freedoms lost till this step. Note that $\hat{y}_{A_{m'},t,i}$ and $y_{D_m,t,i}$, where $m = m', \dots, m' + (r - 1)$, are, respectively, the forecasts of the state $y_{A_{m'},t}$, produced by the i[th] ANN, and of the state $y_{D_m,t}$, generated by the i[th] ANN. In this way, if it is accounted for a wavelet decomposition of level r of y_t $(t = 1, \dots, T)$, then $(r + 1) \times k$ distinct ANNs are needed for separately modeling them.
- Step 3: The 5 best forecasts of each WC in Step 2 are combined by means of an ANN, generically denoted by $f(.)$, in order to produce its combined forecasts:

  $f_{A_{m'}}\left(\hat{y}_{A_{m'},t,1}, \hat{y}_{A_{m'},t,2}, \dots, \hat{y}_{A_{m'},t,5}, \widehat{w}\right) = \hat{y}_{A_{m'},t,C}$, and

  $f_{D_m}\left(\hat{y}_{D_m,t,1}, \hat{y}_{D_m,t,2}, \dots, \hat{y}_{D_m,t,5}, \widehat{w}\right) = \hat{y}_{D_m,t,C}$

  where $\hat{y}_{A_{m'},t,C}$ and $\hat{y}_{D_m,t,C}$ are, respectively, the combined forecasts of $y_{A_{m'},t}$ and $y_{D_{m'},t}$. Indeed, the ANNs $f_{A_{m'}}(.)$ and $f_{D_m}(.)$ provide the predictions $\hat{y}_{A_{m'},t,C}$ and $\hat{y}_{D_m,t,C}$, respectively. Importantly, the ANNs used in Steps 2 and 3 are the same ones described in Section 2.B.
- Step 4: The combined forecasts of each WC in Step 3 are simply added, generating the out-of-sample combined forecasts of solar radiation time series, denoted by $\hat{y}_t$ $(t = t'' + 1, \dots, T + h)$, where t'' is the degrees of freedom lost until Step 3. That is:

$$\hat{y}_t = \hat{y}_{A_{m'},t,C} + \sum_{m=m'}^{m'+(r-1)} \hat{y}_{D_m,t,C}.$$

The steps 1, 2, 3 and 4 are repeated for all wavelet orthogonal basis, and for decomposition level r from 1 to 3. The best forecast is selected.

In this paper, the four steps above are together carried out in an iterative way by means of a computational algorithm (Figure 3) that tests several values for the CWANN parameters exhibit in Steps 1, 2 and 3. As objective function, the minimization of the in-sample (or training) MSE of a given solar radiation time series (i.e., $min \sum_{t''+1}^{T}(y_t - \hat{y}_t)^2$) is adopted. More specifically, by CWANN parameters here, it means: the level of decomposition wavelet r and the wavelet orthogonal basis, in Step 1; and the window length p and the number q of artificial neurons in hidden layer of each ANN, in Steps 2 and 3. Notice that, in Steps 2 and 3, other ANN parameters (as transfer functions and training algorithm) could also be used as "variables" to be optimized. However, in order to avoid increasingly large periods of training, only the window length, the number of nodes in the hidden layer and the maximum number of training iterations were chosen for working out as variables to be numerically adjusted.

Some aspects were accounted for computational algorithm and deserve then to be commented upon. Firstly, since it can be performed an enormous number of different setups between a wavelet level of decomposition r and a wavelet orthonormal basis, and there is no major analytic role in determining what is the best, it follows that an enormous number of empirical tests are needed for obtaining so.

Secondly, concerning the ANN parameter q, there exists no systematic rule in deciding its optimal value such that all values belonging to a determined range (defined by the decision maker) are numerically tested. Thirdly, to choosing an appropriate number of hidden nodes, another important task of ANN modeling of a time series is the selection of the number of lagged observations, p, the dimension of the input vector. This is perhaps the most important parameter to be estimated in an ANN model because it plays a major role in determining the auto-dependence structure of the time series. However, there is also no theory that can be used to guide analytically the selection of an optimal value of p. Fourthly, due to the overfitting effect typically found in neural network modeling, a validation sample was used for choosing the best ANNs. By an overfitted ANN, it means an ANN that has a good fit to the in-sample data, but has poor generalization ability in the out-of-sample period.

Figure 3(a) depicts the wavelet decomposition flowchart, including the loops to search for the best decomposition parameters (filter and level of decomposition) and the call to the ANN routine which makes the CWs forecast. The variables used are *SR* (Solar Radiation time series); *ws* and *we* (window start and window end, limits for the window size range); *ns* and *ne* (neurons start and neurons end, the range for the number of neurons in the hidden layer); *is* and *ie* (maximum number of iterations start and end); *numFilters* is the total number of filters (orthogonal wavelet basis) tested; *SRForecast[f, r]* is a matrix with all tested forecasts, to choose the best forecast; *f* and *r* are indexes to indicate filter and the decomposition level. *Apr* is an approximation CW and *Det* is a detail CW, and *NN* is a calling to the ANN forecasting procedure described in Figure 3 (b). On this figure, the variables *w*, *n* and *i* are also used, as indices of windows, neurons and iterations ranges; *series*, as the CW to be modeled; *fc* as a matrix for the CW forecasts; *cb* as a matrix for the combined CW forecasts and *mlp* is the Multilayer Perceptron neural network calling procedure.

Basically, the procedure (a) makes wavelet decompositions for all possible combinations of the available filters and decomposition levels, using two nested loops. For each CW it calls the NN procedure (b). This NN procedure test all possible combinations of window size, neurons on the hidden layer and number of iterations, making ANN forecasts. Then it selects the 5 best ones and combines them using other two nested loops, and selects the best combination. This combination is then returned as the CW forecast. All CW forecasts are added to form the Solar Radiation forecast for one filter and one decomposition level. At the end, the best Solar Radiation forecast is selected as the final forecast.

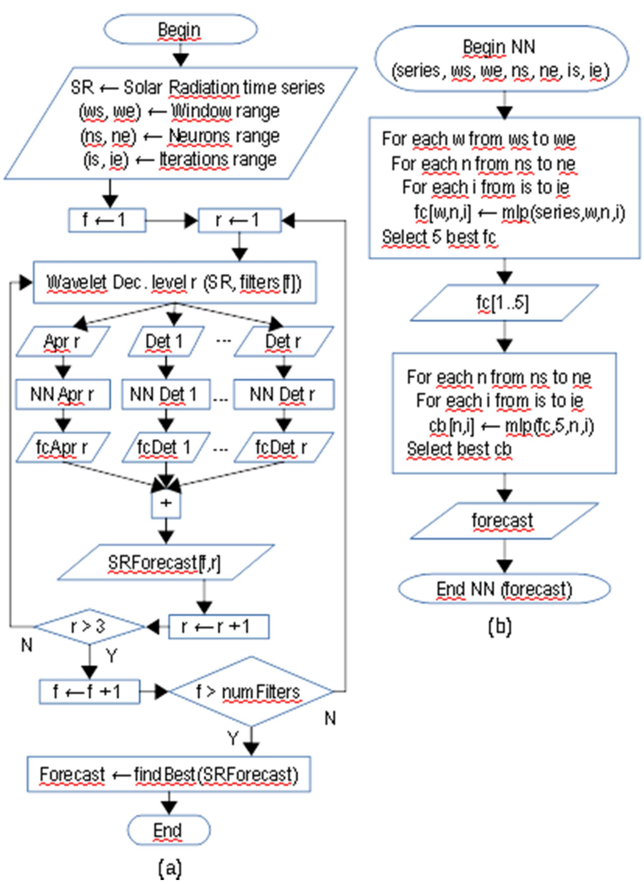

Figure 3. Flowchart of the CWANN method

IV. Numerical Experiments

In order to compare and numerically illustrate the CWANN, the same numerical experiments carried out in [12] were performed in this work. Ten (real) solar radiation time series sampled in ten cities of Brazil on different years were modelled; all of them with hourly data in a period of one year (i.e., 1st Jan to 31th Dec), resulting 8760 observations in W/m^2, from the Solarimetric stations of Sonda Project INPE/CPTEC [25].

A. *Proposed Computational Algorithm*

The computational algorithm described in Section 3 was implemented in R software [26].

In one hand, the orthonormal basis used in the wavelet decompositions were: Haar (db1); Daubechies (db2, db4 and db8); Minimum Bandwidth (mb4, mb8, mb16 and mb24); Fejér-Korovkin (fk4, fk6, fk8, fk14 and fk22); Least Asymmetric or Symlet (la8, la16 and la20); and Battle-Lemarie (bl14 and bl20). The wavelet decomposition levels tested were 1, 2 and 3 (i.e., $r = 1, 2, 3$). All the ANNs used in Steps 2 and 3 were composed with one hidden layer, hyperbolic tangent activation function in the hidden layer, and linear activation function in the output layer. Regarding the learning algorithm, the scaled gradient conjugate (SGC), which simulates the Levenberg-Marquardt algorithm, was used. In all simulations, the input patterns of all ANNs were transformed into a point belonging to the bounded and closed interval [-1,1]. The implemented computational algorithm selects the optimum values to the following parameters: wavelet orthonormal basis, wavelet decomposition level, RNA window size, number of neurons in the hidden layer and maximum number of iterations for each CW and for the combined forecast.

The time series modelling followed the same approach in [12] in order to guarantee a proper comparison of results. Thereby, the training sample was composed of 7008 observations, the next 876 observations were the validation sample, and the 876 remaining data were the test sample.

B. *Modeling For The 10 Time Series*

Table I exhibits the Root Mean Square Deviation (RMSE) and the coefficient of determination R^2 (as [27]) in the test sampling, as well as the best configuration, of the naive predictor (as in [27]), a conventional ANN (as in Section 2.1), the Wavelet ANN proposed by [12] and the CWANN method (proposed method).

Figure 4 illustrates the 876 hourly out-of-sample one-step forecasts for the Cuiabá location by the CWANN method. Note that the forecast values (green) are very close to the reading values (black), and the differences are easier to see during night hours, when solar radiation is close to 0.

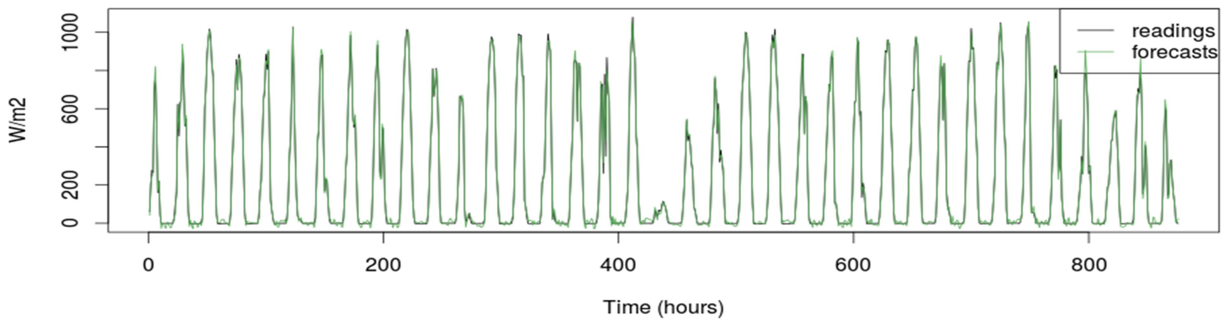

Figure 4. Out-of-sample Cuiabá Forecasts

TABLE I. Wavelet basis, types of ANN, RMSE and R^2 on the test sample for naïve predictor, traditional ANN and hybrid Wavelet-ANN[12] (white background) and by the CWANN method (grey background)

Local	Wavelet basis	Window length	Neurons in the hidden layer	Iterations number	RMSE Wm^{-2}	R^2
Brasília	Naive predictor				143.34	0.7848
	without	12	8		107.88	0.8707
	db32	15	8		91.29	0.9074
	db8	10 to 15	18 to 25	27 to 30	17.05	0.9968
Caicó	Naive predictor				134.68	0.8611
	without	15	19		66.58	0.9648
	db20	15	8		37.61	0.9888
	db8	10 to 15	18 to 25	27 to 30	11.79	0.9989
Campo Grande	Naive predictor				154.21	0.7898
	without	15	10		120.07	0.865
	db20	12	8		76.06	0.9458
	fk8	10 to 16	18 to 25	27 to 30	19.89	0.9964
Cuiabá	Naive predictor				144.84	0.8076
	without	10	19		106.28	0.8908
	db38	10	12		26.14	0.9934
	la20	10 to 25	15 to 25	25 to 30	16.49	0.9974
Florianó-polis	Naive predictor				143.29	0.8302
	without	10	10		109.72	0.8958
	db40	8	15		50.12	0.9783
	la16	10 to 15	18 to 25	27 to 30	19.22	0.9969
Joinville	Naive predictor				111.58	0.8089
	without	11	5		92.24	0.8625
	db32	12	10		84.34	0.885
	la16	10 to 16	18 to 25	27 to 30	15.44	0.9963
Natal	Naive predictor				133.36	0.8738
	without	15	5		57.32	0.9759
	db20	15	13		75.96	0.9577
	bl20	10 to 16	18 to 25	27 to 30	8,68	0.9995
Palmas	Naive predictor				138.54	0.7780
	without	15	10		101.77	0.8727
	db40	10	13		60.3	0.9553
	db8	10 to 15	18 to 25	27 to 30	16.72	0.9966
Petrolina	Naive predictor				121.63	0.8543
	without	15	9		75.11	0.9423
	db15	9	20		82.51	0.9303
	bl14	10 to 16	18 to 25	27 to 30	11.87	0.9986
São Martinho	Naive predictor				140.37	0.8694
	without	15	20		98.87	0.9329
	db13	20	14		19.68	0.9973
	la20	10 to 15	18 to 25	27 to 30	17.11	0.9982

V. CONCLUSIONS

In this paper, a Combination of Wavelet Artificial Neural Networks (CWANN) has been put forward with the aim of producing out-of-sample one-step forecasts of solar radiation. A computational algorithm is proposed for iteratively searching for the optimal values for the CWANN parameters. For evaluating it as well as compare it with other methods, 10 real solar radiation time series of Brazilian system were modeled here.

Regarding the adherence statistics RMSE and R^2, Table 1 shows clearly that the CWANN predictions have achieved greater accuracy power than Naive predictor, ANN and the Wavelet ANN proposed by [12]. Indeed, these results are an important empirical evidence that the optimal numerical adjustment of the non-linear combination of different ANN forecasts by using an ANN integrated with the wavelet decomposition may provide accuracy gains. In addition, the automatic search, not only avoid operational effort, but also contributes to accomplish best forecasts of solar radiation.

Finally, it points out that, even though the mathematical theory associated with the methods (i.e., wavelet decomposition and ANN) that compose the CWANN is relatively complex, they can be straightforwardly implemented in an operational way by using the software R as well as its packages mentioned in the text.

REFERENCES

[1] C. Singh, R. Chaudhary, and R. S. Thakur, "Performance of advanced photocatalytic detoxification of municipal wastewater under solar radiation – A mini review," *International Journal of Energy and Environment*, vol. 2, n. 2, pp. 337-350, 2011.

[2] M. Wittmann, H. Breitkreus, S. Schroedter-Homscheidt and M. Eck, "Case studies on the use of solar irradiance forecast for optimized operation strategies of solar thermal power plants," *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*. vol.1, n. 1, p. 18-27, 2008.

[3] R. Perdomo, E. Banguero and G. Gordillo, "Statistical modeling for global solar radiation forecasting in Bogotá," in *Proc. 2010 IEEE Photovoltaic Specialists Conference (PVSC). Honolulu.*

[4] F. Deng, G. Su, C. Liu and Z. Wang, “Global solar radiation modeling using the artificial neural network technique,” in *Proc. 2010 IEEE Power and Energy Engineering Conf. Asia-Pacific*. *Chengdu*. 2010.

[5] G. Yanling, C. Changzheng and Z. Bo, “Blind source separation for forecast of solar irradiance,” in *Proc. 2012 IEEE Intern. Conf. on Intelligent System Design and Engineering Application. Sanya, Hainan*.

[6] A. Yona and T. Senjyu, “One-day-ahead 24-hours thermal energy collection forecasting based on time series analysis technique for solar heat energy utilization system,” in *Proc 2009 IEEE Transmission and Distribution Conf. and Exposition: Asia and Pacific. Seoul*.

[7] P. L. Zervas, H. Sarimveis, J. A. Palyvos, and N. C. G. Markatos, “Prediction of daily global solar irradiance on horizontal surfaces based on neural-network techniques,” *Renewable Energy,* vol. 33, n. 8, p. 1796-1803. 2008.

[8] N. Zhang and P. K. Behera, “Solar radiation prediction based on recurrent neural networks trained by levenberg-marquardt backpropagation learning algorithm,” in *Proc. 2012 IEEE Innovative Smart Grid Technologies (ISGT). Washington, DC*.

[9] M. Chaabene and B. M. Ammar, B. M, “Neuro-fuzzy dynamic model with Kalman filter to forecast irradiance and temperature for solar energy systems,” *Renewable Energy*. vol. 33, n. 7, p. 1435-1443. 2008.

[10] S. Cao, W. Weng, J. Chen, W. Liu, G. Yu and J. Cao, “Forecast of solar irradiance using chaos optimization neural networks,” in *Proc. 2009 IEEE Power and Energy Engineering Conf. Asia-Pacific. Shanghai.*

[11] H. Zhou, W. Sun, D. Liu, J. Zhao, and N. Yang, “The research of daily total solar-radiation and prediction method of photovoltaic generation based on wavelet-neural network,” in *Proc. 2011 IEEE Power and Energy Engineering Conf. (APPEEC). Wuhan*.

[12] L. A. Teixeira Junior, R. M. Souza, M. L. Menezes, K. M. Cassiano, F. M. P. Pessanha and R. C. Souza, “Artificial neural network and wavelet decomposition in the forecast of global horizontal solar radiation,” *Brazilian Operations Research Society*. vol. 35, n. 1, p. 1-16. 2015.

[13] L. A. Teixeira Junior, M. L. Menezes, K. M. Cassiano, F. M. P. Pessanha and R. C. Souza, “Residential electricity consumption forecasting using a geometric combination approach,” *International Journal of Energy and Statistics*. vol. 1, n. 2, p. 113-125. 2013.

[14] S. Mallat, *A Wavelet Tour of Signal Processing*. 3rd ed. Burlington: Academic Press. 2009.

[15] B. Krishna, Y. R. Satyaji Rao and P. C. Nayak, “Time series modeling of river flow using wavelet neural networks,”. *Journal of Water Resource and Protection*. vol. 3, p.50-59. 2011.

[16] H. Liu, H. Q. Tian, C. Chen and Y. Li, “A hybrid statistical method to predict wind speed and wind power,”. *Renewable Energy*. vol. 35, p. 1857-1861. 2010.

[17] J. P. S. Catalão, H. M. I. Pousinho and V. M. F. Mendes, “Short-term wind power forecasting in portugal by neural networks and wavelet transform,” *Renewable Energy*, vol. 36, p. 1245-1251. 2011.

[18] K. K. Minu, M. C. Lineesh and J. C. Jessy, “Wavelet neural networks for nonlinear time series analysis,” *Applied Mathematical Sciences*. vol. 4, n. 50, p. 2485-2495. 2010.

[19] A. L. F. Alves, C. S. Baptista, A. A. Firmino, M. G. Oliveira and A. C. Paiva, “A comparison of SVM versus Naive-bayes techniques for sentiment analysis in tweets: a case study with the 2013 FIFA confederations cup”, In: *Proc. 2014 ACM 20th Brazilian Symposium on Multimedia and the Web*.

[20] N. Levan, and C. S. Kubrusly, “Multiresolution approximation scale and time-shift subspaces,” *Multidimensional Systems and Signal Processing*, vol. 17, n. 4, p. 343-354. Springer, New York. 2006.

[21] S. S. Haykin, *Neural Networks and Learning Machines*. 3rd Ed. Prentice Hall. 2009.

[22] J. Adamowski and C. Karapataki, “Comparison of multivariate regression and artificial neural networks for peak urban water-demand forecasting: evaluation of different ANN learning algorithms,” *Journal of Hydrological Engineering*, ASCE, vol. 15, p. 729-743. 2010.

[23] I. Daubechies, *Ten Lectures on Wavelets*. 1992 CBMS-NSF Regional Conference Series In Applied Mathematics: SIAM.

[24] J. M. Morris and R. Peravali, “Minimum-bandwidth discrete-time wavelets,” *Signal Processing*. vol. 76, n. 2, p. 181-193. 1999.

[25] E. B. Pereira, F. R. Martins, S. L. Abreu and R. Ruther, “Atlas Brasileiro de Energia Solar”, INPE, 2006.

[26] R CORE TEAM, R: A Language and Environment for Statistical Computing. *R Foundation for Statistical Computing*. Viena, Austria. URL http://www.R-project.org/. 2015.

[27] J. D. Hamilton, J.D. *Time Series Analysis*. Princeton University Press. 1994.