Minimization of Gibbôs Oscillations in Transientsô Simulations using Damping Resistance

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Abstractð **Considering simulations of electromagnetic transients and searching for better results, it is investigated the limits of the change of the representation of the longitudinal parameters in the model of infinitesimal unit of transmission linesô model. It is introduced a damping resistance in parallel to the impedance that represents the longitudinal parameters of transmission lines. Several simulations are made, analyzing how the best relation between the damping resistance and the numeric of infinitesimal unitsô model for minimizing the peak of numeric oscillations or Gibbôs oscillations generated by the trapezoidal rule. The trapezoidal rule is used for solving the linear system applied to the electromagnetic transient simulations in transmission lines.**

Index Termsð **Eigenvalues and eigenfuctions, linear systems, numerical analysis, power system transients simulation, state space methods, time-domain analysis, transmission line modeling.**

I. INTRODUCTION

Numerical methods are used to solve ordinary differential equations. Ordinary differential equations (ODE) are equations with one or more differential relations of an unknown function. They are used in physical models, for example, to calculate the mass flow rate:

$$\frac{dm}{dt} = \rho \cdot A \cdot V \tag{1}$$

It can estimate, with good accuracy, the integral of the function f(x) in the range x_1 to x_2 using numerical methods when f(x) is a mathematical expression whose integral is difficult or impossible to solve or impossible by analytical methods. For this, it can be applied the numerical methods of Euler, Runge-Kutta, Heun, Adams and Dormand-Prince. Elerôs method is a variation of the Taylor series truncating them at first order approximation. It is attractive because its simplicity, but it is necessary a too small integration step to get good accuracy. Heunôs method transforms a set of differential equations into a set of algebraic equations. It is not a method immune to sudden changes in values used for its application. Runge Kuttaôs method allows rapid changes in integration step, but the computational time should be large to avoid rounding errors. Adamsô method is based on simple expressions, but the initial values should be determined by other method.

Modeling linear systems of first order or linearized at a point of interest, Heunôs method is applied to simulations of electromagnetic transients in transmission lines [1]-[6]. In this case, it is not investigate the efficiency and the accuracy of this numerical method related to the other numerical methods. The analysis investigates the introduction of a damping resistance in a simple model applied to obtain the mentioned simulations [7]. So, the damping resistance is introduced in the ´ circuit that represents the infinitesimal unit of transmission lines [7]-[8]. Using a cascade with a great number of these infinitesimal units that is solving using Heunôs method, Gibbôs oscillations leads to undesirable voltage peaks of 25% overvalue without the application of the damping resistance [1]-[10]. If this resistance is used, Gibbôs oscillations become negligible. So, several simulations are carried out applying different values of the damping resistance and the ´ circuitsô number for the mentioned model [6]-[9]. Considering the damping resistance, the ´ circuitsô number and overvalue of the first peak of voltage at the transmission line end, three dimension graphic is used for determining the best values of the damping resistance and the ´ circuitsô number for decreasing the voltage peaks caused by Gibbôs oscillations.

II. HEUNôS METHOD

Heunôs method or trapezoidal integration is a numerical method that aims to transform differential equations into

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equivalent algebraic equations. The integral value in this method is based on the area defined by the curve of f(x) at the interval from x_k to x_{k+1} that it is approximated by the area of a trapezoidal function (Fig. 1) [8].

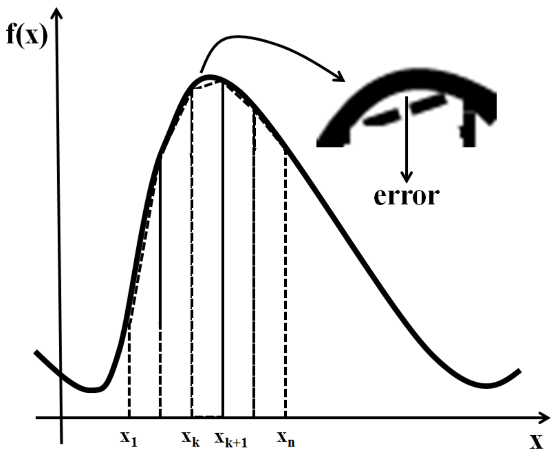

Figure 1. Numeric integration using trapezoidal form (Heunôs method).

Applying the trapezoidal rule, it is obtained [1]-[12]:

$$\int_{x(k)}^{x(k+1)} f(x)\,dx = \frac{\Delta x}{2}\left[f(x_{k+1}) + f(x_k)\right] \quad (1)$$

If y_k point in the vertical axis corresponds to $f(x_k)$ and y_{k+1} corresponds to $f(x_{k+1})$, the trapezoidal rule leads to:

$$y_{k+1} = y_k + \int_{x(k)}^{x(k+1)} f(x)\,dx = y_k + \frac{\Delta x}{2}\left[f(x_k) + f(x_{k+1})\right] \quad (2)$$

The time step (**φx**) is:

$$\Delta x = x_{k+1} - x_k \quad (3)$$

In the next item, this numeric method of integration is applied to electromagnetic transient simulations.

III. TRANSMISSION LINE MODEL

One of the applications of differential equations is related to transmission lines. They can be solved by numerical methods, if the analytical process is not feasible. Analyzing wavesô propagation through the transmission line, this system is decomposed into infinitesimal parts represented by ´ circuits. This makes it possible to monitor how the electrical variables (current and voltage) vary with the time. The model consists of three different ´ circuits: the first, intermediate and last units. In Fig. 2, the intermediate unit is represented, being composed by capacitors, resistors and inductors [1]-[10].

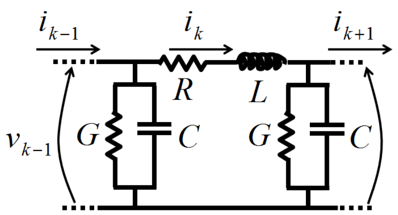

Figure 2. Unit of ´ circuit for transmission line representation.

Based on Fig. 2, the differential relations are obtained:

$$\frac{di_k}{dt} = \dot{i}_k = \frac{1}{L}\left(v_{k-1} - R \cdot i_k - v_k\right) \quad (4)$$

$$\frac{dv_k}{dt} = \dot{v}_k = \frac{1}{C}\left(i_k - G \cdot v_k - i_{k+1}\right) \quad (5)$$

According to (4) and (5), the electrical variables of the k unit of ´ circuit depend on the previous ´ circuitôs voltage and the next electrical ´ circuitôs current. For a transmission line, it is obtained a linear system composed by 2n state variables where n is the number of ´ circuits:

$$x_{2n} = \begin{bmatrix} i_1 & v_1 & i_2 & v_2 & i_3 & v_3 & \cdots & i_n & v_n \end{bmatrix}^T \quad (6)$$

The matrix state equation is:

$$\dot{x} = [A]x + [B]u \quad (7)$$

The *A* matrix is based on differential equations of the cascade of ´ circuits with their differential equations involving current and voltage. The structure of this matrix is:

$$A = \begin{bmatrix} -R/L & -1/L & 0 & \cdots & \cdots & 0 \\ 1/C & -G/C & -1/C & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & 1/C & -G/C & -1/C & 0 \\ \vdots & \ddots & \ddots & 1/L & -R/L & -1/L \\ 0 & \cdots & \cdots & 0 & 2/C & -G/C \end{bmatrix} \quad (8)$$

The *B* vector is related to the systemôs sources:

$$B = \begin{bmatrix} 1/L & 0 & \cdots & 0 \end{bmatrix}^T \quad (9)$$

Introducing the damping resistance, the schematic representation is in Fig. 3 [7].

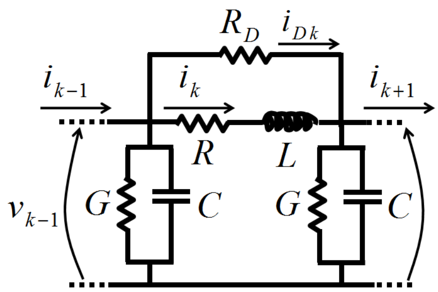

Figure 3. Unit of ´ circuit with the damping resistance.

Based on Fig. 3, it is obtained the following differential relations:

$$\dot{i}_k = \frac{v_{k-1} - R \cdot i_k - v_k}{L} \quad (10)$$

$$\dot{v}_k = \frac{i_k - (2G_D + G)v_k + G_D(v_{k-1} + v_{k+1}) - i_{k+1}}{C} \quad (11)$$

The damping resistance is calculated based on the k_D proportional factor that is a integer value [7]:

$$R_D = k_D \frac{2L}{\Delta t} \quad \leftrightarrow \quad G_D = \frac{1}{R_D} \tag{12}$$

The k_D proportional factor is a integer value. The introduction of R_D leads to changes to the A matrixôs structure. The change to the A_{kk} element of even lines is:

$$A_{kk} = \frac{-(2G_D + G)}{C}, \quad k \text{ is even number} \tag{13}$$

For the even lines, there are new non-null values:

$$A_{k(k-2)} = A_{k(k+2)} = \frac{G_D}{C}, \quad k \text{ is even number} \tag{14}$$

For the A matrixôs last line, the new elements are:

$$A_{2n(2n-2)} = \frac{G_D}{C} \quad and \quad A_{2n\,2n} = \frac{-(2G_D - G)}{C} \tag{15}$$

Applying the trapezoidal rule (Heunôs method), the numeric solution of the linear system in (8) is:

$$x_{t+1} = \left[I - \frac{\Delta t}{2}A\right]^{-1}\left\{\left[I + \frac{\Delta t}{2}A\right]x_t + \frac{\Delta t}{2}\left(Bu_{t+1} + Bu_t\right)\right\} \tag{16}$$

Considering that d is the line length e n is the number of ´ circuits, the values of R, L, G and C are calculated by:

$$R = R'\cdot\frac{d}{n} \qquad L = L'\cdot\frac{d}{n} \qquad G = G'\cdot\frac{d}{n} \qquad C = C'\cdot\frac{d}{n} \tag{17}$$

IV. Results Obtained without the Damping Resistance

Using (8), (9) and (16), it is obtained the simulations without the damping resistance. The flowchart for these simulations is shown in Fig. 4 and the parametersô values are: d=5 km, Rô=0.05 Ý/km, Lô=1 mH/km, Gô=0.556 µS/km, C=11.11 nF/km, çt=50 ns and t_{MAX}=150 ms. The voltage source is a step voltage source. The number of ´ circuits (n) is changed, checking for the influence of this parameter on the voltage peak at the receiving end terminal of the line. The receiving end terminal is opened and, because of this, the voltage at this terminal is the double value of the voltage at the sending end terminal.

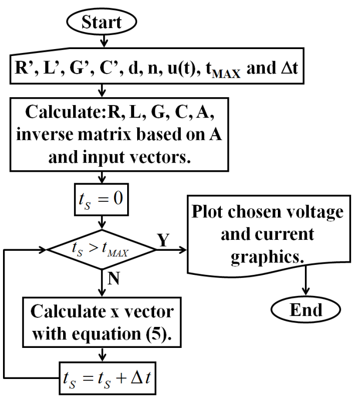

Figure 4. Flowchart for the simulations without the damping resistance.

Using 100 ´ circuits, the voltage peak at the receiving end terminal reaches 2.5 pu. Comparing to the 2 pu value, it is equivalent to the 25 % error caused by Gibbôs oscillations or numeric oscillations (Fig. 5). Considering 200 and 400 ´ circuits, the voltage peaks at the receiving end terminal are also 2.5 pu and the maximum error of Gibbôs oscillations remains in 25 % for all obtained simulations. These results are shown in Figs. 6 and 7. For comparison, Fig. 8 shows the details of the voltage peaks obtained in Figs. 5, 6 and 7.

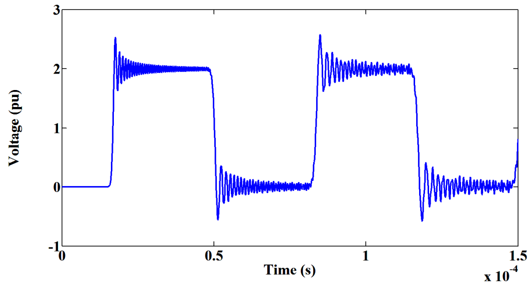

Figure 5. Results of simulation using 100 ´ circuits.

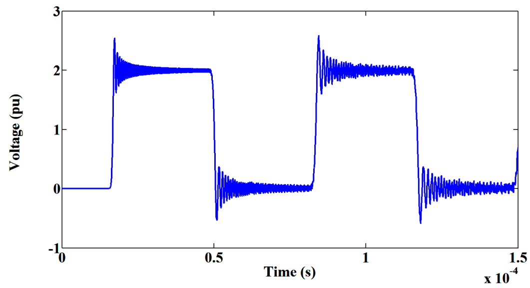

Figure 6. Results of simulation using 200 ´ circuits.

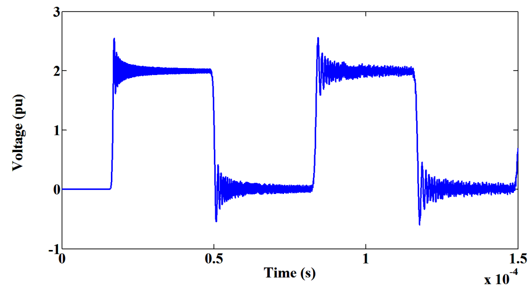

Figure 7. Results of simulation using 400 ´ circuits.

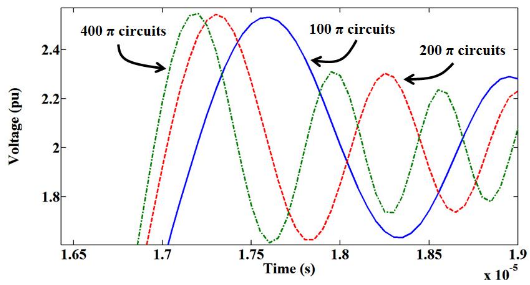

Figure 8. Comparison of the results of simulations without the resistance damping.

Considering the range of the number of ´ circuits (n) from 3 to 400, the applied n values are related to the voltage peaks at the receiving end terminal in Fig. 9. So, the increase of the number of ´ circuits cannot influence the maximum voltage peak error caused by Gibbôs oscillations.

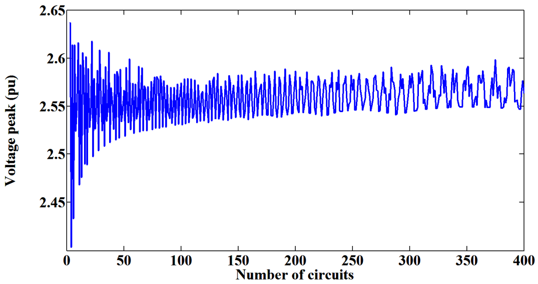

Figure 9. Voltage peaks for different quantities of ´ circuits.

V. Results Obtained with Damping Resistance

Introducing the damping resistance from (12), the flowchart that is used for the results obtained in this item is shown in Fig. 10.

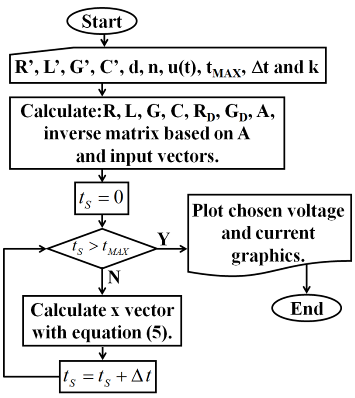

Figure 10. Flowchart for the simulations with the damping resistance.

Using 100 ´ circuits and five different values of k factor for determining the R_D values, the comparisons among the voltage peaks at the receiving and terminal are shown in Fig. 11. So, increasing the k factorôs value, the maximum error caused by Gibbôs oscillations is decreased. Similar conclusions can be related to the results shown in Figs. 12 and 13 that associated to 200 and 400 ´ circuits, respectively.

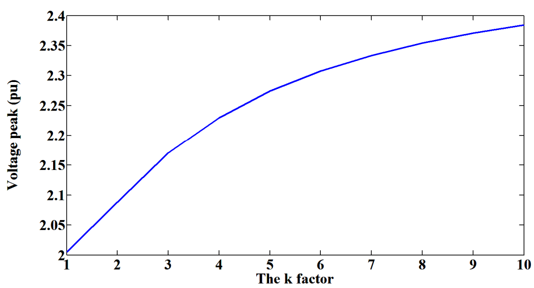

Figure 11. Results of simulations using 100 ´ circuits.

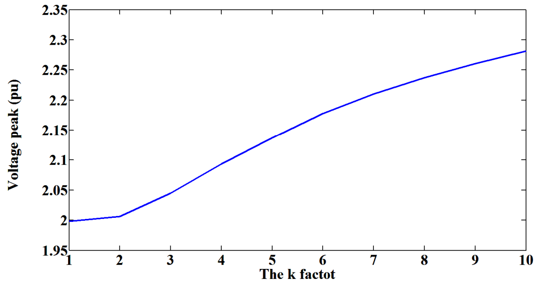

Figure 12. Results of simulations using 200 ´ circuits.

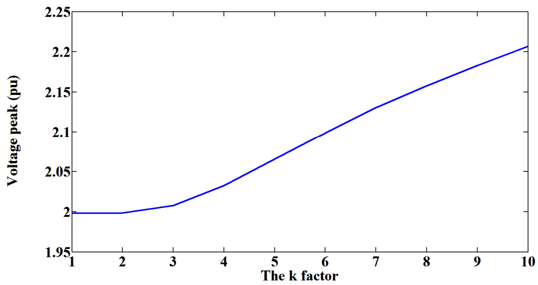

Figure 13. Results of simulations using 400 ´ circuits.

Using the k factor equal to 6 and different quantities of ´ circuits, the comparisons among the voltage peakôs values are shown in Fig. 14. Completing the analyzes and considering the range of the number of ´ circuits from 3 to 400 and the k factor from 1 to 10, the 3D surface is used to analyze the influence of these parameters on the voltage peak at the receiving end terminal of the line (Fig. 15).

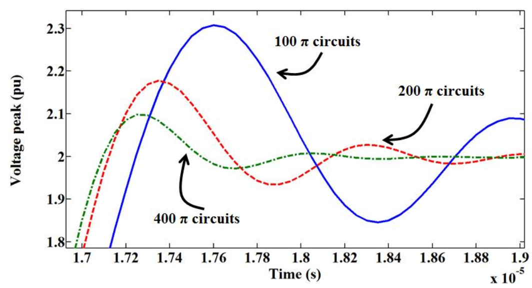

Figure 14. Results of simulations with the resistance damping.

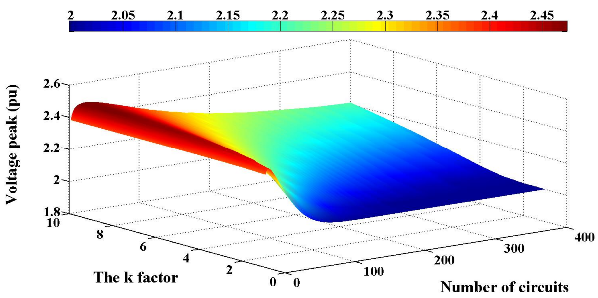

Figure 15. Influence of the factor k and the number of ´ circuits on the voltage peak values at the receiving end terminal line.

Based on Fig. 14, if the k factorôs values are higher than 5, the voltage peakôs values are approximately higher than 2.2 pu, corresponding to errors caused by Gibbôs oscillations higher than 10 %. Analyzing the influence, if the number of ´ circuits is lower than 150 units, the influence of the Gibbôs oscillations on the voltage peakôs values cannot be considered negligible. So, using (12), (13), (14), (15), (16) and the flowchart of Fig. 10, better results of the simulation based on the parameters mentioned in the item IV are obtained applied the number of ´ circuits higher than 200 and the k factorôs values lower than 5. In this case, the k factorôs values are not lower than 1 and they should be integer values

In this case, the damping resistance is included in the matrix structure that represents the entire cascade of ´ circuits. This matrix has several null elements. So, it could be investigated the introduction of the damping resistance concept in the sparse matrix structures for modeling transmission lines during electromagnetic transient phenomena propagation.

VI. Conclusions

Numeric integration methods are useful tools for digital simulations that are related to the first order models of physical phenomena. These methods can be influenced by numeric oscillations or Gibbôs oscillations. For example, if the infinitesimal unit of transmission lines is modeled using the ´ circuit structure and this model is applied for simulating the electromagnetic transientsô wave propagation, the obtained results can be highly influenced by Gibbôs oscillations when the input is composed by high frequency signals. Analyzing the application of cascades of ´ circuits for simulating electromagnetic transient phenomena in transmission lines, it is investigated the results obtained with a change in the structure that represents the transmission line longitudinal parameters. In this case, it is not investigate the efficiency and the accuracy of trapezoidal rule (Heunôs method) related to the other numerical methods. So, it has suggested by other authors the inclusion of a damping resistance in the mentioned structure. Using this suggestion, several simulations are carried out and the results are compared to those results obtained using the model without the damping resistance. Without the resistance damping, increasing the number of ´ circuits of the model, the errors caused by Gibbôs oscillations are not highly decreased. The errors of voltage peaks reach approximately 25 %. Applying the damping resistance, the voltage peaks depend on the combined influence of the damping resistance values and the ´ circuitsô number ones. With the damping resistance, it can be obtained negligible numeric errors. It will be interesting to apply the concept of the damping resistance combined with sparse matrix techniques.

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