A Stochastic Reserve Market Clearing Approach Considering Distributed Generation and Electrical Vehicles

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***Abstract*—The so-called ancillary services, which are necessary to ensure the stability and security of the electrical power system, could be also provided by distributed generators (DGs) due to the well-known benefits that the DGs can bring. The variation in the generation of DGs based on renewable energy sources and operating characteristics of electrical vehicles (EVs) introduce uncertainties, which need to be extensively studied in order to consider their participation as a reliable alternative. In this work, a co-optimization market-based proposal for *energy* and *reserve* considering the presence of DGs and EVs is presented; the uncertainties of DGs with high variation in the generation are addressed through the development of a stochastic model based on the enhanced interval linear programming theory.**

***Index Terms*—Distributed generation, Electrical vehicles, Interval Linear Programming, Distribution systems, Co-optimization market.**

NOTATION

The notation used throughout this paper is reproduced below for quick reference. In constants and variables where it appears, t corresponds to the current time instant.

Sets:

Ω_{Bus} buses of the system
Ω_{Br} branches of the system
Ω_{DG} DGs. Note that Ω_{DG} is a set composed by DGs_{U} (e.g., wind turbines) and DGs_{NU} (e.g., small hydros)
Ω_{EV} EVs
Ω_{Disc} discretizations for linearization of $V_j^{\text{Sq}\pm}$ ($\Omega_{\text{Disc*}}$: without the first element)
Ω_{L} blocks of the piecewise linearization for $P_{ij}^{\pm 2}$ and $Q_{ij}^{\pm 2}$
Ω_{T} study time ($\Omega_{\text{T*}}$:without the first element)

Constants:

$\alpha_t^{\pm}$	cost of *energy* from the main grid bought by the DSO
$\beta_t^{\pm}$	price of *reserve* sold to the main grid by the DSO
$R_{\text{Br},ij}$, $X_{\text{Br},ij}$	resistance and reactance of branch ij
$P_{\text{D},i,t}^{\pm}$, $Q_{\text{D},i,t}^{\pm}$	active and reactive power demanded at node i
$\overline{\Delta}_{\text{V}}$	discretization step of $V_{j,t}^{\text{Sq}\pm}$
$\Delta_{\text{P},l,ij,t}^{\pm}$, $\Delta_{\text{Q},l,ij,t}^{\pm}$	values of the l th block of $\lvert P_{ij,t}^{\pm}\rvert$ and $\lvert Q_{ij,t}^{\pm}\rvert$
$\overline{\Delta}_{\text{S},ij}$	upper bound of each block of the power flow of branch ij
$m_{\text{S},l,ij}$	slope of the l th block of the power flow of branch ij
$R_{\text{Req},t}^{\pm}$	*reserve* requirement
$\underline{P_{\text{DG}}}_{,g}$, $\overline{P_{\text{DG}}}_{,g}$	minimum and maximum active power generation limits of the DG g
$\underline{Q_{\text{DG}}}_{,g}$, $\overline{Q_{\text{DG}}}_{,g}$	minimum and maximum reactive power generation limits of the DG g
$\underline{V}$, $\overline{V}$	minimum and maximum voltage limits
$\overline{I}_{ij}$	maximum current limit in branch ij
$\underline{pf}_{\text{SS}}$	minimum leading and lagging substation power factor limit
$of_{\text{E},g,i,t}^{\pm}$, $of_{\text{R},g,i,t}^{\pm}$	offers of *energy* and *reserve* of the DG g
μ_g^{dec}, μ_g^{inc}	decreasing and increasing ramps of the DG g
Δt	time interval to pass from an active power generation level to another
η	response time in which the *reserve* must be available to restore the system frequency after a disturb
$\overline{P_{\text{EV}}}_{,e,i}$	maximum charging power provided to the EV e at bus i
$E_{\text{EV},e,i,t}^{0}$, $\overline{E_{\text{EV}}}_{,e,i}$	initial energy charge and capacity of the EV e at bus i
$\Delta P_{\text{EV},e,i}$	maximum variation between two time instants of the charging power provided to the EV e at node i
$t_{\text{EV},e,i}^{\text{Con}}$, $t_{\text{EV},e,i}^{\text{Dis}}$	connection and disconnection times of the EV e assigned to node i

Variables:

$PB^{\pm}$	payment burden of the DSO
$FC^{\pm}$	financial compensation to the DGs for *energy* and *reserve* sold to the DSO
$FR^{\pm}$	financial reward of the DSO for *reserve* sold to the main grid
$E_{g,i,t}^{\pm}$, $R_{g,i,t}^{\pm}$	*energy* and *reserve* provided by the DG g at

	node i
$P^{\pm}_{\mathrm{SS},i,t}$, $Q^{\pm}_{\mathrm{SS},i,t}$	active and reactive power provided by the substation at node i
$I^{\mathrm{Sq}\pm}_{ij,t}$	square of the current in branch ij
$P^{\pm}_{ij,t}$, $Q^{\pm}_{ij,t}$	active and reactive power flows in branch ij
$P^{\pm}_{\mathrm{EV},e,i,t}$	charging power provided to the EV e at node i
$n^{\pm}_{\mathrm{EV},e,i,t}$	binary variable associated to the EV e at node i
$V^{\mathrm{Sq}\pm}_{i,\tau}$	square of the voltage at note i
$\pi^{\pm}_{\mathrm{V},u,j,\tau}$	binary variable for linearization of $V^{\mathrm{Sq}\pm}_{j,\tau}$ in discretization u
$P^{\pm}_{\mathrm{C},u,j,\tau}$	power corrections used in the linearization of $V^{\mathrm{Sq}\pm}_{j,\tau}$ in discretization u
$P^{\uparrow\pm}_{ij,\tau}$, $P^{\downarrow\pm}_{ij,\tau}$	non-negative auxiliaries to obtain $\lvert P^{\pm}_{ij,\tau}\rvert$
$Q^{\uparrow\pm}_{ij,\tau}$, $Q^{\downarrow\pm}_{ij,\tau}$	non-negative auxiliaries to obtain $\lvert Q^{\pm}_{ij,\tau}\rvert$
$P^{\pm}_{\mathrm{DG},g,i,t}$, $Q^{\pm}_{\mathrm{DG},g,i,t}$	active and reactive power generated by the DG g at node i
$P_{k,g}$, $Q_{k,g}$	k th point of the linearization of the capability curve of DG g
$\xi^{\pm}_{\mathrm{E},t}$, $\xi^{\pm}_{\mathrm{R},t}$	market price of *energy* and *reserve* for DG g at node i
$n^{\pm}_{\mathrm{E},g,i,t}$, $n^{\pm}_{\mathrm{R},g,i,t}$	binary variables which represent if DG g at node i is providing *energy* and/or *reserve*

I. Introduction

The growing participation of distributed generation and the well-known benefits that the distributed generators (DGs) can bring [1], suggest that they could provide not only energy to satisfy loads and system losses, but also other essential services, known as ancillary services [2], which are necessary to ensure the stability and security of the system.

Specifically, with respect to DGs supporting with active power reserve, some works can be found. In [3], a proposal to evaluate the operating reserve requirements in transmission systems with large amounts of renewable energy sources is presented. An analysis of the impact in the reserve requirements in transmission systems with the interaction between the variability characteristics of the load, wind power generation, solar power generation, and ocean wave power generation is presented in [4].

An approach to solve markets for *energy* (defined here as the active power required to meet loads and losses) and *reserve* (defined here as the active power required to fulfill reserve requirements) could be by creating a joint dispatch structure, which includes both *energy* and *reserve* markets, usually called as co-optimization structure [5].

On the other hand, as result of technological advances and the interest to meet environmental, transportation and energy requirements, the development of Electric Vehicles (EVs) has gained importance in recent years. Studies to analyze the impact of the EVs on the distribution networks have to be conducted in order to preserve the operation of the electrical system within their normal security levels. Factors such as battery technologies, charging profile, market mechanism, policies and operating limits impact on the permitted levels of EV penetration in distribution power systems [6].

Uncertainty factors related to the variation in the generation of DGs based on renewable energy sources (RES) and operating characteristics of EVs, such as, initial energy charge of their batteries and connection/disconnection times, lead to the development of stochastic analysis in order to consider their participation in the electrical power system as a reliable alternative.

In real-world problems, most of the input parameters are characterized by presenting uncertainties. In traditional Linear Programming (LP) problems, these uncertainties are not considered and all parameters are treated as deterministic values. Through the Interval Linear Programming (ILP), these uncertainties are considered intrinsically by representing all model elements by intervals (interval objective function, interval parameters and interval variables). The main advantage of the ILP is the ability to incorporate interval-numbers uncertainties into the linear model without any assumption of probabilistic distributions [7]. An ILP model, which in this work is denoted by the symbol "±", is formed by two LP sub-models: the first sub-model, denoted by the symbol "–", and the second sub-model, denoted by the symbol "+". In this work, an EILP model based on the ILP model, which is presented in detail in [7], is used to account for uncertainties of the DGs and EVs.

In this work, a co-optimization market-based proposal considering the presence of DGs and EVs is presented. Uncertainties related to the DGs based on RES and operating characteristics of the EVs are addressed through the development of a stochastic model based on the Enhanced Interval Linear Programming (EILP) theory [7].

The main contributions of this work are listed below:

1) The participation of the DGs as providers the ancillary service of *reserve* is encouraged.
2) A novel Mixed-integer Linear Programming (MILP) formulation for a co-optimization market clearing process which includes precise models of the DGs and EVs operation.
3) The uncertainty related to DGs based on RES and operating characteristics of EVs is conducted by using the EILP theory, which allows a comprehensive stochastic analysis.

This paper is organized as follows: in Section II, the assumptions and mathematical formulation of the proposed market-based problem considering DGs and EVs is presented; the results and a discussion about several performed test are presented in Section III; and, finally, the conclusions are exposed in Section IV.

II. Assumptions and Formulation of the Market-based Problem

The assumptions and mathematical formulation of the proposed co-optimization market-based problem are presented in next subsections.

A. Assumptions

In order to represent the steady-state operation of a radial distribution system and to perform the market clearing process, some assumptions were made, as presented below:

- Since this approach is directed to primary distribution networks, the distribution system is balanced and represented by a monophasic equivalent.
- The set of DGs are divided into two sub-sets corresponding to the DGs with uncertainties, say DGs_{U} and the DGs without uncertainty, say DGs_{NU}.
- The DSO gets a capacity contract for *reserve* from the TSO in a day basis.
- *Energy* and *reserve* are purchased from all the participating DGs by a single buyer, the DSO, which performs the market clearing. The DSO can also buy *energy* from the main grid.
- The offers of the DGs are presented such that the market is solved with one day in advance, i.e., in a day-ahead basis. These offers consist of prices and capacities of *energy* and *reserve*, represented in the formulation of the market-based problem by the interval parameters $of^{\pm}_{\text{E},g,t}$ and $of^{\pm}_{\text{R},g,t}$ ($\forall g \in \Omega_{\text{DG}}, \forall t \in \Omega_{\text{T}}$), respectively. DGs must also inform the tilt of their decreasing and increasing ramps, say μ^{dec}_{g} and μ^{inc}_{g} ($\forall g \in \Omega_{\text{DG}}$), respectively.
- All DGs providing *energy* and/or *reserve* are paid based on the interval uniform prices, say $\xi^{\pm}_{\text{E},t}$ and $\xi^{\pm}_{\text{R},t}$ ($\forall t \in \Omega_{\text{T}}$), which are the highest priced offers accepted.
- The EVs are monitored using the Advanced Metering Infrastructure (AMI) for EVs charging, which measures, collects and analyzes energy consumption from EVs charging station regularly via communication links [8].
- The EVs are type plug-in hybrid electric vehicle (PHEVs), which utilize rechargeable batteries, that can be restored to full charge by connecting a plug to an external electric power source (usually a normal electric wall socket).

B. Formulation of the Stochastic Co-optimization Market-based Problem

The mathematical formulation of the stochastic co-optimization market-based problem is presented below.

$$\text{Min } PB^{\pm} = \sum_{t\in\Omega_{\text{T}}}\sum_{i\in\Omega_{\text{Bus}}} \alpha^{\pm}_{t} P^{\pm}_{\text{SS},i,t} + \sum_{t\in\Omega_{\text{T}}}\sum_{i\in\Omega_{\text{Bus}}}\sum_{g\in\Omega_{\text{DG}}} \left(\xi^{\pm}_{\text{E},t} E^{\pm}_{g,i,t} + \xi^{\pm}_{\text{R},t} R^{\pm}_{g,i,t}\right) - \sum_{t\in\Omega_{\text{T}}}\sum_{i\in\Omega_{\text{Bus}}}\sum_{g\in\Omega_{\text{DG}}} \beta^{\pm}_{t} R^{\pm}_{g,i,t} \quad (1)$$

Subject to:

$$\sum_{ki\in\Omega_{\text{Br}}} P^{\pm}_{ki,t} - \sum_{ij\in\Omega_{\text{Br}}}\left(P^{\pm}_{ij,t} + R_{\text{Br},ij} I^{\text{Sq}\pm}_{ij,t}\right) + P^{\pm}_{\text{SS},i,t} + \sum_{g\in\Omega_{\text{DG}}} E^{\pm}_{g,i,t} - P^{\pm}_{\text{D},i,t} - \sum_{e\in\Omega_{\text{EV}}} n^{\pm}_{\text{EV},e,i,t} P^{\pm}_{\text{EV},e,i,t} = 0 \qquad \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (2)$$

$$\sum_{ki\in\Omega_{\text{Br}}} Q^{\pm}_{ki,t} - \sum_{ij\in\Omega_{\text{Br}}}\left(Q^{\pm}_{ij,t} + X_{\text{Br},ij} I^{\text{Sq}\pm}_{ij,t}\right) + Q^{\pm}_{\text{SS},i,t} + \sum_{g\in\Omega_{\text{DG}}} Q^{\pm}_{\text{DG},g,i,t} - Q^{\pm}_{\text{D},i,t} = 0 \qquad \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (3)$$

$$V^{\text{Sq}\pm}_{i,t} - 2\left(R_{\text{Br},ij} P^{\pm}_{ij,t} + X_{\text{Br},ij} Q^{\pm}_{ij,t}\right) - \bar{Z}^{2}_{\text{Br},ij} I^{\text{Sq}\pm}_{ij,t} - V^{\text{Sq}\pm}_{j,t} = 0 \qquad \forall ij \in \Omega_{\text{Br}}, \forall t \in \Omega_{\text{T}} \quad (4)$$

$$\underline{V}^2 + \sum_{u\in\Omega_{\text{Disc}}}\left(\pi^{\pm}_{\text{V},u,j,t}\bar{\Delta}_{\text{V}}\right) \le V^{\text{Sq}\pm}_{j,t} \le \underline{V}^2 + \bar{\Delta}_{\text{V}} + \sum_{u\in\Omega_{\text{Disc}}}\left(\pi^{\pm}_{\text{V},u,j,t}\bar{\Delta}_{\text{V}}\right) \qquad \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (5)$$

$$\pi^{\pm}_{\text{V},u-1,j,t} \le \pi^{\pm}_{\text{V},u,j,t} \qquad \forall u \in \Omega_{\text{Disc*}}, \forall j \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (6)$$

$$\pi^{\pm}_{\text{V},u,j,t} \in \{0,1\} \qquad \forall u \in \Omega_{\text{Disc}}, \forall j \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (7)$$

$$0 \le \bar{\Delta}_{\text{V}} I^{\text{Sq}\pm}_{ij,t} - P^{\pm}_{\text{C},u,j,t} \le \bar{\Delta}_{\text{V}} \bar{I}^{2}_{ij}\left(1 - \pi^{\pm}_{\text{V},u,j,t}\right) \qquad \forall u \in \Omega_{\text{Disc}}, \forall ij \in \Omega_{\text{Br}}, \forall t \in \Omega_{\text{T}} \quad (8)$$

$$0 \le P^{\pm}_{\text{C},u,j,t} \le \bar{\Delta}_{\text{V}} \bar{I}^{2}_{ij} \pi^{\pm}_{\text{V},u,j,t} \qquad \forall u \in \Omega_{\text{Disc}}, \forall ij \in \Omega_{\text{Br}}, \forall t \in \Omega_{\text{T}} \quad (9)$$

$$P^{\uparrow\pm}_{ij,t} - P^{\downarrow\pm}_{ij,t} = P^{\pm}_{ij,t} \qquad \forall ij \in \Omega_{\text{Br}}, \forall t \in \Omega_{\text{T}} \quad (10)$$

$$Q^{\uparrow\pm}_{ij,t} - Q^{\downarrow\pm}_{ij,t} = Q^{\pm}_{ij,t} \qquad \forall ij \in \Omega_{\text{Br}}, \forall t \in \Omega_{\text{T}} \quad (11)$$

$$P^{\uparrow\pm}_{ij,t} + P^{\downarrow\pm}_{ij,t} = \sum_{l\in\Omega_{\text{L}}} \Delta^{\pm}_{\text{P},l,ij,t} \qquad \forall ij \in \Omega_{\text{Br}}, \forall t \in \Omega_{\text{T}} \quad (12)$$

$$Q^{\uparrow\pm}_{ij,t} + Q^{\downarrow\pm}_{ij,t} = \sum_{l\in\Omega_{\text{L}}} \Delta^{\pm}_{\text{Q},l,ij,t} \qquad \forall ij \in \Omega_{\text{Br}}, \forall t \in \Omega_{\text{T}} \quad (13)$$

$$0 \le \Delta^{\pm}_{\text{P},l,ij,t} \le \bar{\Delta}_{\text{S},ij} \qquad \forall l \in \Omega_{\text{L}}, \forall ij \in \Omega_{\text{Br}}, \forall t \in \Omega_{\text{T}} \quad (14)$$

$$0 \le \Delta^{\pm}_{\text{Q},l,ij,t} \le \bar{\Delta}_{\text{S},ij} \qquad \forall l \in \Omega_{\text{L}}, \forall ij \in \Omega_{\text{Br}}, \forall t \in \Omega_{\text{T}} \quad (15)$$

$$0 \le P^{\uparrow\pm}_{ij,t},\ P^{\downarrow\pm}_{ij,t},\ Q^{\uparrow\pm}_{ij,t},\ Q^{\downarrow\pm}_{ij,t} \qquad \forall ij \in \Omega_{\text{Br}}, \forall t \in \Omega_{\text{T}} \quad (16)$$

$$\left(\underline{V}^2 + \frac{1}{2}\bar{\Delta}_{\text{V}}\right) I^{\text{Sq}\pm}_{ij,t} + \sum_{u\in\Omega_{\text{Disc}}} P^{\pm}_{\text{C},u,j,t} = \sum_{l\in\Omega_{\text{L}}} m_{\text{S},l,ij}\Delta^{\pm}_{\text{P},l,ij,t} + \sum_{l\in\Omega_{\text{L}}} m_{\text{S},l,ij}\Delta^{\pm}_{\text{Q},l,ij,t} \qquad \forall ij \in \Omega_{\text{Br}}, \forall t \in \Omega_{\text{T}} \quad (17)$$

$$\sum_{i\in\Omega_{\text{Bus}}}\sum_{g\in\Omega_{\text{DG}}} R^{\pm}_{g,i,t} = R^{\pm}_{\text{Req},t} \qquad \forall t \in \Omega_{\text{T}} \quad (18)$$

$$\underline{P_{\text{DG}}}_{,g} \le E^{\pm}_{g,i,t} + R^{\pm}_{g,i,t} \le \overline{P_{\text{DG}}}_{,g} \qquad \forall g \in \Omega_{\text{DG}}, \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (19)$$

$$E^{\pm}_{g,i,t} + R^{\pm}_{g,i,t} \le \frac{P_{1,g}}{Q_{1,g} - \underline{Q_{\text{DG}}}}\left(Q^{\pm}_{\text{DG},g,i,t} - \underline{Q_{\text{DG}}}_{,g}\right) \qquad \forall g \in \Omega_{\text{DG}}, \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (20)$$

$$E^{\pm}_{g,i,t} + R^{\pm}_{g,i,t} \le \frac{P_{2,g} - P_{1,g}}{Q_{2,g} - Q_{1,g}}\left(Q^{\pm}_{\text{DG},g,i,t} - Q_{2,g}\right) + P_{2,g} \qquad \forall g \in \Omega_{\text{DG}}, \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (21)$$

$$E^{\pm}_{g,i,t} + R^{\pm}_{g,i,t} \le \frac{P_{3,g} - P_{2,g}}{Q_{3,g} - Q_{2,g}}\left(Q^{\pm}_{\text{DG},g,i,t} - Q_{3,g}\right) + P_{3,g} \qquad \forall g \in \Omega_{\text{DG}}, \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (22)$$

$$E^{\pm}_{g,i,t} + R^{\pm}_{g,i,t} \le \frac{P_{4,g} - P_{3,g}}{Q_{4,g} - Q_{3,g}}\left(Q^{\pm}_{\text{DG},g,i,t} - Q_{4,g}\right) + P_{4,g} \qquad \forall g \in \Omega_{\text{DG}}, \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (23)$$

$$E^{\pm}_{g,i,t} + R^{\pm}_{g,i,t} \le \frac{P_{4,g}}{Q_{4,g} - \bar{Q}_{g}}\left(Q^{\pm}_{\text{DG},g,i,t} - \overline{Q_{\text{DG}}}_{,g}\right) \qquad \forall g \in \Omega_{\text{DG}}, \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (24)$$

$$E^{\pm}_{g,i,t} \ge 0 \qquad \forall g \in \Omega_{\text{DG}}, \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (25)$$

$$\underline{V}^2 \le V^{\text{Sq}\pm}_{i,t} \le \bar{V}^2 \qquad \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (26)$$

$$0 \le I^{\text{Sq}\pm}_{ij,t} \le \bar{I}^{2}_{ij} \qquad \forall ij \in \Omega_{\text{Br}}, \forall t \in \Omega_{\text{T}} \quad (27)$$

$$P^{\pm}_{\text{SS},i,t}\, tan\left(arccos\left(\underline{pf}_{\text{SS}}\right)\right) \le \left|Q^{\pm}_{\text{SS},i,t}\right| \qquad \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (28)$$

$$\xi^{\pm}_{\text{E},t} = \max\left\{n^{\pm}_{\text{E},g,i,t}\, of^{\pm}_{\text{E},g,i,t}\right\}_{\forall g\in\Omega_{\text{DG}}} \qquad \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (29)$$

$$\xi^{\pm}_{\text{R},t} = \max\left\{n^{\pm}_{\text{R},g,i,t}\, of^{\pm}_{\text{R},g,i,t}\right\}_{\forall g\in\Omega_{\text{DG}}} \qquad \forall i \in \Omega_{\text{Bus}}, \forall t \in \Omega_{\text{T}} \quad (30)$$

$$n^{\pm}_{\mathrm{E},g,i,t} = \begin{cases} 1, & \text{if} \quad E^{\pm}_{g,i,t} > 0 \\ 0, & \text{if} \quad E^{\pm}_{g,i,t} = 0 \end{cases} \qquad \forall g \in \Omega_{\mathrm{DG}}, \forall i \in \Omega_{\mathrm{Bus}}, \forall t \in \Omega_{\mathrm{T}} \quad (31)$$

$$n^{\pm}_{\mathrm{R},g,i,t} = \begin{cases} 1, & \text{if} \quad R^{\pm}_{g,i,t} > 0 \\ 0, & \text{if} \quad R^{\pm}_{g,i,t} = 0 \end{cases} \qquad \forall g \in \Omega_{\mathrm{DG}}, \forall i \in \Omega_{\mathrm{Bus}}, \forall t \in \Omega_{\mathrm{T}} \quad (32)$$

$$\mu^{\mathrm{dec}}_{g} \Delta t \le E^{\pm}_{g,i,t} + R^{\pm}_{g,i,t} - \left(E^{\pm}_{g,i,t-1} + R^{\pm}_{g,i,t-1} \right) \le \mu^{\mathrm{inc}}_{g} \Delta t \qquad \forall g \in \Omega_{\mathrm{DG}}, \forall i \in \Omega_{\mathrm{Bus}}, \forall t \in \Omega_{\mathrm{T}^*} \quad (33)$$

$$0 \le R^{\pm}_{g,i,t} \le \mu^{\mathrm{inc}}_{g} \eta \qquad \forall g \in \Omega_{\mathrm{DG}}, \forall i \in \Omega_{\mathrm{Bus}}, \forall t \in \Omega_{\mathrm{T}} \quad (34)$$

$$0 \le P^{\pm}_{\mathrm{EV},e,i,t} \le n^{\pm}_{\mathrm{EV},e,i,t} \overline{P}_{\mathrm{EV},e,i} \qquad \forall e \in \Omega_{\mathrm{EV}}, \forall i \in \Omega_{\mathrm{Bus}}, \forall t \in \Omega_{\mathrm{T}} \quad (35)$$

$$n^{\pm}_{\mathrm{EV},e,i,t} E^{0}_{\mathrm{EV},e,i,t} \le E^{\pm}_{\mathrm{EV},e,i,t} \le n^{\pm}_{\mathrm{EV},e,i,t} \overline{E}_{\mathrm{EV},e,i} \qquad \forall e \in \Omega_{\mathrm{EV}}, \forall i \in \Omega_{\mathrm{Bus}}, \forall t \in \Omega_{\mathrm{T}} \quad (36)$$

$$E^{\pm}_{\mathrm{EV},e,i,t} = n^{\pm}_{\mathrm{EV},e,i,t-1} E^{\pm}_{\mathrm{EV},e,i,t-1} + n^{\pm}_{\mathrm{EV},e,i,t} P^{\pm}_{\mathrm{EV},e,i,t} \Delta t \qquad \forall e \in \Omega_{\mathrm{EV}}, \forall i \in \Omega_{\mathrm{Bus}}, \forall t \in \Omega_{\mathrm{T}} \quad (37)$$

$$n^{\pm}_{\mathrm{EV},e,i,t-1} P^{\pm}_{\mathrm{EV},e,i,t} = n^{\pm}_{\mathrm{EV},e,i,t-1} \left(1 - \frac{E^{\pm}_{\mathrm{EV},e,i,t-1}}{\overline{E}_{\mathrm{EV},e,i}} \right) \min \left\{ \frac{\overline{E}_{\mathrm{EV},e,i} - E^{\pm}_{\mathrm{EV},e,i,t-1}}{\Delta t}, P^{\pm}_{\mathrm{EV},e,i,t-1} + \Delta P_{\mathrm{EV},e,i}, \overline{P}_{\mathrm{EV},e,i} \right\} \qquad \forall e \in \Omega_{\mathrm{EV}}, \forall i \in \Omega_{\mathrm{Bus}}, \forall t \in \Omega_{\mathrm{T}} \quad (38)$$

$$n^{\pm}_{\mathrm{EV},e,i,t} = \begin{cases} 1, & \text{if} \quad t^{\mathrm{Con}}_{\mathrm{EV},e,i} \le t \le t^{\mathrm{Dis}}_{\mathrm{EV},e,i} \\ 0, & \text{elsewhere} \end{cases} \qquad \forall e \in \Omega_{\mathrm{EV}}, \forall i \in \Omega_{\mathrm{Bus}}, \forall t \in \Omega_{\mathrm{T}} \quad (39)$$

The constraints that represent the steady-state operation of a radial distribution system correspond to expressions (2)-(17) *equation reference goes here*, which are commonly used in the load flow sweep method [9], and they are linearized expressions obtained using the linearization process exposed in [10]. In (18), the required *reserve*, which should be attended by the DGs, is represented. Constraints (19)-(25) correspond to a detailed representation of the operational limits of the DGs, which are linear expressions, related to the capability curves of the SGs [11] and DFIGs [12], as presented in [10]. Voltages in buses must be within the specified limits, according to (26), and currents in branches must be less than the specified maximum values, as shown in (27). The power factor of the substation, both leading and lagging, must be within specified limits, as shown in (28). Constraints (29) and (30) represent the highest priced offers accepted, i.e., the interval uniform market prices of *energy* and *reserve* ($\xi^{\pm}_{\mathrm{E},t}$ and $\xi^{\pm}_{\mathrm{R},t}$, respectively), where $n^{\pm}_{\mathrm{E},g,i,t}$ and $n^{\pm}_{\mathrm{R},g,i,t}$ are defined according to (31) and (32). Expression (33) represents the ability of each DG to pass from an active power generation level to another within a time interval. Constraints to represent the response time after a disturb in which the *reserve* must be available to restore the system frequency are considered through expression (34). The charging power provided to the EVs, as well as the energy stored in their batteries have to be between specified operational limits, according to (35) and (36), respectively. Expression (37) represents the energy stored in the EVs' batteries. Through the constraint (38), the charging power provided to the EVs is defined, where the factor $1 - E^{\pm}_{\mathrm{EV},e,i,t-1} \big/ \overline{E}_{\mathrm{EV},e,i}$ is used to give charging priority to the EVs with the lowest energy stored in their batteries. Finally, the connection/disconnection of the EVs is defined through the constraint (39).

The objective function (1) is composed of three terms: the costs for *energy* provided by the main grid; the costs for *energy* and *reserve* provided by the DGs, say $FC^{\pm}$; and the financial reward of the DSO for *reserve* sold to the main grid, say $FR^{\pm}$.

III. Results and Discussion

Tests of the proposed methodology were performed using data of a modified system based on the IEEE 37-bus distribution test system [13], which is presented in Fig. 1. In this work, the implementations to solve the proposed problem were developed in AMPL (*Algebraic Modeling Programming Language*) [14], using the optimization package CPLEX [15], and the programming language C++ with the compiler g++ 4.4, in the Linux environment.

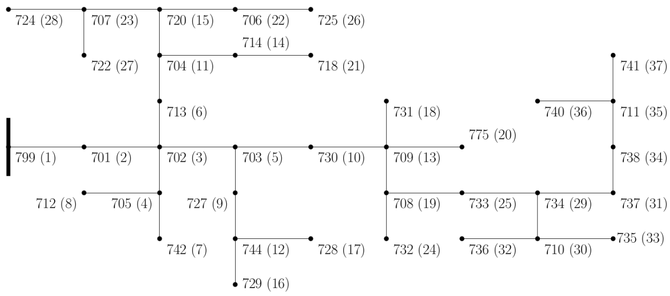

Fig. 1. Modified IEEE 37-bus distribution test system

For all tests, the minimum leading and lagging power factor of the substation were assumed as 0.9, the overcurrent limit in branches was 200 A, and the minimum and maximum voltages limits in buses were 0.93 and 1.30 pu, respectively. The *reserve* requirement, $R^{\pm}_{\mathrm{Req},t}$ (in the format [*value first sub-model*, *value second sub-model*]), was fixed as [90, 110] kW.

The daily load presented in Fig. 2 was assumed for tests, where the lowest values were used for the first sub-model and the highest for the second one. For all tests, it was assumed that the *reserve* provided by the DGs should be available within a time interval of 10 min (i.e., η=10 min). For each $\mathrm{DG_{NU}}$, maximum active power generation limit of 750 kW was assumed; and for each $\mathrm{DG_{U}}$, interval values of maximum active power generation limit of [150, 600] kW were also assumed. For each DG, decreasing and increasing ramps of -8 and 8 kW/min, respectively, were considered. Three $\mathrm{DGs_{NU}}$ (small hydro), coupled to the network through SGs, are supposed as installed in buses 14 ($\mathrm{DG_1}$), 9 ($\mathrm{DG_2}$) and 24 ($\mathrm{DG_3}$); and two $\mathrm{DGs_{U}}$ (wind turbines) installed in buses 18 ($\mathrm{DG_4}$) and 34 ($\mathrm{DG_5}$), coupled to the network through DFIGs, are also supposed.

Supposed interval values of offers of *energy* presented by the DGs are [8.26×10^{-2}, 17.87×10^{-2}] US$/kWh, while the supposed interval values of offers of *reserve* presented by the DGs are [3.44×10^{-2}, 8.38×10^{-2}] US$/kWh. The assumed interval values of costs of *energy* for the DSO (purchased from the main grid) are [12.06×10^{-2}, 20.27×10^{-2}] US$/kWh.

Regarding the prices of *reserve* sold to the main grid by the DSO, these interval values are assumed as [6.98×10^{-2}, 10.66×10^{-2}] US$/kWh.

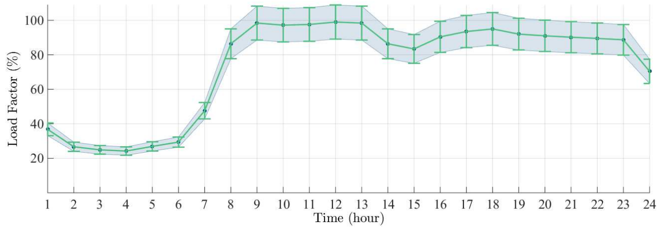

Fig. 2. Daily load for each hour

The numbers of EVs assigned to the buses in the first and second sub-models are presented in Table I. The EVs are supposed to be connected to the network between hours 8 and 17 for the first sub-model, and between hours 6 and 19 hours for the second one. The initial energy charge of the EVs' batteries is supposed between 4.5 and 16.2 kWh for the first sub-model, and between 5 and 18 kWh for the second one.

TABLE I
NUMBER OF EVS ASSIGNED TO EACH BUS (IN THE FORMAT [*VALUE FIRST SUB-MODEL*, *VALUE SECOND SUB-MODEL*])

Bus	2	6	7	8	9	10	12	14	15	16	17	18	20	21	24	25	26	27	28	29	31	32	33	36	37
Number of EVs	[7, 9]	[1, 3]	[1, 3]	[1, 3]	[4, 6]	[1, 3]	[1, 3]	[1, 3]	[4, 6]	[1, 3]	[1, 3]	[1, 3]	[2, 4]	[3, 5]	[1, 3]	[1, 3]	[3, 5]	[1, 3]	[1, 3]	[2, 4]	[1, 3]	[1, 3]	[6, 8]	[2, 4]	[8, 10]

In Table II, the values of $\widehat{PB}^{\pm}$, $\widehat{FC}^{\pm}$, $\widehat{FR}^{\pm}$, $\widehat{PB}^{\pm}-\widehat{FR}^{\pm}$, and the total active power provided by the substation and the DGs, say $\hat{P}^{\pm}_{\mathrm{SS_T}}$ and $\hat{P}^{\pm}_{\mathrm{DG_T}}$, respectively, are presented; and, in Table III, the results corresponding to the market prices of *energy* and *reserve* are shown.

TABLE II
$\widehat{PB}^{\pm}$, $\widehat{FC}^{\pm}$, $\widehat{FR}^{\pm}$, $\widehat{PB}^{\pm}-\widehat{FR}^{\pm}$, $\hat{P}^{\pm}_{\mathrm{SS_T}}$ AND $\hat{P}^{\pm}_{\mathrm{DG_T}}$

$\widehat{PB}^{\pm}$ (US$)	$\widehat{FC}^{\pm}$ (US$)	$\widehat{FR}^{\pm}$ (US$)	$\widehat{PB}^{\pm}-\widehat{FR}^{\pm}$ (US$)	$\hat{P}^{\pm}_{\mathrm{SS_T}}$ (10^3 kW)	$\hat{P}^{\pm}_{\mathrm{DG_T}}$ (10^3 kW)
[3,375.56, 6,820.43]	[3,527.60, 7,099.18]	[152.04, 278.74]	[3,223.52, 6,541.68]	[12,550.83, 21,317.48]	[17,802.75, 18,882.31]

From the stand point of the DSO, the lower extreme result regarding $\widehat{PB}^{\pm}$ corresponds to the most optimistic benefic to the DSO with the highest risk of violating the constrains, while the higher extreme result represents the most pessimistic benefit to the DSO with the lowest risk. The financial compensation to the DGs for *energy* and *reserve* sold to the DSO, i.e., $\widehat{FC}^{\pm}$, is smaller in the most optimistic scenario in 49.69%, than in the most pessimistic one. Although the financial reward of the DSO for *reserve* sold to the main grid, i.e., $\widehat{FR}^{\pm}$, is greater in the most pessimistic scenario than in the most optimistic one, the liquid payment of the DSO, i.e., $\widehat{PB}^{\pm}-\widehat{FR}^{\pm}$, remains as the worst in the most pessimistic scenario.

TABLE III
MARKET PRICES OF *ENERGY* AND *RESERVE* (ALL IN 10^{-2} US$/KWH)

t	1	2	3	4	5	6
$\xi^{\pm}_{\mathrm{E},t}$	[11.62, 17.42]	[11.44, 17.16]	[11.45, 17.17]	[11.60, 17.40]	[11.66, 17.48]	[11.74, 17.60]
$\xi^{\pm}_{\mathrm{R},t}$	[5.50, 8.24]	[5.29, 7.93]	[5.53, 8.29]	[5.42, 8.12]	[5.28, 7.92]	[5.21, 7.81]
t	7	8	9	10	11	12
$\xi^{\pm}_{\mathrm{E},t}$	[11.60, 17.40]	[11.91, 17.87]	[11.54, 17.32]	[11.81, 17.71]	[11.65, 17.47]	[11.74, 17.60]
$\xi^{\pm}_{\mathrm{R},t}$	[5.30, 7.96]	[5.43, 8.15]	[5.57, 8.35]	[5.29, 7.93]	[5.15, 7.73]	[5.53, 8.29]
t	13	14	15	16	17	18
$\xi^{\pm}_{\mathrm{E},t}$	[11.74, 17.62]	[11.84, 17.76]	[11.78, 17.68]	[11.67, 17.51]	[11.51, 17.27]	[11.68, 17.52]
$\xi^{\pm}_{\mathrm{R},t}$	[5.26, 7.90]	[5.58, 8.36]	[5.58, 8.38]	[5.32, 7.98]	[5.24, 7.86]	[5.58, 8.36]
t	19	20	21	22	23	24
$\xi^{\pm}_{\mathrm{E},t}$	[11.82, 17.72]	[11.52, 17.28]	[11.84, 17.76]	[11.52, 17.28]	[11.46, 17.18]	[11.70, 17.54]
$\xi^{\pm}_{\mathrm{R},t}$	[5.42, 8.14]	[5.20, 7.80]	[5.46, 8.18]	[5.22, 7.82]	[5.57, 8.35]	[5.30, 7.96]

In Fig. 3, the middle value of $PB^{\pm}$, which is defined as $M[\widehat{PB}^{\pm}] = (\widehat{PB}^{-}+\widehat{PB}^{+})/2$, and percentages of 25, 50, 75 and 100% of its width value, which is defined as $W[\widehat{PB}^{\pm}] = \widehat{PB}^{+}-\widehat{PB}^{-}$, for all hours of the day is shown. In this figure, note that $W[PB^{\pm}]$ is increased in hours of high demand (between hours 8 and 24). This is due to the fact that this width is mainly affected by the active and reactive power demand of the system and, as these values increase, their corresponding width, i.e., $W[P^{\pm}_{\mathrm{D},i,t}] = P^{+}_{\mathrm{D},i,t}-P^{-}_{\mathrm{D},i,t}$ and $W[Q^{\pm}_{\mathrm{D},i,t}] = Q^{+}_{\mathrm{D},i,t}-Q^{-}_{\mathrm{D},i,t}$, respectively, also increase.

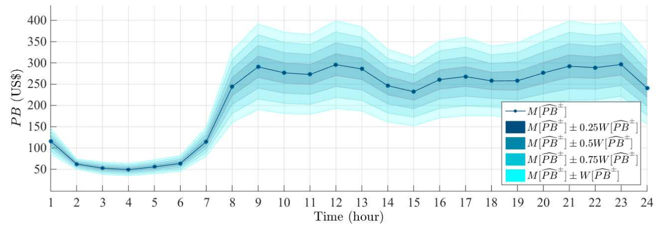

Fig. 3. $\widehat{PB}^{\pm}$ for all hours of the day

In Fig. 4, the middle values of the active power generation of the DGs, which are defined as $M[\hat{P}^{\pm}_{\mathrm{DG}_g}] = (\hat{P}^{-}_{\mathrm{DG}_g}+\hat{P}^{+}_{\mathrm{DG}_g})/2$ $\forall g\in\Omega_{\mathrm{DG}}$, and their corresponding width values, defined as $W[\hat{P}^{\pm}_{\mathrm{DG}_g}] = \hat{P}^{+}_{\mathrm{DG}_g}-\hat{P}^{-}_{\mathrm{DG}_g}$ $\forall g\in\Omega_{\mathrm{DG}}$, are presented. In this figure, note the low variations in the width of the generation of the $\mathrm{DGs_{NU}}$ ($\mathrm{DG_1}$ to $\mathrm{DG_3}$) mainly for hours of high demand. This is because these DGs are required to bring their full capacity for these hours in both, the most pessimistic and the most optimistic scenarios. Note also that the variation in the width of the active power generation of the $\mathrm{DGs_U}$ ($\mathrm{DG_4}$ and $\mathrm{DG_5}$) is more pronounced in hours of low demand and when the EVs are connected to the network, mainly for the $\mathrm{DG_5}$.

In figures 5 and 6, the middle values of $\widehat{P}^{\pm}_{\mathrm{EV}\,e,i,t}$ and $\widehat{E}^{\pm}_{\mathrm{EV}\,e,i,t}$, defined as $M[\widehat{P}^{\pm}_{\mathrm{EV}\,e,i,t}] = (\widehat{P}^{-}_{\mathrm{EV}\,e,i,t}+\widehat{P}^{+}_{\mathrm{EV}\,e,i,t})/2$ and $M[\widehat{E}^{\pm}_{\mathrm{EV}\,e,i,t}] = (\widehat{E}^{-}_{\mathrm{EV}\,e,i,t}+\widehat{E}^{+}_{\mathrm{EV}\,e,i,t})/2$ $\forall e\in\Omega_{\mathrm{EV}}$, respectively, are depicted. As it could be expected, the buses with the most $\widehat{P}^{\pm}_{\mathrm{EV}\,e,i,t}$ and $\widehat{E}^{\pm}_{\mathrm{EV}\,e,i,t}$

requirement would be those with the largest numbers of EVs, say buses 2, 36 and 37; however, this is only true if in each of these buses the total energy required during the charging process of their corresponding EVs is higher than the total energy required by the EVs in other buses. A bus with a few number of EVs to be charged could require more energy than a bus with a largest number of EVs.

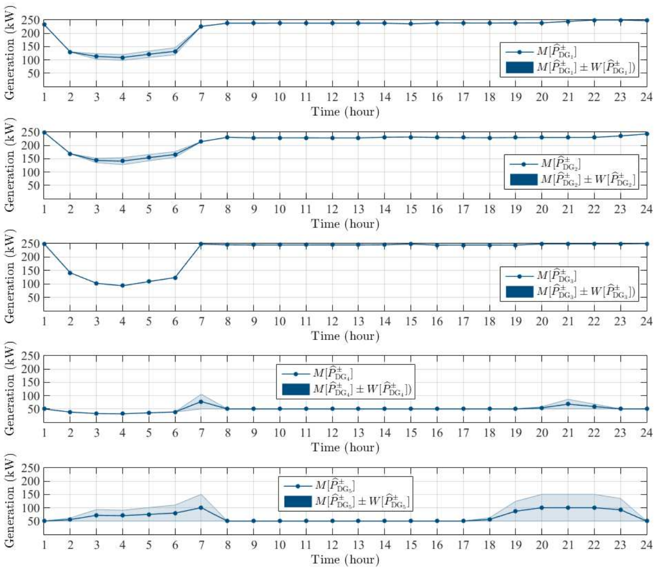

Fig. 4. Active power generated by the DGs

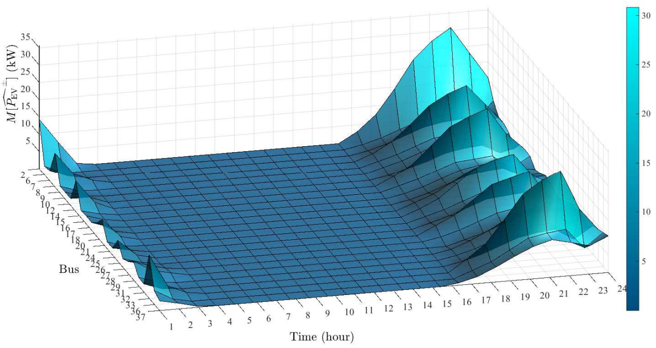

Fig. 5. Active power provided to the EVs

IV. CONCLUSIONS

A novel MILP formulation for a co-optimization market clearing process which includes precise models of the DGs and EVs operation was proposed.

In this proposal, the stochastic behavior related to the main variables of the proposed problem, such as, the penetration, connection/disconnection hours of the EVs, amount of *reserve* requirement, system demand, DGs generation, and DGs and substation *energy* and *reserve* market offers is treated through the development of an EILP model. The proposal allows a comprehensive stochastic analysis in which the DSO can make decisions based on a wide range of options provided by the interval nature of the results.

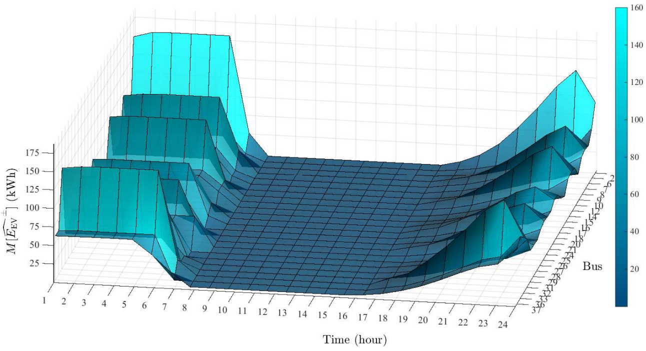

Fig. 6. Energy stored in the EVs' batteries

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