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Methodology for Dimensioning Underground Excavations in Stratified Rocks through Analytical Formulations and Computational Modeling

Arlo Nóbrega de Ávila
Universidade Federal de Ouro Preto, Ouro Preto, Brazil, arloavila@gmail.com

Rodrigo Peluci de Figueiredo
Universidade Federal de Ouro Preto, Ouro Preto, Brazil, rpfigueiredo@yahoo.com.br

SUMMARY: Some minerals, such as coal and potash, are extracted from stratified sedimentary rocks, usually by underground mining. To safely calculate the excavations, geotechnical studies are required in order to ensure that the openings made do not collapse. The dimensions of the excavations are frequently calculated by using a numerical tool such as the elastoplastic finite element software Phase2 (Rocscience Inc.). However, for being an analysis tool, Phase2 does not directly provide the most appropriate dimensions for an excavation project. It should be used only to confirm if a previously developed project is adequate or not. We hereby propose a different method of design for dimensioning underground excavations in stratified rocks. We have assessed the use of an analytical method to calculate the optimal width of an underground excavation. Using Mathcad software (Mathsoft Inc.), we employed mathematical models to design theoretical mining projects for strata with three different types of behavior: (1) embedded beams on rigid supports, (2) simply supported beams on rigid supports, and (3) beams on flexible supports. We compared the solutions from the three models with one another and with the respective results obtained with Phase2. The correlation for each pair of analytical-numerical results proved that the analytical method is more efficient for designing excavations. While the analytical method readily provides the adequate width of the excavations, the numerical method requires a trial-and-error approach, which is a slow process, in which the dimensions of the excavation are varied until the necessary level of stability is achieved. Therefore, a logical and effective strategy for the design of an excavation should include the use of an analytical tool to calculate its dimensions followed by the use of a numerical tool to verify these dimensions.

KEYWORDS: Underground Excavation, Underground Mining, Bedded Rocks, Sizing, Computational Modeling.

1 INTRODUCTION

In underground mines, the voids left by the extraction of rock cause disturbances in the surrounding rock mass. In the decompression zones that are established around the excavations, the rock mass is likely to break, since the rock tends to expand toward the voids created. A good knowledge of the mechanical behavior of the excavated rock can contribute to a safer mining operation as well as to a lower cost with rock supports.

The elastoplastic finite element software Phase2 (Rocscience Inc.) can be used for analysis of an underground excavation stability. However, this computational method has been frequently applied in designing excavations despite it is an analysis tool for projects already developed. By trial and error, various dimensions of the excavations are simulated on the software until an acceptable level of stability is achieved. This is a slow, inaccurate and imprecise technique.

We propose the application of analytical methods that are more efficient than the numerical method mentioned above, to calculate the proper width of an excavation in layered rock mass. Thus, we will consider three possible behaviors for a rock mass of this sort: (1) as embedded beams on rigid supports, (2) as simply supported beams on rigid supports, and (3) as beams on flexible supports. For each of these three behavioral hypotheses, we will present excavation design equations that will be used in theoretical mines planning. The results of this sizing will be compared with each other and with the results of Phase2 software.

2 MECHANICAL BEHAVIOR OF THE ROOF OF EXCAVATIONS IN HORIZONTALLY BEDDED ROCKS

In excavations whose roof is directly below the rock horizontal layering, the thinnest layer near the opening tends to detach from the rock mass, flex down and break in the upper surface of its ends and on the center of the bottom surface. Thicker layers and under horizontal stress layers are more stable. The collapse of a layer leaves cantilevers as supports for the layer above. Then, each layer above the roof forms a progressively shorter beam. The end result of the continuing collapse of the beams is a stable trapezoidal opening (Goodman, 1989). “In the case where the beams thin upwards, the strata will not separate directly above the opening - providing the elastic properties of the beams are equal. Conversely, if the beams thin downwards, strata separation will occur immediately above the roof.” (Hudson and Harrison, 1997).

In this paper, just the tensile ruptures (Figure 1) described above will be considered. However, beyond the tensile rupture, Jeremic (1985) presents other types of roof collapse.

The most important factor in stratified rocks excavations design, in most cases, is the maximum acceptable roof span without support. The stability and the supports demand can be verified by an elastic analysis of beam bending, calculating the maximum tensile stress gravitationally induced by the weight of the layers and their maximum deflections (Hudson and Harrison, 1997).

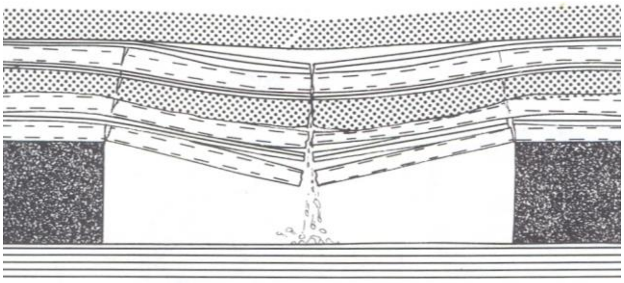

Figure 1 - Roof break by layer tension. Source: Jeremic (1985).

Jeremic (1985) points out that sedimentary rocks are anisotropic. Accordingly, the preferred direction of the openings may have a much greater effect on the stability of the mine than the format of the excavation. The theory of excavations in sedimentary strata tends to rely on the classical approach of elasticity, homogeneity and isotropy of the materials and the stress distribution in an infinite solid beam. It is recognized that a difficulty exists in adopting considerations of inhomogeneity due to oblique anisotropies.

2.1 Mathematical Relations for Rigid Supports

Obert and Duvall (1967) argue that the critical stresses in elastic stratified rocks can be determined by the theory of embedded beams or plates loaded gravitationally. For these authors, there are two cases to consider, depending on the relative values of the load per unit length ratio on the flexural stiffness (*q/EI*) of two beams. If this ratio for the top beam is smaller than that for the lower beam, then each beam acts independently. This is equivalent to layers of the same rock in which the thicker layer lies on top of the thinner one. When the thinner layer lies on top of the thicker one, i.e., the upper layer has a larger ratio, the lower layer will partially support the load of the upper beam.

In the first case above and in situations in which the roofs consist of a single layer, the following equations may be used, respectively, to calculate maximum deflection and maximum tensile strength of each layer:

$$\eta_{max} = \frac{\gamma L^4}{32Et^2} \quad (1)$$

$$\sigma_{max} = \frac{\gamma L^2}{2t} \quad (2)$$

where γ is the unit weight of the rock, L is the length of the layer, t is the layer thickness and E is Young's modulus.

The maximum deflection occurs at the center of the beam and the maximum stress occurs on the upper surface of the beam ends. These formulas can be used for beds inclined up to 10°. As the rocks are much less resistant to tension than to compression and shear, only tensile stresses must be considered in determining the width of an excavation (Obert and Duvall, 1967).

Obert and Duvall (1967) also determine formulations for the second case. However, these equations are not presented in this paper.

The equation (2) may be transformed into the following sizing formula:

$$L_{max} = \sqrt{\frac{2T_0 t}{\gamma F_s}} \quad (3)$$

where T_0 is the tensile strength of the rock and F_s is tensile safety factor, which can be obtained by the formula:

$$F_s = \frac{T_0}{\sigma_{max}} \quad (4)$$

For Obert and Duvall (1967), an acceptable tensile safety factor is in the range of 4 to 8. The lower value is applied for short-lifespan openings and the greater value is applied for openings that must be kept as long lasting openings.

According to Pariseau (2007), the assumption that the beams of roofs of excavations are embedded is inaccurate. The ends of the beams are neither entirely fixed nor free. The author's approach involves both the equations for embedded beams as the equations for simply supported beams. Here are the formulas for simply supported beams:

$$\sigma_{max}(SS) = \frac{3}{4}\frac{\gamma L^2}{t} \quad (5)$$

$$L(SS) = \sqrt{\frac{4}{3}\frac{T_0}{F_s}\frac{t}{\gamma}} \quad (6)$$

$$\eta_{max}(SS) = \frac{5}{32}\frac{\gamma L^4}{Et^2} \quad (7)$$

Pariseau (2007) suggests that it would be more accurate consider the roof as a blade rather than as a beam and, therefore, use $E/(1-v^2)$ in the formulas, rather than E. Thus, a lower deflection would be obtained. However, the difference is less than 10% for Poisson ratio in the range of 0.20 to 0.30. Obert and Duvall (1967) believe that the theory of the blades should be applied only on roofs which length over span ratio is less than two.

2.2 Mathematical Relations for Flexible Supports

Stephansson (1971) presents mathematical solutions for roofs of horizontally stratified rocks based on the theory of elastic beams on elastic supports. Initially, the author introduces the formulas for the modulus of the foundation and for the flexural rigidity. “The modulus of the foundation characterizes the deformability of the elastic support. For homogeneous elastic abutments the modulus is given by:

$$c_u = \frac{E_u}{h(1-v_u^2)} \quad (8)$$

where E_u is the modulus of elasticity, v_u Poisson's ratio, and h the height.” (Höfer and Menzel, 1964, apud Stephansson, 1971).

The flexural rigidity, according to the theory of plates, is given by:

$$K = \frac{EI}{1 - v_1^2} \tag{9}$$

where $I = H^3/12$ is the moment of inertia, E is Young's modulus, and v_1 is Poisson's coefficient of the layer.

The roofs of a layer are divided into two types depending on the ratio between thickness (H) and width (l) of the roof. For thin layers roofs, $H < l/5$, the deformations due to shear stress can be neglected. A monolayer roof of infinite length is considered partially supported by elastic supports and subjected to a uniformly distributed load, $p = H \times \gamma$ (where γ is the specific weight of the layer) and/or any other uniform pressure, e.g., water pressure (Stephansson, 1971). For the layer section $0 \leq x \leq l$, which is above the excavation, the author presents the following equations for the deflection, *y*, and bending moment, *M*:

$$y = \frac{p}{c_u}\left[\alpha^4 x\left(\frac{x^3}{6} - \frac{x^2 l}{3} - \frac{lx}{\alpha} - \frac{l}{\alpha^2}\right) + l\alpha^3\beta(1 + \alpha x)^2 + 1\right] \tag{10}$$

$$M = -\frac{px^2}{2} + \frac{pl}{2}\left(x + \frac{1}{\alpha} - \alpha\beta\right) \tag{11}$$

$$\alpha = \left(\frac{c_u}{4K}\right)^{\frac{1}{4}} \tag{12}$$

$$\beta = \frac{\frac{l^2}{6} + \frac{l}{\alpha} + \frac{1}{\alpha^2}}{(\alpha l + 2)} \tag{13}$$

The stresses are obtained by $\sigma_x = Me/I$, where e is the distance from the neutral axis (Stephansson, 1971).

Within the range $l/5 < H < l/2$, the influence of shear stresses can no longer be neglected. The deflection of the layer is affected by the bending moment and shear, increasing the deflection and tensile stress (Stephansson, 1971). Here are the equations for this situation:

$$y = \frac{px}{2K}\left[\frac{x^3}{12} - \frac{lx}{2}\left(\frac{x}{3} + \frac{1}{\alpha} - \alpha\beta\right) + a(l - x) - \frac{l^3}{24} + \frac{l^2}{2}\left(\frac{l}{4} + \frac{1}{\alpha} - \alpha\beta\right)\right] + y_{II} \tag{14}$$

$$M = -\frac{px^2}{2} + \frac{pl}{2}\left(x + \frac{1}{\alpha} - \alpha\beta\right) + a \tag{15}$$

$$r_1 = \sqrt{\frac{1}{2K}\left[ac_u + \sqrt{a^2(c_u)^2 - 4Kc_u}\right]} \tag{16}$$

$$r_2 = \sqrt{\frac{1}{2K}\left[ac_u - \sqrt{a^2(c_u)^2 - 4Kc_u}\right]} \tag{17}$$

$$y_{II} = \frac{(r_1^2 + r_1 r_2 + r_2^2)\left[\frac{pl}{2K}\left(\alpha\beta - \frac{1}{\alpha} + \frac{1}{r_1 + r_2}\right)\right]}{r_1^2 r_2^2} + \frac{pl}{2Kr_1 r_2(r_1 + r_2)} + \frac{p}{c_u} \tag{18}$$

$$a = \frac{3I}{H(1 - v_1)} \tag{19}$$

Stephansson (1971) also determines equations for multilayer roofs. However, these equations are not presented in this paper.

3 METHODOLOGY AND RESULTS

There were nine excavation situations supposed in a dolomite, stratified and isotropic rock, of high rigidity and high resistance, which occurs in a given Brazilian zinc mine. The adopted rock behaves elastically up to approximately the peak resistance. Thus, such a rock fits perfectly to the assumptions of the theoretical formulations and has the following characteristics:

$\gamma = 0{,}03\ MN/m^3$
$E = 49815\ MPa$
$\nu = 0{,}26$
$T_0 = 11{,}1\ MPa$

Being t_1 the thickness of the lower beam, t_2 the thickness of the upper beam, and L the width of the excavation, we considered the situations presented in Table 1.

Table 1 - Dimensions of the excavations.

EXCAVATION	L (m)	t_1 (m)	t_2 (m)
I	7	1,25	-
II	17	2,25	-
III	20	2,75	-
IV	7	1,25	2,5
V	7	1,25	2
VI	17	2,25	2,5
VII	17	2,25	2
VIII	20	2,75	2
IX	20	2,75	2,5

The excavations IV, V and VI are under two layers with each one acting independently, supporting their own weight. For them, only the calculations for the bottom layer will be presented, which is the most fragile. To calculate the optimal width (L_{max}), the value eight will be adopted for the safety factor. Stephansson's (1971) equations require the height of excavation be determined, that is, the height of the walls, which represent the elastic supports. We adopted a height of 10 meters for all of the excavations. We performed all of the calculations using the Mathcad software, by Mathsoft Engineering & Education Inc.

3.1 Results Obtained through the Analytical Methods

For the situations described above, taking the equations under the approaches of layers behaving like embedded beams on rigid supports, like simply supported beams on rigid supports and like beams on elastic supports, we obtained the results shown in Tables 2 and 3.

Table 2 - Comparison table of safety factors and the optimal spans of the excavations.

EXCAVATIONS	RIGID SUPPORTS				ELASTIC SUPPORTS	
	EMBEDDED BEAMS		SIMPLY SUPPORTED BEAMS			
	F_S	L_{max} (m)	F_S	L_{max} (m)	F_S	L_{max} (m)
I	18,88	10,75	12,59	8,78	18,37	10,69
II	5,76	14,43	3,84	11,78	6,98	16,23
III	5,09	15,95	3,39	13,02	6,19	18,41
IV	18,88	10,75	12,59	8,78	-	-
V	18,88	10,75	12,59	8,78	-	-
VI	5,76	14,43	3,84	11,78	-	-
VII	5,19	13,69	3,46	11,18	7,55	17
VIII	4,08	14,28	2,72	11,66	6,13	20
IX	4,67	15,28	3,11	12,47	6,69	20

Table 3 - Comparative table of maximum deflections and maximum stresses of the excavations.

EXCAVATIONS	RIGID SUPPORTS				ELASTIC SUPPORTS	
	EMBEDDED BEAMS		SIMPLY SUPPORTED BEAMS			
	η_{max} (m)	σ_{max} (Pa)	η_{max} (m)	σ_{max} (Pa)	η_{max} (m)	σ_{max} (Pa)
I	$2{,}89 \times 10^{-5}$	$5{,}88 \times 10^{5}$	$1{,}45 \times 10^{-4}$	$8{,}82 \times 10^{5}$	$1{,}32 \times 10^{-4}$	$6{,}04 \times 10^{5}$
II	$3{,}11 \times 10^{-4}$	$1{,}93 \times 10^{6}$	$1{,}55 \times 10^{-3}$	$2{,}89 \times 10^{6}$	$8{,}32 \times 10^{-4}$	$1{,}59 \times 10^{6}$
III	$3{,}98 \times 10^{-4}$	$2{,}18 \times 10^{6}$	$1{,}99 \times 10^{-3}$	$3{,}27 \times 10^{6}$	$1{,}05 \times 10^{-3}$	$1{,}79 \times 10^{6}$
IV	$2{,}89 \times 10^{-5}$	$5{,}88 \times 10^{5}$	$1{,}45 \times 10^{-4}$	$8{,}82 \times 10^{5}$	$1{,}59 \times 10^{-4}$	-
V	$2{,}89 \times 10^{-5}$	$5{,}88 \times 10^{5}$	$1{,}45 \times 10^{-4}$	$8{,}82 \times 10^{5}$	$1{,}64 \times 10^{-4}$	-
VI	$3{,}11 \times 10^{-4}$	$1{,}93 \times 10^{6}$	$1{,}55 \times 10^{-3}$	$2{,}89 \times 10^{6}$	$9{,}78 \times 10^{-4}$	-
VII	$3{,}45 \times 10^{-4}$	$2{,}14 \times 10^{6}$	$1{,}72 \times 10^{-3}$	$3{,}21 \times 10^{6}$	-	$1{,}47 \times 10^{6}$
VIII	$4{,}97 \times 10^{-4}$	$2{,}72 \times 10^{6}$	$2{,}48 \times 10^{-3}$	$4{,}08 \times 10^{6}$	-	$1{,}81 \times 10^{6}$
IX	$4{,}34 \times 10^{-4}$	$2{,}38 \times 10^{6}$	$2{,}17 \times 10^{-3}$	$3{,}57 \times 10^{6}$	-	$1{,}66 \times 10^{6}$

For excavations VII, VIII and IX, according to the theory of elastic supports, we calculated just the values for the lower layer, which is the most unstable. Stephansson's (1971) equations to calculate the bending moment in the situation of two layers being the upper one of higher flexural rigidity have dimensional compatibility errors. The first term in both equations is dimensioned as a negative power of length, instead of force times length. That fact has made it impossible to calculate the maximum stresses of excavations IV, V and VI, as well as the safety factors and the optimum widths. There is also a dimensional mismatch in the formula for calculating deflection of the situation of double layers in which the flexural stiffness of the upper layer is smaller. In the numerator of the expression, the dimensionless term αx is added to the width *l*. Thus, it was impossible to calculate the maximum deflection of the excavations VII, VIII and IX.

Using Table 3, we identified the most conservative results for each parameter of each excavation. By dividing the results from each of the three methods by the most conservative result, we obtained a factor of variation. The closer to the most conservative result, the closer to one the value of the variation factor will be, as shown in Table 4.

Table 4 - Comparison of the variation of results in relation to the more conservative.

EXCAVATIONS	RIGID SUPPORTS				ELASTIC SUPPORTS	
	EMBEDDED BEAMS		SIMPLY SUPPORTED BEAMS			
	η_{max}	σ_{max}	η_{max}	σ_{max}	η_{max}	σ_{max}
I	0,20	0,67	1,00	1,00	0,91	0,68
II	0,20	0,67	1,00	1,00	0,54	0,55
III	0,20	0,67	1,00	1,00	0,53	0,55
IV	0,18	0,67	0,91	1,00	1,00	-
V	0,18	0,67	0,88	1,00	1,00	-
VI	0,20	0,67	1,00	1,00	0,63	-
VII	0,20	0,67	1,00	1,00	-	0,46
VIII	0,20	0,67	1,00	1,00	-	0,44
IX	0,20	0,67	1,00	1,00	-	0,46

3.2 Results Obtained through the Numerical Method

We carried out computer modeling trying to obey all the assumptions of the theory in a finite element software (Phase2). We considered the materials elastic and subject only to the mass forces. In nature, the rock masses are pre-tensioned by gravity. Therefore, we introduced a first shaping stage in order to allow gravity to act on the model, before excavation. We simulated the hypothesis of interfaces without resistance between layers annulling friction, cohesion and tensile strength of the joints of the models. Figure 2 illustrates the boundary conditions and the mesh used. We obtained the values of stresses by plotting the horizontal stresses of the models, as in figures 3 and 4.

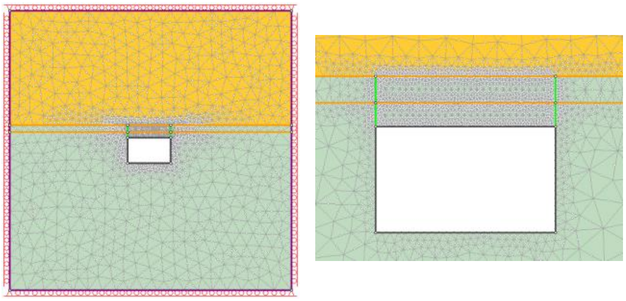

Figure 2 - Kinematics restriction imposed to the external boundaries and typical mesh model (left); and discretization level used in the models (right).

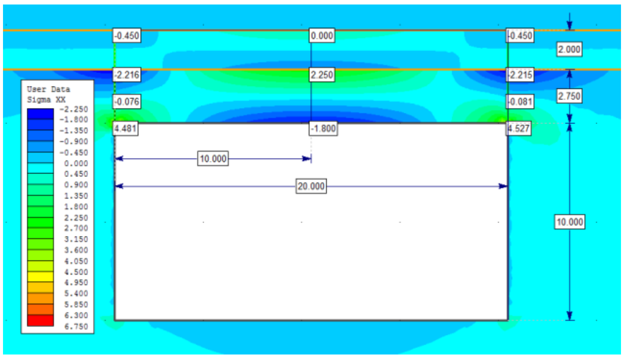

Figure 3 - Horizontal stresses in excavation VIII.

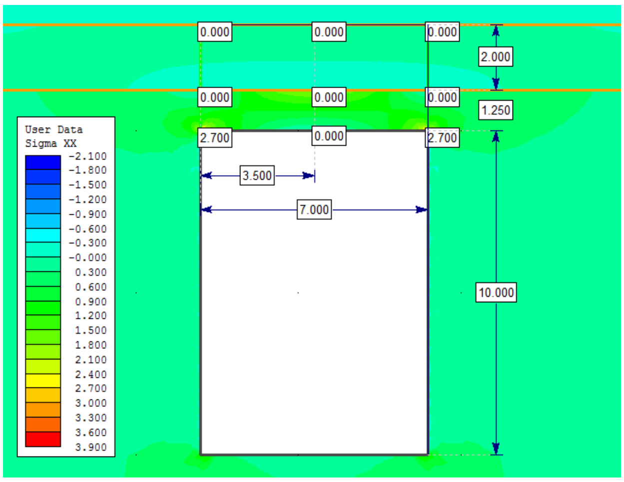

Figure 4 - Horizontal stresses in excavation V.

As with the excavation V (Figure 4), some points of the models showed unexpected tensions (zero or opposite sign). This fact was repeated in various parts of the excavations I and IV and in the upper layers of the excavations VI, VII and IX. It is believed that these inconsistencies should be attributed to the difficulty in defining the mechanical parameters of the interfaces of the layers. We created Table 5 using Table 3 and the values of horizontal stresses calculated by using Phase2.

Table 5 - Comparative table of the results from the analytical and the numerical methods.

EXCAVATIONS	ANALYTICAL METHODS			NUMERICAL METHOD
	RIGID SUPPORTS		ELASTIC SUPPORTS	
	EMBEDDED BEAMS	SIMPLY SUPPORTED BEAMS		
	σ_{max} (Pa)	σ_{max} (Pa)	σ_{max} (Pa)	σ_{max} (Pa)
I	$5{,}88 \times 10^5$	$8{,}82 \times 10^5$	$6{,}04 \times 10^5$	0
II	$1{,}93 \times 10^6$	$2{,}89 \times 10^6$	$1{,}59 \times 10^6$	$1{,}33 \times 10^6$
III	$2{,}18 \times 10^6$	$3{,}27 \times 10^6$	$1{,}79 \times 10^6$	$1{,}48 \times 10^6$
IV	$5{,}88 \times 10^5$	$8{,}82 \times 10^5$	-	0
V	$5{,}88 \times 10^5$	$8{,}82 \times 10^5$	-	0
VI	$1{,}93 \times 10^6$	$2{,}89 \times 10^6$	-	$2{,}25 \times 10^6$
VII	$2{,}14 \times 10^6$	$3{,}21 \times 10^6$	$1{,}47 \times 10^6$	$2{,}06 \times 10^6$
VIII	$2{,}72 \times 10^6$	$4{,}08 \times 10^6$	$1{,}81 \times 10^6$	$2{,}22\times 10^6$
IX	$2{,}38 \times 10^6$	$3{,}57 \times 10^6$	$1{,}66 \times 10^6$	$2{,}35\times 10^6$

4 RESULTS DISCUSSION

Observing Table 4, it is noted that the results for maximum deflection and maximum stress for the hypothesis of the simply supported beams were the most conservative in most situations. The maximum deflections calculated by the hypothesis of embedded beams were the least conservative in all the excavations. Often, the hypothesis of flexible supports showed the lowest values of maximum stress. Therefore, Stephansson's (1971) equations allow the design of wider excavations.

No value found by using the formulas for the maximum deflection and maximum stress differed more than about twice from the value found by using the formulas of other theories, except in one case (see Table 3 and Table 4). There was a large discrepancy in the results of maximum deflection from the theory of embedded beams that in most cases were about five times smaller than the results from other theories. However, the maximum stresses from the theory of embedded beams did not differ much from the other theories, since it was just 50 % lower than the stresses of the simply supported beams theory and always greater than the stresses of the flexible supports theory, reaching a maximum difference of about 50 %.

Comparing the results of the numerical method with the results of analytical methods in Table 5, it is noted that the values are very similar, apart from those found for excavations I, IV and V. These three excavations are the narrowest of all and have in common the seven-meter width. In other excavations, the values of simply supported beam theory were the most differentiated from the numerical method results, reaching a difference of 121 %, in excavation III.

5 CONCLUSIONS

In this work, we presented analytical methods that readily indicate the optimal width of excavations with roof under sub-horizontal bedded rocks. Obtaining the adequate width by using the numerical method involves a lengthy process. In the computational model, the size of the excavation must be changed several times until one that meets the desired stability standards is found.

There was a correlation between the analysis of the excavations performed with Phase2 and the results from the theoretical formulations. This correlation proves that the analytical methods may be employed in the design of excavations. Therefore, it is advisable that Phase2 be used only for verification of pre-defined projects. To design excavations, the application of analytical methods has proved to be more convenient.

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