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Numerical modeling of hydraulic fracturing process in a heterogeneous medium through a coupling scheme between the cohesive fracture and the lattice-Boltzmann models

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SUMMARY: This research discusses the hydraulic fracturing process in a heterogeneous medium. To model this process, the PPR- potential-based cohesive zone model (Park *et. al*.,2009; Park *et. al*., 2012) was implemented in the finite element method (FEM) to model the crack propagation; and the lattice-Bolztmann model (LBM) was used to represent the fluid injected into the fracture. These models are coupled in an iterative process (Mejía Camones *et al*., 2014; Mejía Camones *et. al*., 2015). In the FEM, the crack's boundary is represented by the faces of cohesive elements. The position and velocity of these faces are transferred to the LBM as a boundary condition, where the forces applied on these boundaries, caused by the fluid injected, can be calculated. These forces are transferred to the FEM as external force applied on the element's faces, where the new position and velocity of the boundary of the crack is computed. These values are transferred to the LBM, initializing a new calculation cycle. The utilization of cohesive elements allows introducing different energy and strength fracture values in each element. Thus, the crack propagation in a heterogeneous medium can be modeled. This research shows the results of the crack propagation in heterogeneous medium, comparing with experimental results.

KEYWORDS: Hydraulic fracturing, cohesive fracture model, PPR model, lattice-Boltzmann model.

1 INTRODUCTION

The numerical simulation of the hydraulic fracturing process in geological medium has challenges that should be studied to get a better understanding about the modeling procedure. Two important operational processes are evaluated through numerical modeling in this research: the numerical simulation of crack propagation in rock material, and the numerical simulation of the fluid injected into the crack.

There are methodologies to model the crack propagation in rock materials. Considering the rock as a continuous and homogeneous material, the crack propagation can be modeled through the finite element method (FEM) or through the discrete element method (DEM)

The difficulty in modeling the crack propagation of rock material is to represent numerically the complex distribution of minerals or particles that compose it. A rock material is a heterogeneous medium where each

mineral or particle has its mechanical and fracture properties. Thus, the crack propagation in a heterogeneous medium shows an irregular path. First, some regions near to the crack tip start their softening process. Then, the crack propagates by the region with lowest fracture energy or where the softening process is ended.

In the numerical simulation, a fracture and its softening process can be modeled using interface elements or cohesive elements. These elements, inserted in a FEM mesh, can have independently their own strength and energy fracture. Thus, the use of interface elements allows modeling a crack propagating in a heterogeneous media.

The interface or cohesive elements have been used in the numerical model of crack propagation in rock material through an intrinsic implementation (Mahabadi, 2012) or through an extrinsic implementation (Mejia Camones, 2015). In both of cases, this methodology proved to be capable of represent the irregularities of the crack's path due to the spatial variability of the mechanical fracture properties. The difference of the extrinsic implementation when compared with the intrinsic implementation is that it allows inserting the cohesive element adaptively in the FEM mesh, capturing the softening process. In this case, it is not necessary to insert the cohesive elements at the start of the calculating process.

On the other hand, the numerical simulation of the fluid injected into the crack, to produce the crack propagation, also presents some challenges. In general, for Mode I of fracture type, the lubrication theory can be used. Several analytical (Detournay, 2004; Burger *et. al*., 2005; Adachi & Detournay, 2008) and numerical (Lecampion, 2009; Hunsweek and Shen, 2012; Carrier and Granet, 2012) works have shown good results. However, these methodologies consider the rock as homogeneous material and they do not have applications for the crack propagations under mixed conditions (Mode I-II).

An alternative to improve the numerical simulation of the fluid is to use the lattice-Boltzmann model. In this model, the fluid flow is treated as a discrete medium, where the fluid is represented by particles located in fixed points into a lattice. The particles can move between these points in fixed time intervals. Thus, the time is also discretized. This methodology allows modeling the fluid flow through complex geometries, calculating the pressure acting on the fracture faces and, eventually, allows calculating the drag force of particles suspended into the fluid.

2 NUMERICAL METHODS

2.1 PPR potential-based cohesive model

Finite element method (FEM) is used to simulate the mechanical process of the hydraulic fracturing. The crack propagation is modeled using the PPR potential-based cohesive model (Park et. al., 2009; Park and Paulino, 2012) by means of an extrinsic implementation. In this case, the cohesive elements are adaptively inserted in the mesh to capture the softening process. The consequence of this insertion is the duplication of some nodes during the calculation process. To manage the mesh information and the changes in its topology we use the *Tops* code (Paulino et. al., 2008; Espinha et. al., 2009). The Tops code is a topological data framework developed in C++ that supports these modifications in the mesh and allows optimum information management. To insert the cohesive elements, the tensile traction is checked at all facets of the mesh and is inserted when this tensile traction supersedes the normal or tangential cohesive strengths. The cohesive element allows simulating the softening process through the cohesive traction-separation relationship (Park et. al., 2009) given by:

$$\Psi(\Delta_n,\Delta_t) = \min(\Phi_n,\Phi_t) + \ldots$$
$$\ldots\left[\Gamma_n\left(1-\frac{\Delta_n}{\delta_t}\right)^{\alpha} + \langle\Phi_n-\Phi_t\rangle\right]\left[\Gamma_t\left(1-\frac{|\Delta_t|}{\delta_t}\right)^{\beta} + \langle\Phi_t-\Phi_n\rangle\right] \quad (1)$$

where Ψ is the potential function for cohesive element, Φ_n,Φ_t are the normal and tangential cohesive strengths, Γ_n,Γ_t are the energy

constant in the PPR model, Δ_n, Δ_t are the separation field in local coordinates, δ_n, δ_t are the normal and tangential final crack opening widths and α, β are the shape parameters in PPR model. The gradient of the PPR potential leads to the normal and tangential cohesive tractions along the fracture surface by:

$$T_n(\Delta_n,\Delta_t) = -\alpha\frac{\Gamma_n}{\delta_n}\left(1-\frac{\Delta_n}{\delta_n}\right)^{\alpha-1}\left[\Gamma_t\left(1-\frac{|\Delta_t|}{\delta_t}\right)^{\beta}+\langle\Phi_t-\Phi_n\rangle\right] \tag{2}$$

$$T_t(\Delta_n,\Delta_t) = -\beta\frac{\Gamma_t}{\delta_t}\left(1-\frac{|\Delta_t|}{\delta_t}\right)^{\beta-1}\ldots$$

$$\ldots\left[\Gamma_n\left(1-\frac{\Delta_n}{\delta_n}\right)^{\alpha}+\langle\Phi_n-\Phi_t\rangle\right]\frac{\Delta_t}{|\Delta_t|} \tag{3}$$

The finite element code implemented uses linear triangular elements (T3) and the cohesive element is composed of 4 nodes.

2.2 The lattice-Boltzmann method

Fluid modeling may be accomplished by using the LBM. In this model, space is discretized in cells where the probable number of particles that comprise them, (on time t and with velocity v) is represented by the distribution function $f(x,v,t)$. From this function, fluid macroscopic variables may be calculated. Usually, the solution of LBM is accomplished via two-step process of collision and propagation. The collision phase redistributes the particles on the node (particles collide with each other) due to the collision effect and the propagation phase propagates the particles to neighboring nodes. For the case of the incompressible LBM, the evolution equation is given by:

$$p_\alpha(x+v_\alpha\Delta t, t+\Delta t) = p_\alpha(x,t) - \ldots$$

$$\ldots\frac{1}{\tau^*}(1-B)(p_\alpha(x,t)-p_\alpha^{eq}(x,t)) + B\Omega_\alpha^s \tag{4}$$

$$p_\alpha = c_s^2 f_\alpha \tag{5}$$

$$p_\alpha^{eq}(x,t) = \omega_\alpha\left\{p + p_0\left[3\frac{(v_\alpha\cdot u)}{c^2}+\frac{9}{2}\frac{(v_\alpha\cdot u)^2}{c^4}-\frac{3}{2}\frac{u^2}{c^2}\right]\right\} \tag{6}$$

where the $\tau^* = \tau/\Delta t$ and τ is the time relaxation, f^{eq} is the equilibrium distribution function, x is the position of the node and t is the time. For the two-dimensional case, it is usual to choose a D2Q9 cell that allows nine velocity directions, four on the vertex of the cell, four at the center of each cell side and one in the center of the cell itself. The solids can be included in the mesh. Each cell is identified with a number: "one" for solids and "zero" for fluids. For the interface between solid and liquid, the cells partially saturated (Noble and Torczynsky, 1998) were implemented. The B parameter shown in the Eq. 4 represents a weight function dependent of the solid fraction ε in the cell. The value of the B parameter is given by:

$$B(x,t) = \frac{\varepsilon(x,t)(\tau^*-0.5)}{1-\varepsilon(x,t)+(\tau^*-0.5)} \tag{7}$$

and Ω_α^s is a additional term to modify the pressure function distribution of the solid-liquid interface (Noble and Torczynsky, 1998). Its equation introduces the term v_p that represents the wall velocity:

$$\Omega_\alpha^s(x,t) = p_{-\alpha}(x,t) - p_\alpha(x,t) + \ldots$$

$$\ldots + p_{-\alpha}^{eq}(p,v_p) - p_{-\alpha}^{eq}(x,t) \tag{8}$$

3 NUMERICAL HYDRO-MECHANICAL COUPLING

Two meshes are used in the coupling process between FEM and LBM (Figure 1a). The fracture boundaries are obtained from the finite element mesh. Their positions, which are

computed using FEM, are transferred to the LBM. The force applied on the boundaries, caused by the fluid pressure, can be computed using LBM. After this calculation, these forces are transferred to the FEM and applied as external forces on the faces of the crack. The new position of the crack boundaries is calculated using FEM and then is transferred to the LBM to update the boundary conditions (Figure 1b). Similar to the coupling process between the discrete element method (DEM) and LBM (Velloso, 2010), the coupling process between FEM and LBM has two time steps.

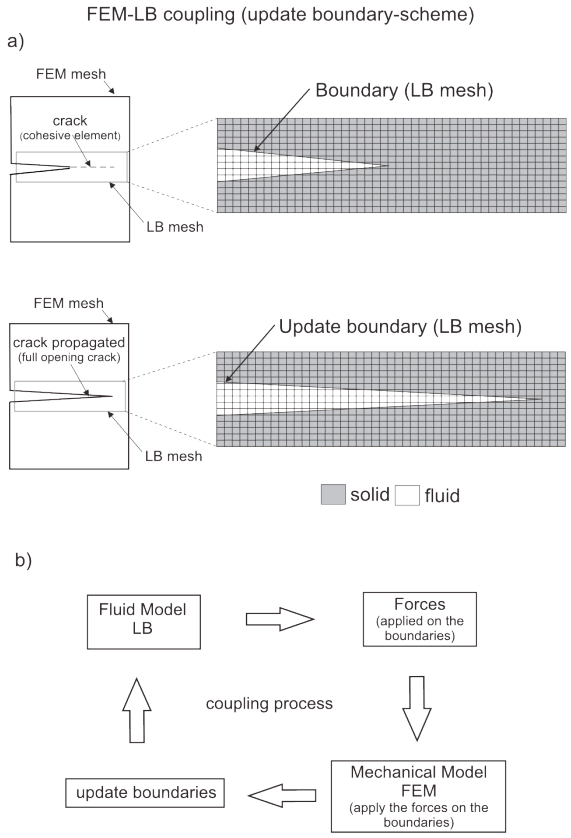

Figure 1. a) The coupling process between FEM and LBM for each subcycle; b) Detail of the two meshes used in the simulation. The LB mesh is created in the zone where the fracture propagations happens.

Normally, the time step for the FEM is smaller than the time step for the LBM. For this feedback-loop for fluid-structure interaction, it is necessary to define a subcycle number. The subcycle number can be described by:

$$n(subcycle) = \Delta t / dt$$

where $n(subcycle)$ is an integer number, Δt is the LBM time step and dt is the FEM time step. Thus, for one step in LBM, n steps of FEM are needed.

4 RESULTS

To model the hydraulic fracturing process in heterogeneous media, we used, as reference, the laboratory test results presented by Gonçalves (2015). Figure 2 shows the model geometry and the localization of the initial cracks. The fluid will be injected into these cracks during the simulation. To model the heterogeneous medium through the FEM-LBM coupling, it is necessary to transfer the mineral position to the FEM mesh. The transference of the mineral position inside the sample is done through a mapping procedure. Using a digital image of the sample, the minerals inside it are defined by the color of pixels (Figure 3a). The mapping process consists in associating the element of the FEM mesh with the color of pixels localized in the same position. Thus, when the mapping process is ended, mechanical and fracture properties are assigned to each element associated to the color of the pixel that represents the mineral. The rock used in the laboratory test is the Barre granite. However, there is none information about the mechanical and fracture properties of the minerals contained in this granite. Another granite with similar mechanical behavior is the Stanstead granite (Mahabadi, 2012) and its mechanical and fracture properties were used in the numerical model. Table 1 shows these properties.

Table 1. Rock properties of the Stanstead granite used in the numerical simulation.

	Quartzo	Feldspar	Biotite
ρ (kg/m^3)	2600	2600	2800
E(GPa)	80	70	20
ν	0.17	0.29	0.2
$\sigma_n=\sigma_t$(MPa)	10	10	7
$\phi_n=\phi_t$(N/m)	40	40	28
$\alpha=\beta$	2	2	2

The contact properties between minerals are an average of those presented on Table 1. Thus,

the contact properties between quartz and feldspar are: $\sigma_n=\sigma_t=8MPa$ and $\phi_n=\phi_t=32N/m$. For the contact between biotite and any other mineral, the properties are: $\sigma_n=\sigma_t=6MPa$ e $\phi_n=\phi_t=24N/m$. The grid of LBM is placed at the central region of the FEM mesh to capture the crack propagation. The fluid is injected into the initial cracks, and it is simulated through applying incremental pressure at constant ratio.

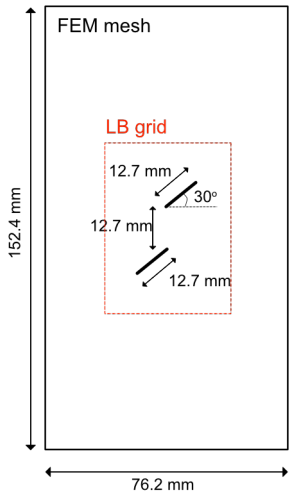

Figure 2. Geometry model and localization of the LB grid.

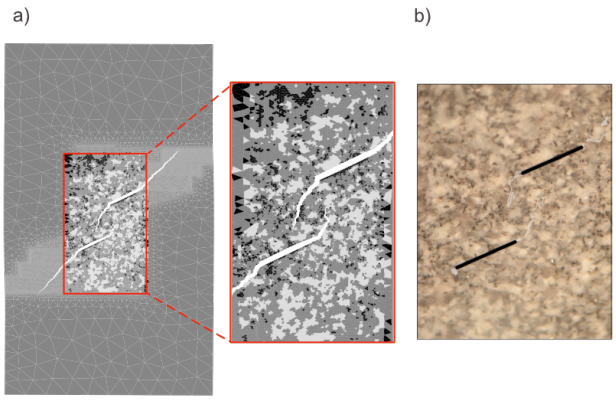

Figure 3. Comparison results among numerical simulation and laboratory test. a) Details of FEM mesh showing the mapping of the minerals inside the rock; b) Laboratory test image used to map the minerals to the FEM mesh.

Figures 3 and 4 show the comparison results between numerical simulation and laboratory test. They show a good approximation. However, the numerical model does not simulate accurately the crack propagation of the fracture tip. It happens due to a difference in the crack propagation velocity in the opposite ends of the fracture. During the numerical simulation, it was observed a higher crack propagation velocity at the external ends of the fractures. The propagation of the external extremes of both fractures reaches the boundaries of the LB grid before completing the coalescence between both cracks (in the middle of the sample). The increase in the velocity of the crack propagation at the external tip can also be observed in the laboratory test, but its intensity is lower when compared to the numerical simulation velocity. Also, the distribution of the elements in the FEM mesh may have influenced preferred directions to the crack propagation. However, numerical simulation results show the same coalescence type as the experimental test, indicating that the modeling is capable of representing the crack propagation of a heterogeneous material.

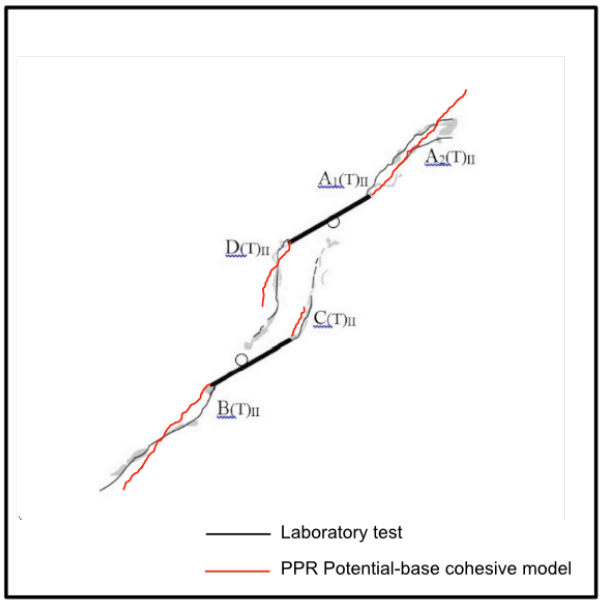

Figure 4. Comparison between the laboratory test result and the numerical result obtained through FEM-LBM coupling.

5 CONCLUSIONS

The coupling methodology proposed is capable to model the hydraulic fracturing process. The use of cohesive elements and PPR model allow modeling the crack propagation in a heterogeneous medium. It is due to these elements has their own fracture process

associated to the fracture properties. The LBM allows the modeling of the fluid injected into the crack independently of the fracture geometry. The comparison results between the numerical simulation and the laboratory test showed a good approximation of the crack paths. Also, the same coalescence type was observed in the results. However, the differences of the crack trajectories found in the numerical simulation can be associated to the mesh dependency, size of the elements and the flow rate of the fluid injected. These differences need to be studied in detail.

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