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Enough Components in Order to Model Cold Heavy Oil Production with Sand

Alejandra Arbelaez Londoño.
Universidad Nacional de Colombia, Medellín Campus, Medellin, Colombia, aarbelal@unal.edu.co

Jose Gildardo Osorio Gallego
Universidad Nacional de Colombia, Medellín Campus, Medellin, Colombia, gosorio@unal.edu.co

Guillermo Alzate Espinosa.
Universidad Nacional de Colombia, Medellín Campus, Medellin, Colombia, gaalzate@unal.edu.co

Abel Naranjo Agudelo.
Universidad Nacional de Colombia, Medellín Campus, Medellin, Colombia, anaranjo@unal.edu.co

SUMMARY: The major heavy oil production technology is Cold Heavy Oil Production with Sand (CHOPS). This is a single well technology, which involves the deliberate initiation and sustaining of sand inflow into the wells using progressive cavity pumps (PCP) to produce at oil high rates with a subsequent high-pressure drawdown around the wellbore. This production strategy of aggressive and continuous sand production enhances noticeably the oil recovery. CHOPS as recovery process has important challenges and high complexity because it responds to two main mechanisms such as the foamy oil and the wormhole formation, both consequences of complex geomechanical aspects as the in situ stresses redistribution, the elastoplastic behavior and the sand liquefaction. The enough components for a 3D single well numerical model for cold heavy oil production with sand are proposed coupling two main elements: a reservoir or fluid flow component and an elastoplastic geomechanical or stress - strain component. The fluid flow component is a multiphase fluid (three phases) composed by oil, gas and water using equations in cylindrical coordinates because this geometry fits the behavior of the flow in the region around the well, where the elastoplastic deformation occurs. The elastoplastic component includes four elements: an incremental stress - strain relation, a yield surface, a flow rule and a hardening / softening law. In addition, a sand production module is also included by the integration of two primary elements: a sanding onset to determine when is obtained the initiation condition for sand production and a sand production criterion to quantify the produced sand. Also, two more components are required to adjust and match a CHOPS model: a foamy oil module to involve the entrained gas drive mechanism from foamy oil crudes and the wormhole module to include the formation and propagation of void space generated by the massive sand production. CHOPS is a newfangled and cost effective alternative to the exploitation of heavy oil reservoirs around the world and especially in Latin America. Modeling cold heavy oil production with sand will lead to understand the complexity of the cold production process and will help also to evaluate how to increase the recovery factor under geomechanical stability conditions and to reduce operational costs of heavy oil reservoirs producing by this technology.

KEYWORDS: Sand production, CHOPS, cold heavy oil production with sand, cold production, sanding.

1 INTRODUCTION

Heavy oil plays a major role in the future of oil industry because of the vast reserves and the production decline of conventional oil. Latin America has the largest heavy oil reserves in countries as Venezuela and Mexico. Other countries as Colombia and Ecuador are making inroads in heavy oil exploitation. Colombia exploits heavy oil fields like Castilla, Chichimene, Rubiales and Apiay, and has high expectations because it is located in the mid moon that connects the heavy oil basins from Venezuela and Ecuador, in the middle looks to project as a sedimentary basin with similar characteristics and large reserves. The main challenge in heavy oil is about extracting, recovering, producing, and selling heavy oils within often changing economic guidelines and with minimal environmental impact. (Arbelaez-Londoño et al., 2014).

Heavy oil reservoirs are weak and unconsolidated sandstones that contain crudes with high density and viscosity. The sand grains production from unconsolidated sandstones under viscous fluid flow is inevitable. Sand production increases successfully productivity in heavy oil reservoirs.

The main heavy oil production technology is Cold Heavy Oil Production with Sand (CHOPS). This is a single well technology, which implicates the deliberate initiation and sustaining of sand inflow into the wells using progressive cavity pumps (PCP) to produce at oil high rates with a subsequent high-pressure drawdown around the wellbore. This strategy of aggressive and continuous sand production enhances noticeably the oil recovery and converts CHOPS as an attractive and profitable alternative to the exploitation of heavy oil reservoirs in Latin America, especially in Colombia. (Arbelaez-Londoño et al., 2014)

Cold production with sands is governed by sand production, mechanism that responds to foamy oil behavior and wormhole formation, both consequences of geomechanics aspects such as elastoplastic behavior, in-situ stress state redistribution and sand liquefaction that generate a complex multiphase fluid flow (oil, gas, water and sand). (Dusseault, 2002)

Modeling CHOPS presents significant challenges because it is a highly coupled geomechanical and fluid flow process, which requires incorporating complex phenomena as to foamy oil and wormhole formation, both consequences of aggressive sand production. Modeling will help to analyze how to increase the recovery factor under geomechanical stability and to reduce operational costs of this technology.

This paper pretends to propose the components of a 3D single well numerical model for cold heavy oil production with sands integrating the complex multiphase fluid flow, the elastoplastic behavior and the sand production. Also, two more components are proposed: foamy oil module and wormhole module.

2 COMPONENTS OF THE MODEL

A 3D single well numerical model for cold heavy oil production with sand requires two main components to be coupled: a reservoir or fluid flow model to be implemented by the finite-difference method and an elastoplastic geomechanical or stress-strain model to be implemented by the finite element method.

Furthermore, a sand production module, component to include the aggressive and continuous sanding, integrated by two elements: the sanding onset to predict the production conditions to initiate the sand influx and the sanding criterion to quantify the sand production rate.

Additionally, two more components are needed to adjust the phenomena associated to CHOPS. These components are the foamy oil module and the wormholes module. The foamy oil module includes the foamy crude flow using PVT properties, inflow performance and relative permeability curves. The wormhole formation includes the formation and propagation of void spaces produced by the massive sanding.

2.1 Fluid-Flow Model.

The fluid-flow model assumes isothermal and multiphase fluid composed by oil, gas and water. The physical model uses equations in cylindrical coordinates because this geometry fits the flow behavior in the region around the well, where the elastoplastic deformation occurs.

Four basic relations constitute this model: fluid mass conservation, solid mass conservation, Darcy's law, and the equation of state. (Naranjo, 2009)

2.1.1 Fluid mass conservation

$$\nabla \cdot (\phi \mathbf{v}_{oSC}) = \frac{1}{V_T}\frac{\partial}{\partial t}\left(\frac{\phi V_T S_o}{B_o}\right) + q_o \quad (1)$$

$$\nabla \cdot (\phi \mathbf{v}_{wSC}) = \frac{1}{V_T}\frac{\partial}{\partial t}\left(\frac{\phi V_T S_w}{B_w}\right) + q_w \quad (2)$$

$$\nabla \cdot \left(\sum R_{si} \phi \mathbf{v}_{iSC}\right) = \frac{1}{V_T}\frac{\partial}{\partial t}\left(\sum R_{si}\frac{\phi V_T S_i}{B_i}\right) + q_g \quad (3)$$

for $i = o, w, g$, i.e, oil, water and gas, where $\mathbf{v}_{iSC}$ is the real velocity vector at standard conditions, R_{si} is the gas solubility ($R_{sg} = 1$), B_i is the formation volume factor, S_i is the saturation, V_T is the total volume of the infinitesimal element, ϕ is the formation porosity; and q_o, q_w and q_g are the oil, water and gas source or sink term expressed as volumetric flow rate at standard conditions per unit of bulk volume ($q > 0$ for production and $q < 0$ for oil injection).

2.1.2 Solid mass conservation.

$$\nabla \cdot [\rho_s(1-\phi)\mathbf{v}_s] + \frac{1}{V_T}\frac{\partial}{\partial t}[\rho_s(1-\phi)V_T] + \widetilde{q_s} = 0 \quad (4)$$

where ρ_s is the solid density, $\mathbf{v}_s$ is the solid real velocity vector and $\widetilde{q_s}$ is the inflow or outflow solid mass through the source or sink per total volume unit per time unit.

2.1.3 Darcy's law.

For oil, water and gas phases, the general form:

$$\mathbf{v}_{iSC} = \mathbf{v}_s - \frac{\boldsymbol{k}_i}{\phi \mu_i B_i}\nabla p_i \quad (5)$$

where $\boldsymbol{k}_i$, is the permeability tensor, μ_i is viscosity, p_i, is the pressure and B_i is the formation volume factor, for for $i = o, w, g$, i.e, oil, water and gas.

2.1.4 Equation of state.

The oil and water phases are handle as a slightly compressibility fluid and, then, the oil and water compressibilities, c_o and c_w, are assumed to remain constant over the range of pressure of interest and can be expressed as:

$$c_i = -\frac{1}{B_i}\frac{\partial B_i}{\partial p_i} = constant \quad (6)$$

The gas phase is considered as a real gas, i.e., the gas compressibility c_g is pressure dependent.

2.1.5 Other relations.

Saturations.

The oil, water and gas saturations must satisfy the following constraint:

$$S_o + S_w + S_g = 1 \quad (7)$$

with $0 \leq S_o, S_w, S_g \leq 1$

Capillary Pressures.

The oil, water and gas pressures can be related through the concept of capillary pressure:

$$p_{cow} = p_o - p_w \quad (8)$$

$$p_{cgo} = p_g - p_o \quad (9)$$

where p_{cow} is the oil-water capillary pressure and p_{cgo} is the gas-oil capillary pressure.

2.1.6 Governing Equations.

Equations (1) through (9) can be combined resulting in the following fluid-flow equations:

For oil phase,

$$\nabla \cdot \left(\frac{k_o}{\mu_o B_o}\nabla p_o\right) = \frac{1}{V_T}\frac{\partial}{\partial t}\left(\frac{\phi V_T S_o}{B_o}\right) + q_o - \frac{Q_s \phi}{\Delta V_T(1-\phi)} \quad (10)$$

For water phase,

$$\nabla \cdot \left(\frac{k_w}{\mu_w B_w}\nabla p_w\right) = \frac{1}{V_T}\frac{\partial}{\partial t}\left(\frac{\phi V_T S_w}{B_w}\right) + q_w - \frac{Q_s \phi}{\Delta V_T (1-\phi)} \quad (11)$$

For gas phase,

$$\nabla \cdot \left(\sum R_{si}\frac{k_i}{\mu_i B_i}\nabla p_i\right) = \frac{1}{V_T}\frac{\partial}{\partial t}\left(\sum R_{si}\frac{\phi V_T S_i}{B_i}\right) + q_g - \frac{Q_s \phi}{\Delta V_T (1-\phi)}\sum R_{si} \quad (12)$$

where Q_s is the sand volumetric rate.

2.2 Elastoplastic Model.

The strain-stress model for heavy oil reservoirs is a 3D elastoplastic model to be implemented by Finite Element Method (FEM).

The basic assumptions are which the total strain increment may be decomposed into elastic and plastic parts, with only the elastic part contributing to the stress increment by means of an elastic law.

This model is based on three basic relations (Chen and Baladi, 1985): (i) the stress equilibrium equation, (ii) the strain-displacement equation, and (iii) the constitutive relation of the material to be modeled, elasticity or elastoplasticity and defines the strain-stress-pressure relations.

The elastoplastic calculations include an incremental stress-strain relation, a yield surface, a flow rule and a hardening / softening law.

2.2.1. Basic relations.

Initially, the porous media behaves elastically according to Hooke's law and later the material yields mechanically, and a new stress-strain relation is required to describe this behavior.

The stresses equilibrium equation is solved. F is related with the body forces, which in this case are assumed to be zero.

$$\sigma_{ij,j} + F_j = 0 \quad (13)$$

where $\sigma_{ij} = \sigma_{ji}$.

Using the Hooke's law for an homogenous porous media, the relationship between stresses, strains and pore pressure is linear and assume that all the deformation are recoverable. This relationship with isotropic elastic properties can be expressed by:

$$\sigma_{ij} = 2G\varepsilon_{ij}^e + \left(\lambda\varepsilon_v^e + \alpha p_p\right)\delta_{ij} \quad (14)$$

where σ_{ij} is the stress tensor, where is G is the shear modulus, ε_{ij}^e is the elastic deformation, p_p is the pore pressure, α is Biot's coefficient, λ is the Lame's constant and δ_{ij} is the Kronecker's delta function.

The incremental strain is assumed linearly divided into two parts: elastic and plastic:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p \quad (15)$$

2.2.2. Yield surface.

The elastoplastic condition displays both elastic and plastic properties, typically as result of being stretched beyond an elastic limit. Thus, the material behavior is elastic up to a transition point called yield point, in which the behavior changes from elastic to plastic (irreversible deformation). (Davis and Selvadurai, 2002)

$$f = f\left(\sigma_{ij}, \varepsilon_{ij}^p\right) = 0 \quad (16)$$

where f is the yield function and ε_{ij}^p is the plastic strain related to strain hardening / softening.

The yield criterion, often expressed as yield surface, is a hypothesis concerning the limit of elasticity under any combination of stresses, i.e., it defines a surface where plastic deformation occurs.

The yield surface or failure envelope determines the plastic behavior: perfectly plastic material or plastic material with hardening / softening. So that the stress state reaches the failure envelope, the afterload generates plastic strains. For perfectly plastic materials, the failure envelope is function of the stresses and does not change with time, but for plastic materials with hardening / softening the failure envelope is also function of stresses and plastic strains, i.e., the failure envelope change with the load history.

The yield criteria can be defined in terms of shear or tensile because both processes can generate the failure or yield.

2.2.3. Failure criteria.

The simplest and more used criterion is the Mohr Coulomb failure criterion, which defines the failure in terms of the maximum stress the minimum stress. (Davis and Selvadurai, 2002)

$$\sigma_1 = N_\varphi \sigma_3 + 2\tau_0\sqrt{N_\varphi} \quad (17)$$

where τ_0 is the cohesion, φ is the friction angle, and:

$$N_\varphi = \frac{1+\sin\varphi}{1-\sin\varphi}$$

The Mohr Coulomb failure envelope must be limited by the tensile stress at failure, which it is defined as the maximum stress that a material can withstand while being stretched or pulled before failing and is known as the tensile strength,

$$\sigma_3 = -\sigma^t \quad (18)$$

where σ^t is the tensile strength and for this reason it is a positive value. When the tensile strength of the material is unknown, an extrapolation of the Mohr Coulomb failure envelope is required, getting the maximum value of the tensile strength as,

$$\sigma^t_{max} = \frac{\tau_0}{\tan\varphi} \quad (19)$$

2.2.4. Yield functions.

Two failure functions are defined as (Itasca, 2011):

$$f^s = -\sigma_1 + \sigma_3 N_\varphi + 2\tau_0\sqrt{N_\varphi}$$

$$f^t = \sigma^t + \sigma_3 \quad (20)$$

where f^s is the shear failure function and f^t is the tensile failure function. It is clear that when the stress state has overcome the Mohr Coulomb failure criterion by shear $f^s \leq 0$, and similarly by tensile failure $f^t \leq 0$.

The function h delimits the space of tensile failure and shear failure, and is expressed by:

$$h = \sigma_3 + \sigma^t + a^p(\sigma_1 - \sigma^p) \quad (21)$$

where,

$$a^p = \sqrt{1 + {N_\varphi}^2} + N_\varphi$$

$$\sigma^p = \sigma^t N_\varphi + 2\tau_0\sqrt{N_\varphi}$$

2.2.5. Potential functions.

Similarly, the potential functions can be defined as (Itasca, 2011):

$$g^s = -\sigma_1 + \sigma_3 N_\psi \quad (22)$$

where g^s is the shear potential function, ψ is the dilation angle, and:

$$N_\psi = \frac{1+\sin\psi}{1-\sin\psi}$$

The associated flow rule for tensile failure is derived from the tensile function:

$$g^t = \sigma_3 \quad (23)$$

2.2.6. The flow rule.

The flow rule specifies the direction of the plastic strain increment vector as normal to the potential surface. It is called associated if the potential and yield function coincides, non associated otherwise. (Han et al., 2005).

The plastic strain increment can be defined as:

$$d\varepsilon^p_{ij} = \Lambda \frac{dg}{d\sigma_{ij}} \quad (24)$$

where $\bar{g}$ is the plastic potential surface and Λ is the plasticity scale multiplier. For associated flow, $\bar{g}$ is set equal to the yield criterion $\bar{f}$, which is a function of both stresses and strains.

Consequently, comparing to perfect plasticity where the failure surface is only associated to the stresses $\bar{f} = f(\sigma_{ij})$, the consistency condition becomes:

$$d\,\bar{f} = \frac{\partial \bar{f}}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial \bar{f}}{\partial \varepsilon_{ij}^{p}} d\varepsilon_{ij}^{p} = 0 \qquad (25)$$

Considering the plastic strain increment into the consistency condition:

$$\Lambda = -\frac{\frac{\partial \bar{f}}{\partial \sigma_{ij}} d\sigma_{ij}}{\frac{\partial \bar{f}}{\partial \varepsilon_{ij}^{p}} \frac{\partial \bar{f}}{\partial \sigma_{ij}}} \qquad (26)$$

The total strain increment $d\varepsilon_{ij}^{p}$, resulting from elastoplastic strain can be defined as:

$$d\sigma_{ij} = 2G d\varepsilon_{ij} + \left(\frac{1}{3} + \frac{2G}{9K}\right) dJ_1 \delta_{ij} - 2G\Lambda \frac{\partial \bar{f}}{\partial \sigma_{ij}} \qquad (27)$$

with,

$$dJ_1 = 3K \left(dI_1 - \Lambda \frac{\partial \bar{f}}{\partial \sigma_{ij}} \delta_{ij}\right)$$

Then,

$$\Lambda = \frac{\left\{\frac{\partial \bar{f}}{\partial \sigma_{ij}}\right\}^T \left\{2G d\varepsilon_{ij} + \left(K - \frac{2}{3}G\right) dI_1 \delta_{ij}\right\}}{2G \left\{\frac{\partial \bar{f}}{\partial \sigma_{ij}}\right\}^T \frac{\partial \bar{f}}{\partial \sigma_{ij}} + \left(K - \frac{2}{3}G\right) \left\{\frac{\partial \bar{f}}{\partial \sigma_{ij}}\right\}^T \frac{\partial \bar{f}}{\partial \sigma_{ij}} \delta_{ij} - \left\{\frac{\partial \bar{f}}{\partial \varepsilon_{ij}^{p}}\right\}^T \frac{\partial \bar{f}}{\partial \sigma_{ij}}} \qquad (28)$$

Finally, the elastoplastic stress increment can be determined as:

$$d\sigma_{ij} = \left[D^e - \frac{D^e \frac{\partial \bar{f}}{\partial \sigma_{ij}} \left\{\frac{\partial \bar{f}}{\partial \sigma_{ij}}\right\}^T D^e}{\left\{\frac{\partial \bar{f}}{\partial \sigma_{ij}}\right\}^T D^e \frac{\partial \bar{f}}{\partial \sigma_{ij}} - \left\{\frac{\partial \bar{f}}{\partial \sigma_{ij}}\right\}^T \frac{\partial \bar{f}}{\partial \sigma_{ij}}}\right] \qquad (29)$$

with,

$$D^e = 2G + \left(K - \frac{2}{3}G\right) \delta_{ij}$$

2.2.7. Hardening / Softening Parameter.

The shear hardening parameter ε^{ps} quantifies the level of plastic strain by shear as (Han et al., 2005):

$$d\varepsilon^{ps} = \left\{\frac{1}{2}\left(d\varepsilon_1^{ps} - d\varepsilon_m^{ps}\right)^2 + \frac{1}{2}\left(d\varepsilon_2^{ps} - d\varepsilon_m^{ps}\right)^2 + \frac{1}{2}\left(d\varepsilon_3^{ps} - d\varepsilon_m^{ps}\right)^2\right\}^{\frac{1}{2}} \qquad (30)$$

where $\varepsilon_j^{ps}, j = 1{,}3$ are the principal plastic shear strains, and:

$$d\varepsilon_m^{ps} = \frac{1}{3}\left(d\varepsilon_1^{ps} + d\varepsilon_2^{ps} + d\varepsilon_3^{ps}\right)$$

For plastic strain for a tensile failure, the tensile hardening e^{pt}can be expressed by:

$$d\varepsilon_m^{pt} = d\varepsilon_1^{pt} \qquad (31)$$

where ε_1^{pt} is the tensile plastic strain in the direction of the major principal stress and $\varepsilon_i^{pt}$is the plastic principal strain.

2.3 Sand Production Module.

A sand production module is also involved to model the sanding process and integrates two primary elements: a sanding onset and a sand production criterion.

2.3.1. Sanding onset.

The sanding onset determines the initiation point for sand production in terms of a production criterion such as a pressure gradient or a production rate. The onset sanding prediction consists of evaluating a yield criterion at the stress state to determine the pressure gradient or production rate at which the sanding occurs. In this work, the yield criterion to be used is Mohr Coulomb criterion.

2.3.2. Sand production criterion.

Araujo et al. (2015) propose a new criterion to quantify the sand production rate based on the rock plastic deformation using experimental data for a sand production test, in which the vertical and the radial stresses are increased independently, and a high radial flow rate is produced.

$$Q_s = \begin{cases} 0 & , \quad \varepsilon^{ps} < \varepsilon^{ps}_{cr} \\ \frac{\iota_s}{\rho_s}\frac{A_e}{A_c}\frac{d\varepsilon^{ps}}{dt} & , \quad \varepsilon^{ps} > \varepsilon^{ps}_{cr} \quad \& \quad v_{fo} > v_{fo\,min} \end{cases} \quad (32)$$

where ε^{ps}_{cr} is the minimum plastic strain needed to initiate the sanding and $v_{fo\,min}$ is the minimum foamy oil velocity required to erode the sand grains, and ι_s is an experimental parameter, which relates the sand production and the plastic strain increments at the cavity surface. In addition, A_c is the area of the cavity, which is prone to sanding during the expermental test, and A_e is the area of the infinitesimal volume element.

2.4 Foamy Oil Module.

Kumar and Mohadevan (2012) proposed a first approach to include the foamy oil behavior during production, i.e., the entrained gas drive mechanism. Thus, this module defines two parameters such as the endpoint entrained-gas fraction and the apparent bubble point, and integrates three elements: (i) the inflow performance as a foamy oil with three phases: dissolved gas, oil component, and entrained gas, (ii) the fluid characterization changing the PVT properties for a foamy oil crude, and (iii) the relative permeability curves in terms of pressure. This module woks adjusting the fluid flow component.

2.5 Wormhole Module.

The wormhole can be defined as a cavity or space created during the massive sanding, i.e, a volume in which there is no grain-to-grain contact, a cavity in situ that is not actually empty and is filled with sand fluidized in foamy oil. The wormholes grow around the wellbore define ng a liquefied zone, where sand liquefies and foamy oil with fluidized sand flows. This zone has porosity greater than 50% and an extremely high permeability. (Arbelaez-Londoño et al., 2014)

The coupling of all model components can generate this zone by its own, with a wormhole propagation criterion in terms of a critical flow rate or a critical pressure at the tip, by which a sanding onset condition is reached and depends on the formation cohesion and friction angle. (Wang and Chen, 2004). Additionally, the produced sand volume inside each wormhole is calculated using material balance once the length of the wormhole is determined. Thus, the wormhole module is an adjustment of the production module.

3 IMPLEMENTATION OF THE COMPONENTS.

The three phases fluid-flow component will be implemented by finite difference method and the elastoplastic stress-strain component will be implemented by finite element method. These two main components will be coupled using the term of ΔV_p, which can be define as function of the stresses and pressures. Figure 1 presents how the model components interacting to integrate the CHOPS model.

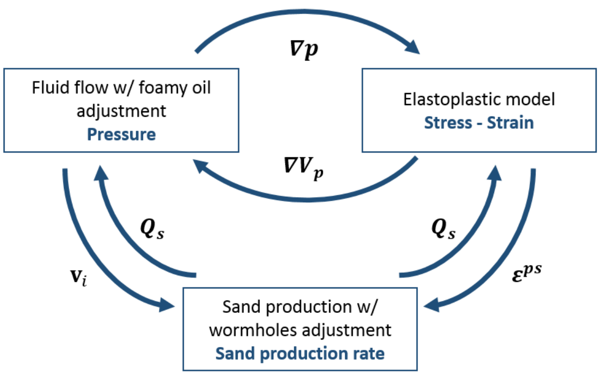

Figure 1. Implementation of the model components.

CONCLUSIONS

The main components of a 3D single well numerical model for cold heavy oil production with sands are proposed, integrating the complex multiphase fluid flow (oil, water and gas) and the elastoplastic behavior and the sand production.
The sand production module integrates a sanding onset to determine the initiation of the sand production, and a sand production criterion to

quantify the produced sand. Also, two more components are projected to adjust the CHOPS model: a foamy oil module and wormhole module.

NOMENCLATURE

α	=	Biot's poroelastic constant.
A	=	Area.
B	=	Formation volume factor.
c	=	Compressibility.
δ	=	Kronecker's delta function.
ε	=	Strain.
ρ	=	Density.
σ	=	Stress.
f	=	Yield function.
g	=	Potential function.
G	=	Shear modulus.
ι	=	Sand production parameter (experimental).
I, J	=	Principal strain invariants.
$k, \mathbf{k}$	=	Permeability scalar, vector.
K	=	Elastic bulk modulus.
μ	=	Fluid viscosity.
λ	=	Lame´s constant.
Λ	=	Plasticity scale multiplier.
p	=	Fluid pressure.
φ	=	Friction angle.
ψ	=	Dilation angle.
ϕ	=	Effective porosity.
Q	=	Volumetric flow rate.
q	=	Volumetric flow rate per unit of volume.
$\tilde{q}$	=	Mass rate per unit of bulk volume.
Rs	=	Gas solubility.
S	=	Saturation.
t	=	Time.
τ	=	Cohesion.
$\mathbf{v}$	=	Real velocity vector.
V	=	Volume.

Superscripts

e	=	Elastic.
p	=	Plastic.
s	=	Shear.
t	=	Tensile.
T	=	Transposed.

Subscripts

b	=	Bulk.
c	=	Capillary.
c	=	Cavity.
cr	=	Critical.
e	=	Infinitesimal volume element.
fo	=	Foamy oil.
g	=	Gas.
ij	=	Tensor indexes.
max	=	Maximum.
min	=	Minimum.
m	=	Mean.
o	=	Oil.
p	=	Porous.
s	=	Solid.
SC	=	Standard conditions.
T	=	Total
w	=	Water.

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