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Comparison Between Deterministic and Probabilistic Stability Analysis, Featuring a Consequent Risk Assessment

Isabela Moreira Queiroz
Walm Engenharia e Tecnologia Ambiental, Belo Horizonte, Brazil, belamq@yahoo.com.br

SUMMARY: Slope stability analyses are largely carried out by deterministic methods and evaluated through a single safety factor. Although it is known that the geotechnical parameters can present great dispersal, such analyzes are considered fixed and known. The probabilistic methods, in turn, incorporate the variability of input key parameters (random variables), resulting in a range of values of safety factors, thus enabling the determination of the probability of failure, which is an essential parameter in the calculation of the risk (probability multiplied by the consequence of the event). Between the probabilistic methods, there are three frequently used in geotechnical society: FOSM (first order and second moment), estimate points (Rosenblueth) and Monte Carlo. In this paper is presented a comparison between the results from deterministic and probabilistic analyzes (FOSM method, Monte Carlo and Rosenblueth) applied to a hypothetical slope. The end was held to evaluate the behavior of the slope and consequent risk analysis, which is used to calculate the risk and analyze their mitigation and control solutions. It can be observed that the results obtained by the three probabilistic methods were quite close. Notes that the calculation of the risk makes it possible to list the structures in degrees of priority to the implementation of mitigation measures. Therefore, it is recommended to do a good assessment of the geological-geotechnical model and incorporating uncertainty in viability, design, construction, operation and closure by means of risk management.

KEYWORDS: Risk assessment, probabilistic methods, slope stability, risk management.

1 INTRODUCTION

The study to analyze the stability of the slope can be done through deterministic or probabilistic methods. Normally a stability analysis is performed to determine the conditions for the project in such a way to ensure the required minimum safety.

The probabilistic analysis differs from deterministic, mainly because it considers the variability of the parameters. Most of the input data in a slope stability analyses are not known with precision. The variability is due to the dispersion of the results of tests or to natural variability in the value of the grandeur that exists from one point to another in the slope. It is, therefore, a distribution of values for each parameter, which causes are considered random variables. Thus, it is concluded that the safety factor itself is a random variable which depends on many input variables and has its own distribution.

There are three commonly used probabilistic methods in geotechnical mean: FOSM (first order and second moment), Estimate points (Rosenblueth) and Monte Carlo.

The Monte Carlo method establishes that the probability distribution functions of the independent variables are initially known. In the absence of these, usually adopts a normal distribution. Has the advantage of obtaining the distribution of safety factor (dependent variable). As disadvantage the method demands time, large computational effort and specific programs (Hammersley & Handscomb, 1964).

As the First Method Order Second Moment (FOSM) presented by Christian el al. (1992), is based on the truncation of the Taylor series for the dependent variable (safety factor). The average value of the dependent variable is calculated from the average values of the independent variables. The standard deviation is calculated from the input parameters and variances of the derivatives of the dependent variable for each independent variable. An advantage of this method is the quantification of the influence of each independent variable in the variance of the safety factor. As a disadvantage, not gets a distribution of safety factor and adopt assumptions on this distribution. In addition, the maximum probability of failure is not always related to the surface of rupture for the minimum safety factor (Tobutt & Richards, 1979).

And lastly would be the method of estimate (Rosenbleuth, 1975). The Rosenbleuth method, the knowledge of the distribution functions of the independent variables, using only their values calculated in so-called estimate points (average standard deviation and mean less standard deviation) the dependent variable is calculated for these points, obtaining a sample of what you can calculate your average and standard deviation. The method is easy to apply. You must, however, take a distribution for the safety factor (usually normal) and it is assumed that the distribution of each independent variable is symmetric.

While the deterministic approach adopts the safety factor as stability index in the balance problem of the slope, the probabilistic methods adopt as such the probability of failure, which is a data input on risk analysis.

The risk assessment process generally involves the scope definition and selection of the method of analysis, definition and identification of hazard conditions, estimate the probability of failure and consequence, risk estimation, documentation, verification and analysis update. The methods of risk assessment can be qualitative or quantitative. The qualitative methods rely on descriptive forms or sort numerical scales to describe the greatness of probability and consequence, whereas the set quantitative uncertainties, based, therefore, on numeric values of probability and consequence (Hartford & Baecher, at, 2004 apud Melo, 2014).

This paper compares the deterministic and probabilistic analyses applied to hypothetical slope stability. And finally a risk assessment carried out by quantitative method.

2 CASE STUDY

The slope that was analyzed in this paper has about 40.0 m and an inclination of approximately 1V:2H. The independent variables (*Xi*) considered for probabilistic analysis are (c’) cohesion, friction angle (φ') and the weight (γ).

The parameters ($\overline{X}_i$) used were refereed to deterministic mean safety factor of about 1.5. Although fictitious values adopted are quite representative. The standard deviations (σ[Xi]) have been determined based on the typical literary variation coefficients (Table 1).

Table 2 summarizes the values adopted. Figure 1 show the geological-geotechnical model adopted, considering the reduced water level.

Table 1. Typical values of coefficient of variation

Parameter	Coefficient of variation (%)
Specific Weight	03 (02 to 08)
Cohesion	40 (20 to 80)
Effective Friction angle	10 (04 to 20)
Cohesion Not Drained	30 (20 to 50)

Table 2. Geotechnical parameters of the materials used.

Lithology	Cohesion (kPa)			Friction angle (°)			Weight (kN/m^3)		
	Mean	CV	Standar deviation	Mean	CV	Standar deviation	Mean	CV	Standar deviation
Sandy Silt	5,0	40%	2,0	29,0	10%	2,9	21,1	3%	0,6
Pebble Sand	5,0	40%	2,0	34,0	10%	3,4	29,0	3%	0,9
Residual Soil	10,0	-	-	34,5	-	-	20,0	-	-
Saprolite	10,0	-	-	39,0	-	-	21,0	-	-

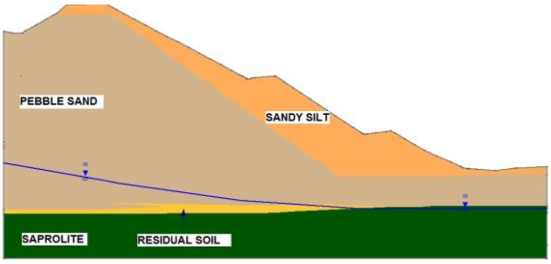

Figure 1. Geological-geotechnical model used.

3 CALCULATION METHODOLOGY

Stability analyzes were carried out with the assistance of Slide 6.0 program, developed by the company Rocscience. The safety factor was calculated by the method GLE/Morgenstern-Price.

The probability of failure was calculated using probabilistic methods of Monte Carlo, FOSM (First Order Second Moment ") and Rosenblueth (Method of estimate).

The application of these three probabilistic methods requires the existence of a mathematical formulation (empirical, analytical or numerical) that relates the fault indicator (safety factor-dependent variable) with the input data (material properties, charging, water level-independent variables) (Assis et. al., 2013).

Considering the safety factor as a random variable, can compute the probability of failure as the probability of the FS to be less than or equal to 1, IE (P [FS $\leq$ 1]).

The probability of failure is being increasingly used as a criterion for acceptance in the last 35 years, although it is still viewed with varying degrees of enthusiasm and skepticism (Wesselloo & Read, 2009). These authors point out the following advantages in calculating the probability of failure:

- Allows the variation of the resistant forces and mobilizing forces and help to establish the level of confidence in the project. The reliability of the structure is its probability of success. Therefore, if the probability of rupture of a slope is 20%, their reliability is of 80%;
- Is an essential parameter in the quantification of risk value (R), defined by the following equation:

$$R = P_f \text{ x (consequence of failure)} \qquad (1)$$

According to Melo (2014), risk assessment is based on the use of the information available to estimate the relative risk to individuals or populations, or properties environments, due to conditions of danger. It involves the breakdown or decomposition of the system and source of risk in its fundamental parts.

Technically, the risk assessments can be qualitative or quantitative. Qualitative analysis is used in a way that is descriptive or numerical sorting scales to present the magnitude of potential consequences and their likelihood of occurrence. As the quantitative analysis is based on numeric values of the potential consequences and their probabilities, assuming that such values are a valid representation of the real magnitude of the consequences and the probability of the scenario studied (Melo .2014).

The probability of failure is represented by the area under the curve of probability distribution, contained the left of FS = 1.0, as it illustrates the Figure 2. In this figure, are represented the probability distributions of two cases A and b. the case presents an average safety factor equal to 1.5 and a standard deviation of 0.25 and if B has an average safety factor of 2.0 and standard deviation of 0.85. According to Figure 2, you can see that the probability of the case is lower than in case B, so, the project presents a lower possibility of undesirable events (break) (Maia, 2003).

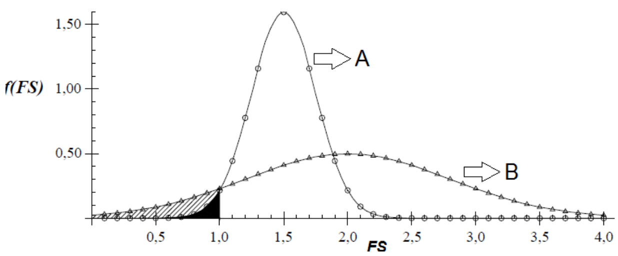

Figure 2. Statistical distributions with different safety factors and standard deviation.

The evaluation of the probability distribution of the safety factor (dependent variable) and the calculation of the probability of rupture are carried out through the application of probabilistic methods, which the most important are Monte Carlo methods, FOSM, and method of estimate (Rosenblueth method).

3.1 Monte Carlo Method

The Monte Carlo method was first used as a research tool in the development of the atomic bomb during World War II (Hammersley & Handscomb, 1964 apud Maia, 2003). It is an iterative procedure which covers four steps, described below (Willie & Mah, 2004):

- Estimate of the probability distributions for each of the input parameters considered as variables;
- Generation of random values for each parameter;
- Calculation of the operating forces and sturdy and resistant forces verification are larger than the active;
- Repeat the process N times (N > 100) and then determining the likelihood of breakage Pf according to the following equation:

$$P_f = \frac{N-M}{N} \quad (2)$$

Where:
M is the number of times that the resistant force exceeded the operating forces (i.e., the factor of safety is greater than 1.0).

3.2 FOSM Method

The FOSM method ("First Order Second Moment") it was initially used in the steel industry projects (Kottegoda & Rosso, 1974 apud Maia, 2003). Is a method that uses the first terms of the Taylor series expansion of the function of performance (i.e. safety factor) to estimate the expected value (mean) and the variance of the function of performance. Is called a second time because the variance is a form of second moment, being the result of higher order statistics used in the analysis (Baecher & Christian, at, 2003).

Assuming multiple random variables, independent of each other, the equations of the FOSM method correspond to:

$$E[F] = F\left(\bar{x}_1, \bar{x}_2, ...\bar{x}_N\right) \quad (3)$$

$$V[F] = \sum_1^n \left(\frac{\delta F_i}{\delta X_i}\right)^2 \times V[X_i] \quad (4)$$

Where:
E [F] = average value expected to F;
V [F] = variance of F, equal to the square of its standard deviation;
(δ) F = F Variation that occurs when range from a value of (δ) xi each of n parameters xi;
(δ) Xi = rate of change of the variables involved in the study;
V [Xi] = variance of each xi.

If the number of random variables is equal to n, this method requires the making of n + 1 tests.

3.3 Rosenblueth Method

The method of estimate or Rosenblueth method it was created in 1975. In this method, the distribution of a random variable Xi is concentrated on two particular points located by:

$$X_{i+} = \bar{X}_i + \sigma_i \quad (5)$$

$$X_{i-} = \bar{X}_i - \sigma_i \quad (6)$$

Where:
$\bar{X}_i$ and σ are be the mean value and the standard deviation of the distribution of the variable Xi.

"The method of estimation, for the case of n variables correlated, need 2^n estimated values for each combination of the particular points Xi and Xi+ Xi-. After the completion of the combination of the particular points, starts the probabilistic procedure through deterministic calculations, for this group of 2^n values. So, for each new probabilistic process step, the deterministic calculations are powered by a group of new values (data) input to be used in the next calculation "(Maia, 2007).

The Figure 3 illustrates the procedure for calculating the Rosenblueth method. It should be noted that this method assumes that the distribution of safety factor is normal and should be used when the random variables follow the normal distribution (Rocscience, 2013).

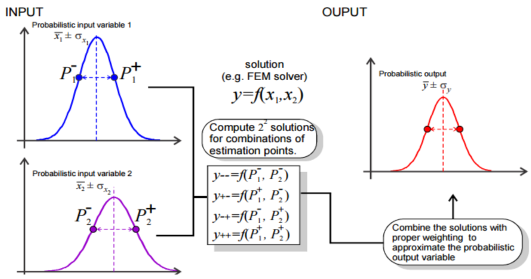

Figure 3. Diagram of the procedure for calculating the Rosenblueth method.

4 RESULTS

4.1 FOSM

The first analyzes by the FOSM method uses the mean values of random variables, resulting in an average safety factor deterministic. The other analyzes apply a rate of increase of 5% to 10% over the average value of each random variable. From the application of (4), the percentage of contribution of each random variable on the variance of the safety factor. For the calculation of the probability of failure, it was considered a normal distribution for the safety factor and was used the following equation in Microsoft Excel:

DIST. NORM. N (x; average; standard deviation; cumulative) (7)

Where the value of x corresponds to the minimum value adopted for the safety factor. There is also the logical value "cumulative" must be set to true for a cumulative distribution function.

The results presented in this paper consider the calculation of probability of failure, that is, Pf [FS ≤ 1.0]. Here are the results obtained by applying the FOSM method.

The Figure 4 shows the critical surface of rupture and the minimum safety factor obtained for the slope and the Figure 5 illustrates the results of applying the FOSM method. In Figure 5 shows that the random variable that most contributed to the variation of the safety factor was the friction angle of sandy silt (71.8%). The cohesion of the sandy silt contributed 28.1% and specific weight with only 0.1%.

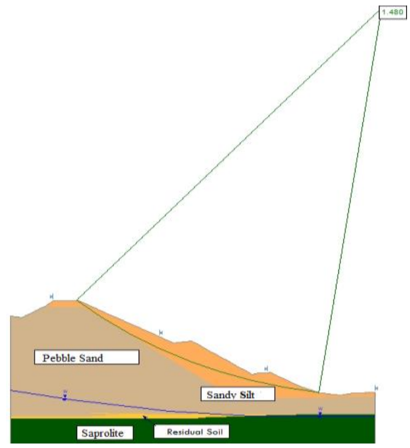

Figure 4. FSmean = 1.48 (Deterministic).

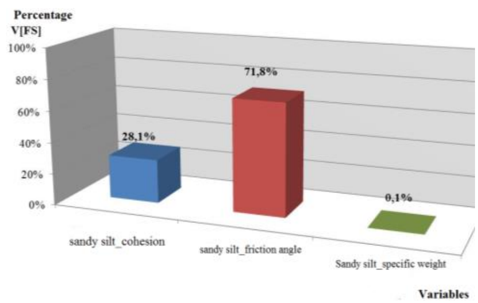

Figure 5. Percentage of influence of random variables in the variation of the safety factor of slope.

The Table 3 displays the value of the probability of rupture determined by the FOSM method, adopting a normal distribution for the safety factor.

Table 3. Summary of the results of analyses by applying the FOSM method.

FSmean	σ [FS]	Pf [FS ≤ 1.0]	Statistical Distribution
1,48	0,15	$7x10^{-4}$	Normal

Where: σ [FS] = standard deviation of the safety factor; Pf [FS ≤ 1.0] = probability of rupture.

4.2 Monte Carlo

The Slide program 6.0 allows probabilistic analyzes using the Monte Carlo method, where the user can choose to use the options "Global Minimum" or "Overall Slope.

For the analyzes presented in this paper, was opted for the "Global Minimum" option, where the probabilistic analysis is performed on the surface of slipping which presents the overall minimum safety factor, calculated by a deterministic stability analysis.

The safety factor is then calculated N times (where N is the number of iterations) to this rupture surface, using a different set of values (input data), generated randomly for each analysis. The probability of failure is the number of analyzes that resulted in a safety factor less than 1.0 divided by the total number of iterations. The analyze option "Global Minimum" assumes that the probability of failure calculated for the critical surface with minimum safety factor is representative of the probability of failure for the slope as a whole (Rocscience, 2013).

The number of iterations performed to achieve convergence by the Monte Carlo Method depends on the level of trust (α) admitted to the case study and can be estimated according to the following equation:

$$N = \left(\frac{Z^2_{\alpha/2}}{4\alpha^2}\right)^n \qquad (8)$$

Where:
N = number of attempts;
Z = parameter of reliability;
α = Tolerance;
n = number of variables.

In this way, to 03 random variables and a tolerance level (α) equal to 10%, the minimum required number of iterations by the Monte Carlo method, estimated by (8), would be approximately 310,000.

In cases where the number of iterations was insufficient for the convergence and/or none or very few safety factors, values calculated during simulation by Monte Carlo Method, were less than 1.0, the area below the histogram of the distribution of safety factor was calculated by the (7), presented earlier.

The Table 4 shows the result obtained by Monte Carlo Method and the Figure 6 shows the histogram of the resulting safety factor of iterations performed on Slide 6.0 program.

Table 4. Test result by applying the Monte Carlo method.

FS mean	σ [FS]	Pf [FS $\leq$ 1.0]	Statistical Distribution
1,48	0,15	$8x10^{-4}$	Normal

Notes: σ [FS] = standard deviation of the safety factor; Pf [FS $\leq$ 1.0] = probability of rupture.

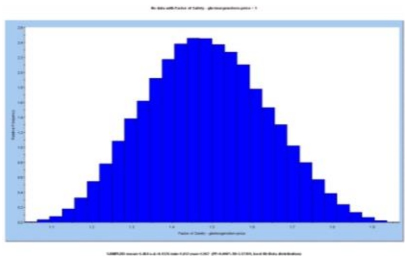

Figure 6. Histogram of the safety factor.

4.3 Rosenblueth

For the application of the method of Rosenblueth, were considered the three random variables used in previous analyzes, resulting in 8 combinations and values of FS presented in Table 5.

Table 5. Rosenblueth method-Combinations.

N° of Calculation	Particular Points			Values			FS
	c' silt	(ϕ) silt	γ silt	c' silt	(ϕ)' silt	(γ) silt	
1	+	+	+	7,0	31,9	21,7	1,638
2	-	+	+	3,0	31,9	21,7	1,550
3	+	-	+	7,0	26,1	21,7	1,398
4	-	-	+	3,0	26,1	21,7	1,247
5	+	+	-	7,0	31,9	20,5	1,635
6	-	+	-	3,0	31,9	20,5	1,558
7	+	-	-	7,0	26,1	20,5	1,412
8	-	-	-	3,0	26,1	20,5	1,254

The Table 6 presents the result obtained by applying the method of Rosenblueth.

Table 6. Test result by applying the method of Rosenblueth.

FSmean	σ [FS]	Pf [FS ≤ 1.0]	Statistical Distribution
1,46	0,16	$2x10^{-3}$	Normal

Where: σ [FS] = standard deviation of the safety factor; Pf [FS ≤ 1.0] = probability of rupture.

4.4 Summary of the results of the analyses

Table 7 shows a comparison between the results of the various probabilistic methods. It is observed that, in general, the probability values obtained by the three probabilistic methods were quite close.

The values obtained for the FOSM and Monte Carlo methods, for example, has the same order of magnitude. Considering the Rosenblueth method, the order of magnitude of values is greater than the FOSM method to the Monte Carlo method.

Table 7. Summary of the results Obtained by probabilistic methods.

Method	E [FS]	σ [FS]	Pf [FS ≤ 1.0]	Distribution function
FOSM	1,48	0,15	$7x10^{-4}$	Normal
Monte Carlo	1,48	0,15	$8x10^{-4}$	Normal
Rosenblueth	1,46	0,16	$2x10^{-3}$	Normal

5 CONCLUSION

It should be emphasized that a great advantage of the probabilistic analysis is to reduce the level of conservatism, as it can be considered the variability of geotechnical properties of the materials and not just a single value.

Was analyzed a slope in which predominated the effect of friction angle of sandy silt. The slope was reviewed by three probabilistic methods (FOSM, Monte Carlo and Rosenblueth). In general, can be highlighted the following points.

The importance of accurately determine the standard deviation of each variable in the problem. Small errors in the coefficient of variation of a variable, can lead to significant errors in the probability of failure in a slope that depends strongly of this variable.

At first, the specific weight cannot be discarded. This parameter affects both resistive efforts, as active. In this way can positively or negatively affect the safety factor.

It is emphasizing the importance of strict determination of the surface of rupture, once all other variables are calculated around this surface.

Start the analysis by the FOSM method, so, is obtained the random variables to be used in the other two methods (Monte Carlo and Rosenblueth), which are the variables that contributed most in variation of the safety factor.

Considering all the cases analyzed, the one that presents the highest probability value corresponds to the method of Rosenblueth (Pf = 2×10^{-3}).

This way, having assessed the probability of failure, it would be possible to perform the calculation of risk. But to be able to establish the risk, it is not enough just to identify and quantify the failure, since risk it is understood by the combination of the probability of failure with consequences. In the case studied, to reach a risk analysis of the slope, is still necessary an analysis of these consequences. For example, what would be the harm done with the break of slope? If a rupture occurs, could result in damage to infrastructure works, destruction of homes, deaths etc., so as to carry out the risk assessment in relation to the criterion of acceptability and tolerability, which may be individual or societal level (Pinto, 2008).

The acceptability of the individual risk of breach is on the order of 10^{-6} (United Kingdom; Morris et al.) the 10^{-6} to 10^{-8} (Australia; ANCOLD, 1994) per person per year. The individual risk, usually associated with the probability of loss of a human life, is a suggestive shape to represent the risk, as it allows their immediate comparison with different types of accident (Pinto, 2008).

The acceptability of the societal risk is calculated by FN curves, established in terms of the number of victims, N, and the corresponding annual probability of rupture (or frequency, F, accumulated by slope per year), with an expected value of victims equal to or greater than N the criteria for acceptability and tolerability represent the maximum permissible limits for the risk and are represented in the following graphic, where the annual probability of failure is on the vertical axis and the consequence, both in monetary cost and loss of life, on the horizontal (Baecher & Christian, at, 2003).

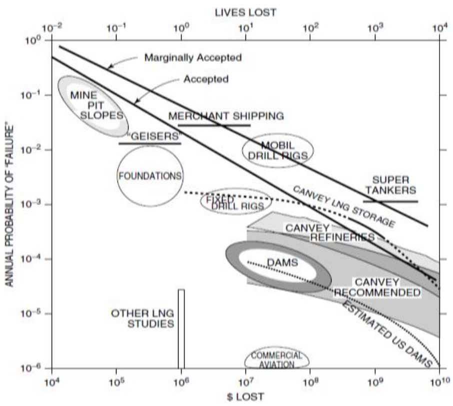

Figure 7. Acceptable levels and marginally acceptable risk.

The study of the consequences was not performed, so the risk assessment couldn't, effectively, be evaluated. Therefore, the risk was not appreciated in relation to any criterion of tolerance necessary to decide whether the risk should be mitigated.

Another important note is that values of probability of rupture are annual and must be updated whenever changes occur in the conditions assumed in the analyses or a greater knowledge of the properties and geotechnical parameters.

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