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Prediction of the Shear Behaviour of Clean Joints in Soft Rocks using Perceptron

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SUMMARY: One of the factors that most affects the safety of structures in or on rock masses is the shear behaviour of rock discontinuities. Some analytical models predicting the shear behavior of rock joints have been developed on the basis of experimental data obtained from direct shear tests performed under different boundary stiffness conditions. However, the use of these analytical models sometimes become difficult due to lack of laboratory test results, or even the difficulty of obtaining some of their parameters. The objective of this paper is to present a model that can be used to predict the shear behaviour of soft clean joints using the artificial neural network (ANN) known as perceptron. Results from direct shear tests conducted on idealized saw-tooth synthetic rock joints under different boundary conditions were considered in the development of the proposed ANN model. The following parameters were considered as input for the ANN model training: boundary normal stiffness, asperity height, initial asperity angle, initial normal stress and the horizontal displacement The output of the ANN model is the shear stress at a particular horizontal displacement. The developed ANN model has an A:5-15-5-1 architecture, where each number represents the number of neurons per layer. The coefficients of correlation between the actual test results and the model output values used to evaluate the behaviour of the neural model during training and validation were 0.999 and 0.997, respectively. The shear strength obtained by applying this kind of model can be considered more advantageous for use in practice than that obtained from the analytical models. One of the advantages of this tool is that, once the synaptic weights and bias of the model are known, the prediction of the shear behaviour of the clean joints of soft rocks can be made using a simple spreadsheet with only parameters which represent the initial roughness of the joint and the boundary conditions (CNL or CNS), rather than several model parameters. Besides, it was observed that the results obtained from the proposed ANN model presented a better fit to the experimental data when compared to the results of some of the existing analytical models.

KEYWORDS: rock, clean joints, shear behaviour, perceptron.

1 INTRODUCTION

It is known that the economic and safe design of structures in rock masses depends on a reasonable prediction of their shear behaviour which is usually governed by the conditions of the rock discontinuities. A number of analytical models predicting the shear behaviour of rock joints have been developed on the basis of experimental data obtained from direct shear tests performed under both CNL (Constant Normal Load) and CNS (Constant Normal Stiffness) conditions (Patton, 1966; Barton, 1973, Indraratna et al., 1998, 1999, 2010; Indraratna and Haque, 2000; etc.). According to these models, the shear behaviour of clean rock joints is influenced by several factors, such as, the normal stiffness and initial normal stress acting on the joint surfaces, some features of the intact rock matrix, e.g., the basic friction angle and the compressive strength, and also some characteristics related to the joint roughness, among others. Hence, the shear behaviour of the clean joints can be considered typically as a multivariate, nonlinear and complex phenomenon.

Despite the satisfactory results, the use of analytical models sometimes becomes difficult due to lack of laboratory test results, or even the difficulty of obtaining some of their parameters. In this context, and considering the powerful of artificial neural networks (ANN) in modelling phenomena in Civil Engineering (Dantas Neto et al., 2014; Araújo, 2015; Dantas Neto et al., 2016; etc.), the objective of this work is to present a non-conventional model that can be used to predict adequately the shear behaviour of soft clean joints using an ANN technique known as perceptron.

Some advantages of using the proposed ANN model are: its simplicity, once the prediction of the shear behaviour of the clean rock joints can be made using a simple spreadsheet with only parameters, rather than several model parameters; and the accuracy with which the results of the model fit the experimental data.

2 LITERATURE REVIEW

2.1 Shear Behaviour of Clean Joints of Rock Masses

In the beginning, the models used to describe the shear strength of rock discontinuities were based on limit equilibrium condition with parameters obtained from direct shear tests conducted under CNL condition (Patton, 1966; Barton, 1973, etc.). According to these models, the peak shear strength of the clean joints is function of the normal stress (σ_n) acting on the joint surface, which is considered constant in tests conducted under CNL condition, the basic friction angle (ϕ_b) of the rock, and the roughness of the joint profile, represented by either the initial asperity angle (i_0), or the value of the JRC (Joint Roughness Coefficient).

Haque (1999), Indraratna et al. (1998, 1999, 2010), and Indraratna and Haque (2000) are some authors who present results of direct shear tests conducted on the idealized saw-tooth joint of synthetic rock under both CNL and CNS condition. The results obtained by these authors show that the higher the normal stiffness (k_N), initial normal stress (σ_{n0}) and roughness of the joint profile, the higher the values of dilation and shear stresses during shearing. The authors show that the normal stiffness has a significant role on the shear behaviour of clean rock joints. Thus, models which consider only the CNL boundary condition can underestimate the shear strength of the rock masses (Indraratna et al., 1998).

Indraratna and Haque (2000) present an analytical model (Equation 1) to represent the shear-displacement behaviour of the clean joints in soft rocks using the energy principle of Seidel and Haberfield (1995).

$$\tau_h = \left(\sigma_{n0} + \frac{k_N \delta_v}{A_j}\right)\left(\frac{\tan\phi_b + \tan i_0}{1 - \tan\phi_b \tan i_h}\right) \qquad (1)$$

where:

- τ_h: shear stress at the horizontal displacement h;
- σ_{n0}: initial normal stress acting on the joint surface;
- k_N: external/boundary normal stiffness acting on the joint;
- δ_v: dilation of the joint;
- A_j: joint surface area;
- ϕ_b: basic friction angle;
- i_0: initial asperity angle;
- i_h: inclination of the tangent to the dilation x horizontal displacement curve at any value of h.

The value of dilation presented in Eq. 1 is proposed to be expressed in the form of a Fourier series as:

$$\delta_v = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2\pi nh}{T}\right) + b_n \sin\left(\frac{2\pi nh}{T}\right)\right] \qquad (2)$$

where:
a_0, a_n, and b_n: Fourier coefficients;
n: number of harmonics;
h: horizontal displacement;
T: period considered for the Fourier series.

The values of the a_0, a_n, b_n e T are determined by interpolating the results obtained from laboratory tests performed using the direct shear apparatus. Eqs. 3, 4 and 5 present the expressions used to define the values of all Fourier coefficients to represent the function f(x) showed in Figure 1.

$$a_0 = \frac{1}{T}\int_a^b f(x)dx \tag{3}$$

$$a_n = \frac{1}{T}\int_a^b f(x)\cos\left(\frac{2\pi nx}{T}\right)dx \tag{4}$$

$$b_n = \frac{1}{T}\int_a^b f(x)\sin\left(\frac{2\pi nx}{T}\right)dx \tag{5}$$

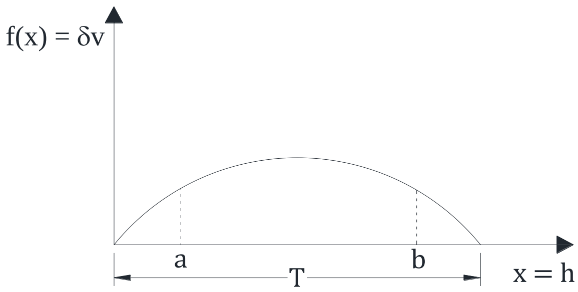

Figure 1. Typical Dilation Curve of a Saw-tooth Joint (modified from Indraratna and Haque, 2000).

Despite the good results provided by the analytical method presented in Equation 1, its application sometimes becomes difficult due the availability of the direct shear test results necessary to obtain the results of dilation to determine all Fourier coefficients.

2.2 Multilayer Perceptron

The multilayer perceptron is an important class of ANN used for engineering modeling. It is a feedforward multilayer neural network constituted by the input, hidden (one or more) and output layers. The input layer contains all nodes responsible for receiving the external signal. The function of the hidden layers is to increase the ANN capacity of extracting the most complex behaviour from the studied phenomenon. The output layer contains the artificial neurons which represent the response of the model to the external signal introduced in the input layer.

Figure 2 shows the schematic structure of an artificial neuron which represents the fundamental unit of processing data in a multilayer perceptron, with all elements according to the model proposed by McCulloch and Pitts (1943).

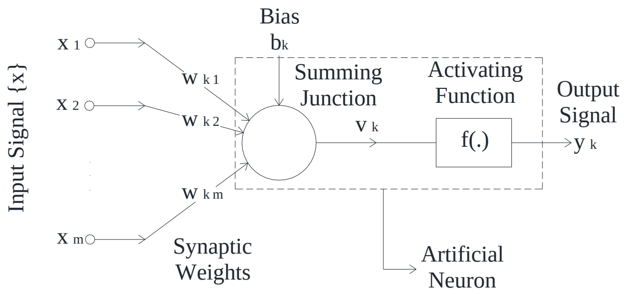

Figure 2. Representation of an Artificial Neuron (modified from Amancio, 2013).

The following expressions show the calculation of the signal y_k of the artificial neuron represented in Figure 2:

$$u_k = \sum w_{km} x_m = \{w\}^T\{x\} \tag{6}$$

$$v_k = u_k + b_k = \{w\}^T\{x\} + b_k \tag{7}$$

$$y_k = f(v_k) = f(\{w\}^T\{x\}) + b_k) \tag{8}$$

Where:
u_k: linear combiner output;
v_k: induced local field;
b_k: bias or threshold;
f(.): activation function;
{x}: input data vector;
{w}: synaptic weights of vector that connects the neuron k to the input data vector {x}.

According to Haykin (2001), the learning of an ANN consists of two steps: training and validation. In training, all synaptic weights and bias are adjusted based on the knowledge stored in an experimental data set. The validation consists of testing the ANN behaviour with data not used during the training.

The adjustment of the synaptic weights and bias in the multilayer perceptron is usually performed using the Error Backpropagation Algorithm developed by Rumelhart et al. (1986). In this algorithm, the signal of error obtained by comparing the outputs of the ANN to the targets existing in the training data set is used to change all parameters of the network.

It is important to mention that many authors have already presented studies regarding the use of ANN in Geotechnical Engineering using the Error Backpropagation Algorithm (Nejad et al., 2009, Dantas Neto et al. (2014, 2016); Amancio et al., 2014, etc.). The satisfactory results obtained by these authors has encouraged the application of the perceptrons in the development of the model to predict the shear behaviour of the clean joints of soft rocks as a simple alternative to the analytical methods previously mentioned.

3 MATERIAL AND METHODS

3.1 Definition of the Input Variables

The input variables considered in the development of the ANN model for predicting the shear behaviour of the soft clean rock joints are the same of those used by the analytical model presented by Indraratna and Haque (2000). The proposed ANN model intends to be a tool which allows the prediction of the shear strength of the clean joints at any horizontal displacement easily, without the need of the direct shear test results used to determine the Fourier coefficients from the interpolation of the dilatancy curves.

Eq. 9 shows the representation of the ANN model considered to predict the shear strength (τ_h) acting on the clean joint at any horizontal displacement and its input variables.

$$\tau_h = f(x_1, x_2, x_3, x_4, x_5) \tag{9}$$

where:

x_1 = k_N: boundary normal stiffness, in kN/mm;
x_2 = i_0: initial asperity angle, in degrees;
x_3 = a: asperity height, in mm;
x_4 = σ_{n0}: initial normal stress, in MPa;
x_5 = h: horizontal displacement, in mm.

3.2 Data Collection

This work has considered the results presented in Haque (1999), Indraratna et al. (1998, 1999) and Indraratna and Haque (2000) of direct shear tests conducted on synthetic idealised saw-tooth joints under CNL (k_N = 0 kN/mm) and CNS (k_N = 8.5 kN/mm) boundary conditions. Samples of 250 mm in length, 75 mm in width and 150 mm in height were tested with the large scale shear apparatus available at the University of Wollongong, in Australia.

Table 1 shows the values of the initial angle and height of asperities of the considered idealized saw-tooth joints. The synthetic material used to model the rock presents a basic friction angle (ϕ_b) of 37.5º and a compressive strength of 12 MPa.

Table 1. Geometric Characteristics of Idealized Saw-Tooth Clean Joints (Haque, 1999).

Joint Description	i_0 (degrees)	a (mm)
Type I	8.5	2.5
Type II	18.6	5.0
Type III	26.5	8.0

3.3 Training and Validation

The training and validation of the ANN model were performed using the software QNET2000, which uses the Error Backpropagation Algorithm to adjust the values of synaptic weights and bias. Equation 10 presents the sigmoid function used to compute the signal of all artificial neurons in QNET2000 (Amancio, 2013).

$$f(x) = \frac{1}{1+e^{-x}} \tag{10}$$

In order to use the Error Backpropagation Algorithm, it is necessary to normalize all data existing in the training and validation data set. In this work all input variables and targets were normalized between 0.15 and 0.85. The normalization of all variable is performed using Equation 11 and values presented in Table 2.

$$\frac{x_{nor} - 0.15}{0.85 - 0.15} = \frac{x - x_{min}}{x_{max} - x_{min}} \qquad (11)$$

Where:
x_{nor}: normalized value of the variable x;
x_{max}: maximum value of variable x;
x_{min}: minimum value of the variable x.

Table 2. Maximum and Minimum Values of Input for Normalization

	Input Variables				
	k_N (kN/mm)	i_0 (degree)	a (mm)	σ_{n0} (MPa)	h (mm)
Min	0	8.5	2.5	0.16	1.0
Max	8.5	26.5	8.0	2.43	19.0

To improve the convergence of the Backpropagation Algorithm during the training, the values of the learning rate (η) were considered varying automatically in the range of 0.001 to 0.15. The momentum constant (α) of 0.8 was adopted.

The criterion used to define the best ANN models take into account the one with the highest coefficient of correlation among all architecture tested in the validation data set.

4 RESULTS AND DISCUSSION

Figure 3 illustrates the architecture A:5–15–5–1 of the best ANN model obtained for predicting the shear strength of the idealized clean joints in soft rocks according to Eq. 9. The coefficients of correlation obtained after 2,000,000 iterations were 0.998 and 0.996 in the training and validation phases, respectively. In this situation, the best performance of the model was not obtained at iteration corresponding to the maximum values of the coefficient of correlation. The model with parameters obtained after 11 million iterations did not present a satisfactory capability of generalization which is a fundamental requirement during a ANN modelling.

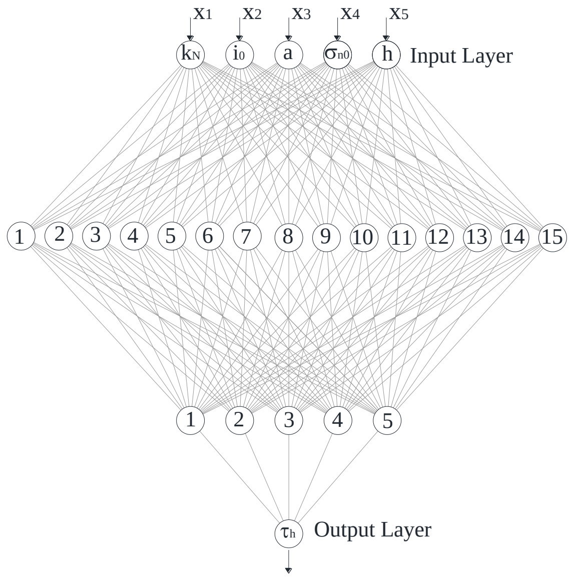

Figure 3. Architecture of the ANN model for shear strength prediction of clean joint in soft rocks.

Figure 4 shows the convergence of the Backpropagation Algorithm during training and validation phases. It is important to mention that the values of correlation for the model represented in Figure 4 are higher than many other predicting models previously developed in Geotechnical Engineering (Amancio, 2013; Dantas Neto et al., 2014, etc.). That indicates how powerful this technique can be in modeling the shear behaviour of the rock joints.

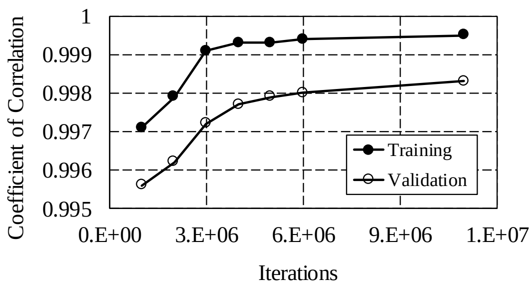

Figure 4. Variation of the coefficient of correlation during the training and validation.

Figure 5 presents the comparison between the outputs of the ANN model and the target values available in the training and validation data set. Here, it can be observed that the results obtained with the ANN model are fitting good all used experimental data.

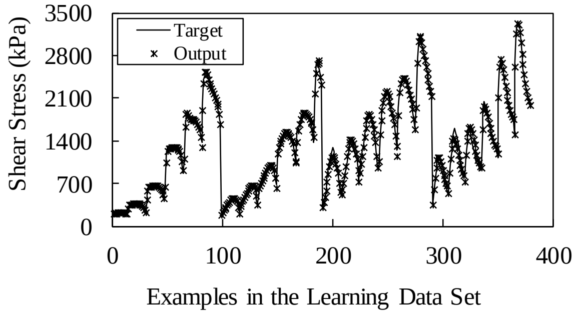

Figure 5. Comparison between targets and outputs in both training and validation data set.

Table 3 presents the node input interrogator provided by QNET2000. These results show that the characteristics of joint roughness (48%) and the initial normal stress (29%) are the inputs that most influence the response of the ANN model. Therefore, the ANN model is capturing one of the most important features of the behaviour of the clean joints, the influence of roughness on dilation which has a significant role in the development of the stresses during shearing (Haque, 1999; Indraratna and Haque, 2000, etc.).

Table 3. Node Input Interrogator

Input	k_N	i_0	a	σ_{n0}	h
Contribution (%)	9.5	28	20	29	13.5

Despite the importance of the boundary normal stiffness in the shear behaviour of clean joints, the node input interrogation obtained for this variable can be considered lower than expected. The reason for that may be the use of limited values of k_N, i.e. only two, representing the CNL (k_N = 0 kN/mm) and CNS conditions (k_N = 8.5 kN/mm) in the training and validation data sets.

According to Haykin (2001), the values of synaptic weights and bias represent all knowledge about a particular phenomenon after the ANN model training. Table 4, 5 and 6 present the values of all parameters which can be used in the Eqs. 6, 7 and 8 to predict the shear strength of clean joints in soft rocks at any horizontal displacement h, if the values of all other inputs are known.

Table 4. Synaptic Weights and Bias of Neurons of Hidden Layer 1.

Inputs		Input Layer					Bias
		x_1	x_2	x_3	x_4	x_5	
Hidden Layer 1	1	-1.3405	0.8234	-0.9751	-0.6212	0.8808	-0.26732
	2	3.2088	-3.1910	-0.3757	-8.0904	-0.7712	1.86613
	3	0.7013	0.6588	0.5532	1.1809	0.2635	0.20593
	4	-3.3389	1.9613	1.7554	5.5046	-2.5545	-0.17471
	5	-1.1131	1.4427	2.5379	-2.9314	0.1572	-0.28136
	6	-0.0796	2.8384	2.0355	-1.5220	0.8411	0.1196
	7	-1.6085	-0.6243	-0.1757	0.5389	3.1312	0.18287
	8	-2.4534	1.0787	2.6808	-0.5649	2.7823	-1.74139
	9	0.3320	0.3605	-0.2597	-1.5452	8.1101	-2.11211
	10	-0.1635	-1.7771	0.8915	0.2713	-5.8908	1.27046
	11	-0.1724	1.1956	-2.2197	7.9246	-0.2729	1.3107
	12	1.4295	1.2646	-0.5873	1.4278	5.2835	-0.71049
	13	1.4837	0.9730	1.1953	0.6994	-0.8441	-0.26883
	14	-1.4756	0.0695	-1.0070	-0.1238	0.3425	0.16808
	15	-2.8184	1.9695	2.4226	-0.7218	0.5123	-2.70457

Table 5. Synaptic Weights and Bias of Neurons of the Hidden Layer 2.

Inputs		Hidden Layer 2				
		1	2	3	4	5
Hidden Layer 1	1	-1.46342	-0.83985	0.70942	-0.19428	-0.78811
	2	-5.11166	0.74631	-2.63848	-1.2129	-1.56235
	3	-0.13639	0.47373	-0.35952	-0.38704	1.12842
	4	-1.61559	-1.17322	-1.00389	0.61235	0.10506
	5	2.55386	-2.87818	-1.3177	4.93206	-3.33347
	6	1.59998	-1.97876	0.40559	-1.26563	-0.82604
	7	-2.21396	0.34107	1.7562	1.07667	0.0156
	8	-0.47447	2.57574	2.83273	-0.03325	-1.89269
	9	2.21814	8.64486	2.02429	-1.33174	-0.32552
	10	7.70225	2.38435	0.56384	-6.89385	-1.65138
	11	-0.02797	-0.86443	-3.00123	3.33445	1.59874
	12	-4.65859	-0.37504	0.74802	-0.82344	-0.22065
	13	1.70571	0.65702	0.61521	-0.68997	1.34187
	14	-1.45344	-0.56661	0.05034	-0.71538	-0.82981
	15	-0.22163	1.89924	-0.02711	-2.80901	-1.2051
Bias		-0.4153	0.59148	-0.41224	-0.74457	0.51681

Table 6. Synaptic Weights and Bias of Neurons of the Output Layer.

Inputs	Hidden Layer 2					Bias
	1	2	3	4	5	
Output	-8.9899	-5.9353	-4.8542	8.2909	3.7535	3.02358

In order to evaluate the ANN model, let's consider the shear behaviour of Type II joint (i_0 = 18.5 and a = 5 mm) considering a CNS boundary condition (k_N = 8.5 kN/mm) and an initial normal stress of 0.3 MPa. Figure 6 compares the laboratory test results to those obtained with the proposed ANN model and by applying the analytical model presented in Equation 1.

It can be observed that the ANN model results fit the laboratory data better than the analytical model presented by Indraratna and Haque (2000). Besides, some features of the shear behaviour of clean joints are adequately captured by the ANN model, e.g., the decrease of shear stresses after peak due the degradation of asperities referred by Indraratna et al. (1999).

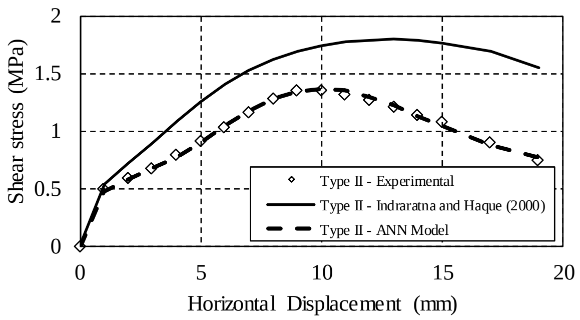

Figure 6. Shear Behaviour of Type II Joint (σ_{n0} = 0.3 MPa)

5 CONCLUSIONS

This paper presents an ANN model to predict the shear strength of idealised saw-tooth clean joints in soft rocks. The input variables of the model are the normal stiffness (k_N), and the initial normal stress (σ_{n0}) acting on the joint surface; the both initial angle (i_0) and height (a) representing the roughness; and the horizontal displacement (h). The model presents an architecture A:5-15-5-1, where each one of these values corresponds to the number of artificial neurons in its layers.

The coefficients of correlation in the training and validation phases were 0.998 and 0.996, respectively, after 2,000,000 iterations. It is important to note that these values are higher than many other presented by other researchers who used the ANN technique in Geotechnical Engineering. That indicates how powerful the artificial neural networks can be in the development of models in rock mechanics, especially those related to the rock joint behaviour.

The main advantages of the proposed ANN model are its simplicity and accuracy. Its uses allows the prediction of the shear behaviour of clean rock joints by using a simple spreadsheet with only parameters which represent the initial roughness of the joint and the boundary conditions (CNL or CNS), rather than several model parameters. It was also observed that its results better fitting to the experimental data when compared to the results of some existing analytical models.

Despite the satisfactory behaviour, the use of the proposed ANN model is still limited, once it was developed considering a limited data obtained from laboratory tests conducted on synthetic clean joints representing the soft rocks. However, in order to fully represent the shear behaviour of all clean joint a global model considering the uniaxial strength of the intact rock should be developed. Nevertheless, the results indicate the ANN presents itself as a powerful technique in the modeling of rock joints.

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REFERENCES

Amâncio, L.B. (2013) *Previsão de recalques em fundações profundas utilizando redes neurais artificiais do tipo perceptron.* Dissertação de Mestrado, Programa de Pós-Graduação em Engenharia Civil, Departamento de Engenharia Hidráulica e Ambiental, Universidade Federal do Ceará, 300p.

Amâncio, L.B.; Dantas Neto, S.A.; and Anjos, G.J.M. (2014). Modelo de Previsão de Recalques em Estacas Hélice Contínua Utilizando Redes Neurais Artificiais.

COBRAMSEG 2016, Goiânia, Brasil, 9p., CD ROM.
Araújo, C.B.C (2015) *Aplicação das redes neurais artificiais do tipo perceptron na estimativa de recalques em estacas.* Dissertação de Mestrado, Programa de Pós-Graduação em Engenharia Civil, Departamento de Engenharia Hidráulica e Ambiental, Universidade Federal do Ceará, 227p.
Barton, N. (1973) Review of a New Strength Criterion for Soft Rocks, *Engineering Geology*, 7, pp. 287-332.
Dantas Neto, S.A.; Silveira, M.V.; Amâncio, L.B.; and Anjos, G.M. (2014) Pile Settlement Modeling with Multilayer Perceptrons, *Electronic Journal of Geotechnical Engineering*, v. 19, pp. 4517-4528. http://www.ejge.com/2014/Ppr2014.426ma.pdf.
Dantas Neto, S.A.; Silveira, M.V.; Anjos, G.M.; and Moura, A.S. (2016) Settlement Prediction Model for Continuous Flight Auger, Steel and Bored Piles Using Artificial Neural Networks, *Revista Geotecnia*, Sociedade Portuguesa de Geotecnia, v. 136, 21p.
Haque, A. (1999) *Shear Behaviour of Soft Rock Joints Under Constant Normal Stiffness*, Doctor of Philosophy Thesis, Department of Civil and Mining Engineering, University of Wollongong, 271p., http://ro.uow.edu.au/theses/1266.
Haykin, S. (2001). Neural Networks – A Comprehensive Foudation, *Prentice Hall International Inc.*, 2nd Edition, USA, 842p.
Indraratna, B.; Haque, A.; and Aziz, A. (1998) Laboratory Modelling of Shear Behaviour of Soft Joints under Constant Normal Stiffness Condition, *Journal of Geotechnical and Geology Engineering*, 16, pp.17-44.
Indraratna, B.; Haque, A.; and Aziz, A. (1999) Shear Behaviour of Idealised Infilled Joints under Constant Normal Stiffness, *Géotechnique*, 49(3), pp.331-355.
Indraratna, B.; Oliveira, D.A.F.; and Brown, E.T. (2010) A Shear-Displacement Criterion of Soil-Infilled Rock Discontinuities, *Géotechnique*, 60(8), pp.623-633.
Indraratna, B. and Haque, A. (2000) Shear behaviour of rock joint, *Rotterdam: Balkema*, 164p.
McCulloch, W.S. and Pitts, W. (1943) A Logical Calculus of the Ideas Immanent in Nervous Activity, *Bulletin of Mathematical Biophysics*, Vol. 5, pp. 115-133.
Nejad, F.P.; Jaksa, M.B.; McCabe, B.A. (2009) Prediction of pile settlement using artificial neural networks based on standard penetration test data. *Computers and Geotechnics*, Vol. l, pp. 1125-1133.
Patton, F.D. (1966) Multiple Modes of Shear Failure in Rocks, *Proceedings of 1st Congress of ISRM*, Lisbon, Portugal, pp. 509-513.
Rumelhart, D. E., Hinton, G. E., and Williams, R. J. (1986) Learning representations by back-propagating errors, *Nature*, 323, 533-536.
Seidel, J.P. and Haberfield, C.M. (1995) The application of energy principles to the determination of the sliding resistance of rock joints. *Rock Mechanics & Rock Engineering*, 28(4), pp. 211-226.