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FUZZY ROBUST DESIGN OF MECHANICAL SYSTEMS

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Abstract. *This contribution aims at aid the development of robust optimization and decision making techniques of mechanical system when these are subject to uncertain information. In this case a novel approach is proposed regarding a fuzzy logic optimization strategy. The uncertain information of the considered mechanical systems is treated as fuzzy variables. Consequently, the associated optimization problem is described as a fuzzy function that maps the fuzzy inputs. The proposed technique is applied to the design of a robust welded cantilever beam. This numerical study illustrates the versatility and convenience of the proposed fuzzy logic optimization strategy.*

Keywords*: robust optimization, fuzzy logic, uncertainty analysis.*

1. INTRODUTION

Usually, in the design/operation of mechanical systems a deterministic model is formulated so that the dynamic behavior of the process can be evaluated. These deterministic models considerer parameters known and well defined. However, whether by lack of knowledge or due to design/operational fluctuations (e.g. manufacturing errors, temperature variations) or due to damage, uncertainties are inherent aspects of mechanical systems. Thus, the deterministic approach can be regarded as outdated and should be replaced by one that takes into account such uncertainties.

Among the innumerous approaches for modeling uncertain quantities in mechanical systems, the stochastic approach and the fuzzy logic are among the most widely used. The stochastic methodology based on the probability theory has a longer history of applications in mechanical systems. On the other hand, the intuitive fuzzy logic approach based on fuzzy sets (Zadeh, 1965) and the possibility theory (Zadeh, 1968) has recently gained more attention and relevant applications can be found in the literature (Möller and Beer 2004; Lara-Molina et al. 2014; Cavalini et al. 2015). Both methodologies are well suited for modeling uncertain or inaccurate quantities; however, due to its mathematical simplicity the fuzzy logic seems to be more appropriate for an optimization procedure.

Robustness is a widely used term with different meanings, but in general, it is understood as a value or solution that may not be the best for one particular set but is the better for several uncertain scenarios. In a robust optimization procedure, the goal is to find a solution that is the least sensitive as possible to the model uncertainties. The pioneer of robust optimization is G. Taguchi (Taguchi, 1984),who proposed the insertion of noise factors in the cost function thus defining a mean square deviation that should be maximized regarding the design variables.

Most of the known robust optimization approaches are based on stochastic techniques, mainly based on the statistical moments of the problem (Paenke et al. 2006; Beyer and Sendhoff, 2007; Borges et al. 2010). These techniques usually lead to large combinatorial problems, especially if they rely on sampling methodologies such as the Monte Carlo Simulation, which can impose serious disadvantage to the optimization task. Therefore, a simpler mathematical scheme as the fuzzy logic may prove to be more advantageous for robust optimization.

In this sense, the present work aims at contributing to the development of fuzzy robust optimization techniques, and for such, a new robust optimization concept is proposed. The uncertainties are treated as fuzzy variables so that the optimization model becomes a fuzzy function that maps the fuzzy inputs. Therefore, a fuzzy evaluation of the optimization problem can be conceived through the evaluation of the pessimistic and optimistic values of the fuzzy sets (i.e., the uncertain information).This fuzzy evaluation generates a fuzzy response that serve as robustness metric for the proposed design. This new methodology was applied in the welded cantilever beam design, considering uncertainties that affect the design variables.

This paper is organized as follows. First the key concepts of fuzzy logic are discussed, and then the fuzzy robust optimization technique is presented and evaluated in the design problem considered. Finally, a few remarks are made and further research work is proposed.

2. FUZZY LOGIC

Introduced by Zadeh (1965), the fuzzy set theory formulated the graded membership notion represented by the membership function. In a classical sense, a set is a collection of elements that belong to a definition. Any element either belong or not to the set. In a fuzzy set this *Boolean* notion of membership is replaced by a graded notion, i.e., an element can belong, not belong or partially belong to the set.

Dubois and Prade (1997) presented the three basic interpretations of the membership function of a fuzzy set: the degree of similarity, in which the membership function quantifies how similar an element of the set is compared to a prototype element; the degree of preference, where the membership function quantifies the preference or the feasibility of an element of the set; or a degree of uncertainty, in which the membership function is the degree of possibility that a determined parameter u has the value of an specific element x of the set.

Based on the concept of degree of uncertainty, a fuzzy set can be understood as the union of the pessimist (lower bounds) and optimist (upper bounds) values of an uncertain information considering each level of possibility (i.e., α-levels of the membership function).It is important to point out that the membership function in this view, is a possibility distribution, not a probability distribution as considered when the stochastic theory is applied. Possibility is the measure of whether an event can happen, while probability is a measure of whether an event will happen. Therefore, the possibility distribution of a given parameter u quantifies the possible values that this parameter can assume; while the probability distribution quantifies the chances that the parameter u has to assume a certain value x.

In the fuzzy set theory, the uncertainties are described as fuzzy inputs and the mathematical model represented as a fuzzy function that maps these inputs. The so-called α-level optimization (Möller and Beer 2004) is a method for the mapping of fuzzy variables. In this case, the fuzzy inputs are discretized by means of α-cuts creating a crisp subspace that is the search space for an optimization problem. The optimization process is carried out in order to find the minimum and maximum values of the model output for each α-level (i.e., lower and upper limits of the correspondent α-level, respectively).The α-level optimization is an efficient methodology for mapping fuzzy inputs. However, the effectiveness of the method is highly related to the performance of the selected optimization technique.

3. FUZZY ROBUST OPTIMIZATION

In general, robust optimization is performed relying on the stochastic theory, where a set of samples of the design variables (i.e., a population) is generated and the robustness is evaluated by means of the statistical analysis of this set. However, this approach usually creates a large combinatorial problem imposing high computational costs.

Fuzzy uncertainty analysis through the α-level optimization requires fewer evaluations of the cost function, presenting lower computational cost as compared with the stochastic approach. Therefore, the development of fuzzy metrics to evaluate the robustness of a given design leads to an efficient tool for robust optimization.

In the case of a robust optimization problem, for a given design (a configuration of the design variables, $\boldsymbol{x}$) a fuzzy uncertainty analysis is performed generating a fuzzy output. This fuzzy output is the union of the pessimist and optimist responses, in which the α-level boundaries represent the lower and the upper bounds of all possible optima. It can be inferred that a robust solution will have the smallest difference between these bounds (smallest sensibility to the uncertainties influence) and therefore a smallest area of its fuzzy output.

Thus, a fuzzy response minimization scheme can be formulated aiming at finding the robust optimum,which is represented by the samallest area of the fuzzy output (see Fig. 1).

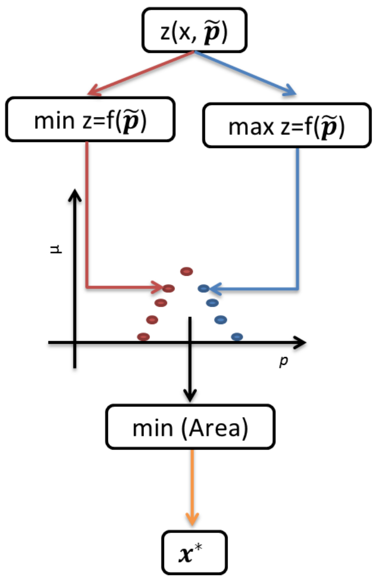

Figure 1. Flowchart for the fuzzy response minimization procedure

4. FUZZY ROBUST OPTIMIZATION

In order to ascertain the efficiency of the proposed methodology, the conceived fuzzy robust optimization procedure was applied in the robust design of a welded cantilever beam.

The formulation and discussion of the welded beam design problem can be found in Rao (1996) and in Lobato and Steffen Jr. (2014). Mathematically, the problem can be formulated as a cost function represented by Eq. (1) as follow:

$$\min f(\boldsymbol{X}) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2) \tag{1}$$

subject to the constraints set presented in Eq. (2-10).

$$g_1(\boldsymbol{X}) = \tau(\boldsymbol{X}) - \tau_{max} \leq 0 \tag{2}$$

$$g_2(\boldsymbol{X}) = \sigma(\boldsymbol{X}) - \sigma_{max} \leq 0 \tag{3}$$

$$g_3(\boldsymbol{X}) = x_1 - x_4 \leq 0 \tag{4}$$

$$g_4(\boldsymbol{X}) = 0.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5 \leq 0 \tag{5}$$

$$g_5(\boldsymbol{X}) = 0.125 - x_1 \leq 0 \tag{6}$$

$$g_6(\boldsymbol{X}) = \delta(\boldsymbol{X}) - \delta_{max} \leq 0 \tag{7}$$

$$g_7(\boldsymbol{X}) = P - P_c(\boldsymbol{X}) \leq 0 \tag{8}$$

$$g_8(\boldsymbol{X}) \text{ to } g_{11}(\boldsymbol{X}): 0.1 \leq x_i \leq 2, \quad i = 1,4 \tag{9}$$

$$g_{12}(\boldsymbol{X}) \text{ to } g_{15}(\boldsymbol{X}): 0.1 \leq x_i \leq 10, \quad i = 2,3 \tag{10}$$

where $\tau(\boldsymbol{X}) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}$, $\tau' = \frac{P}{\sqrt{2}x_1x_2}$, $\tau'' = \frac{MR}{J}$, $M = P\left(L + \frac{x_2}{2}\right)$, $R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1+x_3}{2}\right)^2}$, $J = 2\left\{\frac{x_1x_2}{\sqrt{2}}\left[\frac{x_2^2}{12} + \left(\frac{x_1+x_3}{2}\right)^2\right]\right\}$, $\sigma(\boldsymbol{X}) = \frac{6PL}{x_4x_3^2}$, $\delta(\boldsymbol{X}) = \frac{4PL^3}{Ex_3^3x_4}$, $P_c(\boldsymbol{X}) = \frac{4.013\sqrt{EG(x_3^2x_4^6/36)}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$, $P = 6000$ lb, $L = 14$ in., $E = 30\text{x}10^6$psi, $G = 12\text{x}10^6$psi, $\tau_{max} = 13600$ psi, $\sigma_{max} = 30000$ psi, and $\delta_{max} = 0.25$ in.

Regarding the uncertainty scenario, an uncertainty level of $\pm 10\%$ affecting the design variables was considered. Therefore, for each evaluation of a possible design (x), a lower bound ($\underline{x} = 0.9x$) and an upper bound ($\overline{x} = 1.1x$) were generated, so the sensibility of the proposed design could be assessed. Table 1 presents the results obtained following the deterministic optimization approach and the results obtained through the proposed methodology.

Table 1. Results comparison for the robust design of a welded cantilever beam.

	Approach	
	Deterministic	Fuzzy Response Minimization
X_1	0.2444	0.3460
X_2	3.0402	2.5036
X_3	8.2913	9.2128
X_4	0.2444	0.2715
Cost Function (Nom.)	1.8617	2.3171
Cost Function (Pes.)	$2.95\text{x}10^8$	2.9828
Cost Function (Opt.)	2.3128	2.8936

Figure 2 illustrates the fuzzy response of each design, the robustness metric used to evaluate the sensibility of the proposed the design.

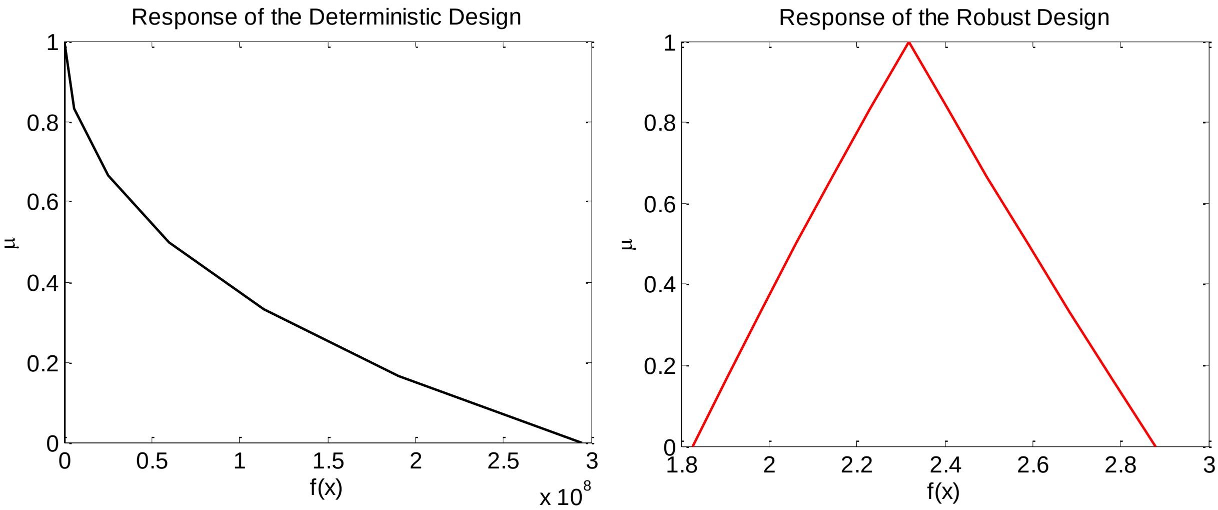

Figure 2. Fuzzy response of both proposed designs.

The deterministic design presented a variation in order of 10^8 (difference between the smallest response (1.8617) and the largest response ($2.95x10^8$)), while the design obtained by the proposed fuzzy response minimization procedure showed a variation around 0.7 (in order of 10^{-1}). This results demonstrates that the fuzzy response minimization design found a very robust configuration (a design insensitive to the system fluctuations), whereas the deterministic design obtained a design extremely sensitive to the system fluctuations.

5. FINAL REMARKS

This paper aimed at contributing to the development of simpler robust optimization methodologies. To do so, a new robust optimization procedure, namely fuzzy response minimization was conceived and tested in the robust design of an engineering problem from the literature, i.e., the welded cantilever beam design.

Considering a robust optimum as the value that presents better results in as many different scenarios as possible, the results show that the presented procedure is very efficient at finding robust optima with low computational cost associated.

Fuzzy robust optimization techniques are very promising for robust optimization due to its simple mathematical formulation, which leads to fast computation. However, depending on the problem features it will still imposes a large number of evaluations of the cost function; consequently, a new goal in the development of fuzzy robust optimization procedures should be the reduction of cost function evaluations, which are very time consuming.

6. REFERENCES

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8. RESPONSABILITY FOR THE INFORMATION

The authors are solely responsible for the information included in this paper.