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IMPLEMENTATION AND VERIFICATION FOR FLOWS WITH HEAT TRANSFER ON THE IMERSPEC METHODOLOGIES

Denise Kinoshita, denikino@doutorado.ufu.br
Felipe P. Mariano, fpmariano@eeec.ufg.br
Leonardo de Queiroz Moreira, leonardo_queiroz_moreira@yahoo.com.br
Aristeu Silveira-Neto, aristeus@mecanica.ufu.br

Resumo: *A new numerical methodology combining Fourier pseudo-spectral and immersed boundary methods - IMERSPEC – has been developed for fluid flow problems modeled using the Navier-Stokes, mass and energy equations, for incompressible flows. The numerical algorithm consists in a Fourier pseudo-spectral methodology using the collocation method, where every kind of thermal boundary condition can be modeled using an immersed boundary method (Multi Direct Forcing Method). The IMERSPEC methodology was presented by Mariano et al. (2010). A new model for boundaries conditions of first, second and third were developed, implemented and verified, using synthesized solutions for Navier-Stokes and energy equation. Preliminary results are presented in the present paper.*

Palavras-chave: *boundary condition, Fourier pseudo-spectral method, immersed boundary method, energy equation*

1. INTRODUCTION

In the last two decades a lot of effort has been spent by the fluid dynamic scientific community to address two crucial but conflicting key issues in the science of computational fluid dynamics (CFD). These are associated with the need to model increasingly complex boundary conditions in one hand, and, at the same time, requiring high accuracy Ferziger and Peric (1996). The great majority of engineering and geophysical fluid flow problems are characterized by very complex geometries that arise mainly from the irregular domain frontiers. This is often associated with the presence of moving and deformable geometries.

The methods used to solve flow problems over complex and moving geometries require moving and deforming meshes or even remaking the mesh after a given number of time steps. These procedures cause the methodologies to have a high computational cost and have a more complex implementation Nakahashi et al. (2003).

The immersed boundary methodology has been developed since 1970 by several researchers Peskin (1972), Goldstein et al. (1993), Lima e Silva et al. (2003), Mittal and Iaccarino (2005). This method reach a portion of the requirements described above; specifically, it can handle complex and moving geometries, using Cartesian mesh.

This methodology was applied by Mariano et al. (2010) to solve the flow over a driven cavity and over a backward facing step. The cavity flow was also simulated by Botella and Peyret (1998), using a Chebyshev collocation method. This work was developed and applied for isothermal flows. Flows with heat transfer effects was simulated using immersed boundary methodology for boundary conditions of first, second and third type by Jungwoo and Haecheon (2004), Jeong et al. (2010), Wang et al. (2009), Pan (2006) and Young et al. (2009).

In special Wang et al. (2009) used the multi-direct forcing scheme to ensure the temperature Dirichlet boundary conditions at the immersed boundary and the finite difference scheme was applied to solve heat transfer problems, the results showed second-order spatial accuracy when applied to solve the Taylor-Green vortices.

An accurate Navier-Stokes equations solver allows to obtain a better solution of a given flow problem in a given grid, as compared with a less accurate numerical method. In terms of high order accuracy, the family of the so called spectral methods Canuto (2007) has been virtually unsurpassed. Spectral methods are characterized by exponential convergence to the exact solution with increasing grid size.

Within the family of spectral methods, the classical Fourier pseudo-spectral collocation method is probably the most impressive, due to its extremely high accuracy and its low computational cost. Moreover, the pressure terms in the Navier-Stokes equations can be lumped together with the non-linear term, for incompressible flows. So, the Fourier pseudo-spectral collocation method does not requires the solution of a pressure Poisson equation. It results in an unusually fast time stepping procedure. These classical methods, however, are in general not applicable for over complex geometries. The Fourier collocation method, in particular, can only be used in flows with periodic boundary conditions.

The goal of the present work is to show a new methodology for incompressible flows with heat transfer and with three kinds of boundary conditions. It has been looked to combine the accuracy and low computational cost of the classical Fourier pseudo-spectral method Canuto (2007) with flexibility in handling complex geometries allowed by the immersed boundary methods. Was introduced, specifically, the *IMERSPEC* method Mariano et al. (2010*)*, which combines a classical Fourier pseudo-spectral method, with an immersed boundary method. The main goal is to take into account the effects arising from the presence of complex boundaries. Also, any spatial derivative is computed with spectral accuracy. In the present paper, a new model for boundaries conditions of first (Dirichlet), second (Newman) and third (Robin) types were developed, implemented and verified, using synthesized solutions for the Navier-Stokes and energy equations.

2. MATHEMATICAL MODELING

The presented methodology is based on the merging process of the immersed boundary method with a classical Fourier pseudo-spectral method. The equations in physical space, the pseudo-spectral and immersed boundary methods are described in Mariano et al. (2010). In this paper, only the models for several kinds of boundary conditions will be presented.

The mathematical model for incompressible flows of Newtonian fluids, with heat transfer is established with the energy, the Navier-Stokes and mass equations. These equations present source terms that model the boundary conditions for momentum and heat transfer. These equations are presented below:

$$\vec{\nabla} \cdot \vec{V} = 0, \tag{1}$$

$$\rho \frac{\partial \vec{V}}{\partial t} = \overrightarrow{RHS} + \vec{f}, \tag{2}$$

$$\rho C_p \frac{\partial T}{\partial t} = RHS_T + f_T, \tag{3}$$

$$\overrightarrow{RHS} = -\rho \vec{\nabla} \cdot \left(\vec{V}\vec{V}\right) - \vec{\nabla} p + \mu \nabla^2 \vec{V} + \vec{f}_{ss}, \tag{4}$$

$$RHS_T = -\vec{\nabla} \cdot \left(\vec{V} T\right) + \kappa \nabla^2 T + f_{Tss}, \tag{5}$$

where $\vec{f}(\vec{x},t)$ is the source term for Navier-Stokes equations and $f_T(\vec{x},t)$ is the term source for the energy equation respectively. The terms $\vec{f}_{ss}(\vec{x},t)$ and $f_{Tss}(\vec{x},t)$ are the source term related to the synthetized solutions for Navier-Stokes and energy equations. These source terms model the boundary conditions as well as others kind of physical effects.

The main goal of the present work is to present a new model for the IMERSPEC methodology Mariano et al. (2010) extended for flows with heat transfer effects. In the following topic the models for $f_T(\vec{x},t)$ source terms are presented. Particularly, a model for the energy source term is proposed for the boundary conditions of first, second and third types.

2.1. Boundary conditions for the energy equation

Three types of boundary conditions will be proposed.

2.2.1 Dirichlet boundary condition (first type)

For this type of boundary condition the temperature $T(\vec{x},t)$ is given over the immersed boundary, which gives the reference temperature, defined as:

$$T_{ref}\left(\vec{X},t\right) \equiv T_{\Gamma}\left(\vec{X},t\right), \tag{6}$$

where $T_{ref}\left(\vec{X},t\right)$ is used in order to calculate the forcing term $f_T\left(\vec{X},t\right)$. In the present paper, this forcing term is calculated using the direct forcing, with a procedure similar to that used for the velocity. So, if we discretize Eq. (3) using the Euler time discretization method, as a guise of demonstration, we obtain:

$$\rho C_p \frac{T^{n+1}(\vec{x}) - T^n(\vec{x})}{\Delta t} = RHS_T^n(\vec{x}) + f_T^{n+1}(\vec{x}). \tag{7}$$

Adding and subtracting $T^{*n+1}(\vec{x})$ (guess of the temperature) on the left hand side of Eq. (7), it gives:

$$\rho C_p \frac{T^{n+1}(\vec{x}) - T^n(\vec{x})}{\Delta t} + \rho C_p \frac{T^{*n+1}(\vec{x}) - T^{*n+1}(\vec{x})}{\Delta t} = RHS^n(\vec{x}) + f_T^{n+1}(\vec{x}). \tag{8}$$

This equation can be decomposed in the following two equations:

$$\rho C_p \frac{T^{*n+1}(\vec{x}) - T^n(\vec{x})}{\Delta t} = RHS_T^n(\vec{x}), \tag{9}$$

$$f_T^{n+1}(\vec{x}) = \rho C_p \frac{T^{n+1}(\vec{x}) - T^{*n+1}(\vec{x})}{\Delta t}. \tag{10}$$

Equation (10) can be rewritten for a material particle placed at the immersed boundary:

$$F_T^{n+1}(\vec{X}) = \rho C_p \frac{T^{n+1}(\vec{X}) - T^{*n+1}(\vec{X})}{\Delta t}, \tag{11}$$

where $T^{n+1}(\vec{X}) = T_{ref}^{n+1}(\vec{X})$ is given by the physical characteristic of each problem. On the other hand $T^{*n+1}(\vec{X},t)$ is obtained by the interpolation of $T^{*n+1}(\vec{x})$, which is given by the solution of Eq. (9). This interpolation is given by the following equation:

$$T^{*n+1}(\vec{X}) = \int_\Omega T^{*n+1}(\vec{x})\delta(\vec{x} - \vec{X})d\vec{x}. \tag{12}$$

where $\vec{x} \in \Omega$ and $\vec{X} \in \Gamma$. Known $F_T^{n+1}(\vec{X})$, given by Eq. (11), it can be distributed using the following equation:

$$f_T(\vec{x}) = \int_\Gamma F_T(\vec{X})\delta(\vec{x} - \vec{X})d\vec{X} = \int_\Gamma \rho C_p \frac{T^{n+1}(\vec{x}) - T^{*n+1}(\vec{x})}{\Delta t} d\vec{X}. \tag{13}$$

2.1.2 Newman boundary condition (second type)

For this type of boundary condition, the heat flux is given, or equivalently, the temperature gradient is known.

$$\vec{q}''(\vec{X},t) = -k_f \vec{\nabla} T(\vec{X},t). \tag{14}$$

We are looking to calculate $f_T^{n+1}(\vec{x})$ using the boundary condition given by Eq. (14). Applying the gradient operator to Eq. (10), it gives:

$$\vec{\nabla} f_T^{n+1}(\vec{x}) = \frac{\rho C_p}{\Delta t}\left(\vec{\nabla} T^{n+1}(\vec{x}) - \vec{\nabla} T^{*n+1}(\vec{x})\right). \tag{15}$$

This equation can be applied to a material particle that is placed beside an immersed boundary Γ:

$$\vec{\nabla} F_T^{n+1}(\vec{X}) = \frac{\rho C_p}{\Delta t}\left(\vec{\nabla} T^{n+1}(\vec{X}) - \vec{\nabla} T^{*n+1}(\vec{X})\right) = -\frac{\rho C_p}{\Delta t}\left(\frac{\vec{q}''^{n+1}(\vec{X})}{k_f} + \vec{\nabla} T^{*n+1}(\vec{X})\right). \tag{16}$$

The quantity $\frac{\vec{q}''^{n+1}(\vec{X})}{k_f}$ is given by the physical nature of the problem. On the other hand, $\vec{\nabla} T^{*n+1}(\vec{X})$ is obtained by the interpolation of $\vec{\nabla} T^{*n+1}(\vec{x},t)$ which is calculated after the solution of Eq. (9):

$$\vec{\nabla} T^{*n+1}(\vec{X}) = \int_\Omega \vec{\nabla} T^{*n+1}(\vec{x})\delta(\vec{x} - \vec{X})d\vec{x}. \tag{17}$$

Eq. (16) can be rewritten for a material particle placed at $\vec{x} \in \Omega_{PhD}$, given, approximately, by:

$$\vec{\nabla} f_T^{n+1}(\vec{x}) = \int_\Gamma \vec{\nabla} F_T^{n+1}(\vec{X})\delta(\vec{x} - \vec{X})d\vec{X} = \int_\Gamma \frac{\rho C_p}{\Delta t}\left(\frac{\vec{q}''^{n+1}(\vec{X})}{k_f} + \vec{\nabla} T^{*n+1}(\vec{X})\right)\delta(\vec{x} - \vec{X})d\vec{X}. \tag{18}$$

Applying the divergent operator to Eq. (18), the following equation is obtained:

$$\nabla^2 f_T^{n+1}(\vec{x}) = \vec{\nabla} \cdot \underbrace{\int_\Gamma \frac{\rho C_p}{\Delta t} \begin{pmatrix} \dfrac{\vec{q}\,''^{n+1}(\vec{X})}{k_f} + \\ \vec{\nabla} T^{*n+1}(\vec{X}) \end{pmatrix} \delta(\vec{x} - \vec{X}) d\vec{X}}_{\vec{I}^{n+1}(\vec{x})} = \vec{\nabla} \cdot \vec{I}^{n+1}(\vec{x}). \tag{19}$$

There are many methods to solve Eq. (19). Particularly, it is possible to solve this equation using the Fourier spectral method. It is possible even for non-periodical problems, as shown by Mariano et al. (2010). Transforming this equation to the Fourier space, it gives:

$$\hat{f}_T^{n+1}(\vec{k},t) = -\frac{1}{k^2} TF\left\{\vec{\nabla} \cdot \vec{I}^{n+1}(\vec{x},t)\right\}. \tag{20}$$

So, from Eq. (20), $\hat{f}_T^{n+1}(\vec{k})$ or $f_T^{n+1}(\vec{x})$, can be obtained and used to solve Eq. (3).

2.1.3. Robin boundary condition (third type)

This is a hybrid boundary condition, which takes into account at the same time the temperature and its gradient at the immersed boundary. A differential equation can be obtained making an energy balance over the boundary. So, the heat flux given by diffusion in side of the boundary must be equals to the heat flux given by convection on the other side of the boundary. It gives the following equation:

$$-k_f \vec{\nabla} T(\vec{X},t) = h_f \left(T(\vec{X},t) - T_\infty\right)\vec{n}(\vec{X}) dA(\vec{X}), \tag{21}$$

which can be rewritten as follow:

$$\vec{\nabla} T(\vec{X},t) + \lambda T(\vec{X},t)\vec{n}(\vec{X}) = \lambda T_\infty \vec{n}(\vec{X}), \tag{22}$$

where $\lambda = \frac{h_f}{k_f} dA$. From Eq. (10) we have:

$$\lambda f_T^{n+1}(\vec{x}) = \frac{\lambda \rho C_p}{\Delta t}\left(T^{n+1}(\vec{x}) - T^{*n+1}(\vec{x})\right) \tag{23}$$

and

$$\vec{\nabla} f_T^{n+1}(\vec{x}) = \frac{\rho C_p}{\Delta t}\left(\vec{\nabla} T^{n+1}(\vec{x}) - \vec{\nabla} T^{*n+1}(\vec{x})\right). \tag{24}$$

Equation (23) can be rewritten for a material particle placed at an immersed boundary. The resultant equation is still multiplied by the normal vector $\vec{n}$. This gives the following equation:

$$\lambda F_T^{n+1}(\vec{X},t)\vec{n} = \frac{\lambda \rho C_p}{\Delta t}\left(T^{n+1}(\vec{X},t) - T^{*n+1}(\vec{X},t)\right)\vec{n}(\vec{X},t). \tag{25}$$

Similarly, Eq. (24) is rewritten at the immersed boundary, resulting:

$$\vec{\nabla} F_T^{n+1}(\vec{X}) = \frac{\rho C_p}{\Delta t}\left(\vec{\nabla} T^{n+1}(\vec{X}) - \vec{\nabla} T^{*n+1}(\vec{X})\right). \tag{26}$$

Adding Eqs. (25) and (26), results:

$$\vec{\nabla} F_T^{n+1}(\vec{X}) + \lambda F_T^{n}(\vec{X})\vec{n} = \frac{\rho C_p}{\Delta t}\begin{bmatrix} \left(\vec{\nabla} T^{n+1}(\vec{X}) - \vec{\nabla} T^{*n+1}(\vec{X})\right) + \\ \lambda\left(T^{n+1}(\vec{X}) - T^{*n+1}(\vec{X})\right)\vec{n}(\vec{X}) \end{bmatrix}. \tag{27}$$

Equation (27) can be rewritten in a convenient form:

$$\vec{\nabla} F_T^{n+1}\left(\vec{X}\right) + \lambda F_T^{n+1}\left(\vec{X}\right)\vec{n} = \frac{1}{\Delta t}\begin{bmatrix} \underbrace{\left(\rho C_p \vec{\nabla} T^{n+1}\left(\vec{X}\right) + \lambda \rho C_p T^{n+1}\left(\vec{X}\right)\vec{n}\left(\vec{X}\right)\right)}_{=\lambda\rho C_p T_\infty \vec{n}} - \\ \left(\rho C_p \vec{\nabla} T^{*n+1}\left(\vec{X}\right) + \lambda \rho C_p T^{*n+1}\left(\vec{X}\right)\vec{n}\left(\vec{X}\right)\right) \end{bmatrix}. \quad (28)$$

So, using Eq. (22), it gives:

$$\vec{\nabla} F_T^{n+1}\left(\vec{X}\right) + \lambda F_T^{n+1}\left(\vec{X}\right)\vec{n}\left(\vec{X}\right) = \frac{1}{\Delta t}\left[\lambda\rho C_p T_\infty \vec{n}\left(\vec{X}\right) - \begin{pmatrix} \rho C_p \vec{\nabla} T^{*n+1}\left(\vec{X}\right) + \\ \lambda\rho C_p T^{*n+1}\left(\vec{X}\right)\vec{n}\left(\vec{X}\right) \end{pmatrix}\right]. \quad (29)$$

Equation (29) can be formally rewritten in the Ω domain, giving approximately:

$$\vec{\nabla} f_T^{n+1}(\vec{x}) + \lambda \int_\Gamma F_T^{n+1}\left(\vec{X}\right)\vec{n}\left(\vec{X}\right)\delta\left(\vec{x}-\vec{X}\right)d\vec{X} = \frac{1}{\Delta t}\int_\Gamma \begin{bmatrix} \lambda\rho C_p T_\infty \vec{n}\left(\vec{X}\right) - \\ \begin{pmatrix} \rho C_p \vec{\nabla} T^{*n+1}\left(\vec{X}\right) + \\ \lambda\rho C_p T^{*n+1}\left(\vec{X}\right)\vec{n}\left(\vec{X}\right) \end{pmatrix} \end{bmatrix} \delta\left(\vec{x}-\vec{X}\right)d\vec{X}. \quad (30)$$

Applying the divergent operator to Eq. (30), it gives:

$$\nabla^2 f_T^{n+1}(\vec{x}) + \vec{\nabla}\cdot\vec{I}_1^{n+1}(\vec{x}) = \vec{\nabla}\cdot\vec{I}_2^{n+1}(\vec{x}), \quad (31)$$

where

$$\vec{I}_1^{n+1}(\vec{x}) = \lambda\int_\Gamma F_T^{n+1}\left(\vec{X}\right)\vec{n}\left(\vec{X}\right)\delta\left(\vec{x}-\vec{X}\right)d\vec{X}\ , \quad (32)$$

and

$$I_2^{n+1}(\vec{x}) = \frac{1}{\Delta t}\int_\Gamma \begin{bmatrix} \lambda\rho C_p T_\infty \vec{n}\left(\vec{X}\right) - \\ \begin{pmatrix} \rho C_p \vec{\nabla} T^{*n+1}\left(\vec{X}\right) + \\ \lambda\rho C_p T^{*n+1}\left(\vec{X}\right)\vec{n}\left(\vec{X}\right) \end{pmatrix} \end{bmatrix} \delta\left(\vec{x}-\vec{X}\right)d\vec{X}. \quad (33)$$

Equation (31) can be solved by several methods. Particularly it is possible to be solved using a Fourier spectral method. In this case the periodical boundary condition must be used, as commented before. Applying the Fourier transform to Eq. (32), it gives:

$$-k^2 \hat{f}_T^{n+1}\left(\vec{k}\right) + \vec{k}\cdot\hat{\vec{I}}_1^{n+1}\left(\vec{k}\right) = \vec{k}\cdot\hat{\vec{I}}_2^{n+1}\left(\vec{k}\right), \quad (34)$$

from which the following equation is obtained:

$$\hat{f}_T^{n+1}\left(\vec{k}\right) = \frac{1}{k^2}\left[\vec{k}\cdot\hat{\vec{I}}_1^{n+1}\left(\vec{k}\right) - \vec{k}\cdot\hat{\vec{I}}_2^{n+1}\left(\vec{k}\right)\right]. \quad (35)$$

The solution of Eq. (31) requires the integral $\vec{I}_1^{n+1}(\vec{x})$, which requires $F_T^{n+1}\left(\vec{X}\right)$. So it must be explicitly given and $F_T^{n+1}\left(\vec{X}\right)$ is replaced by $F_T^{n}\left(\vec{X}\right)$.

3. RESULTS

3.1. The Green Taylor problem with thermal effects, without immersed boundary

Figure (1) shows the temperature field and the immersed boundary inside the global domain.

The global error, is shown by Fig. (2), for the boundary conditions of first, second and third type. It can be seen that the boundary conditions keep the accuracy of the spectral method when the immersed boundary methodology is used. The rate of convergence was approximately 8 to the N_xxN_y=8x8, 16x16 and 32x32 collocations points, Fig. (2) (a). Above this resolution, it reaches round-off truncation error Fig. (2) (b).

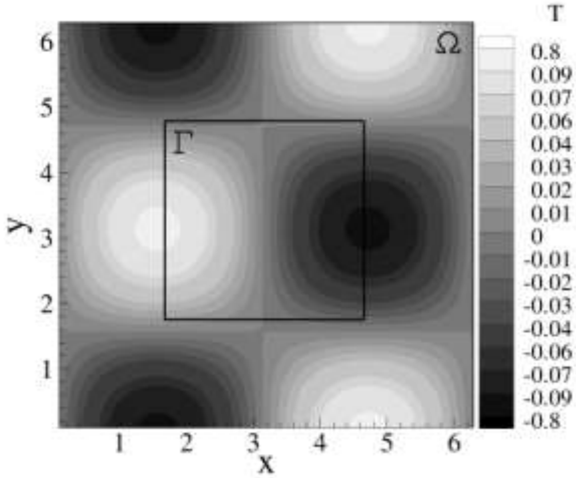

Figure 1. The temperature field; immersed boundary Γ inside the complete domain Ω .

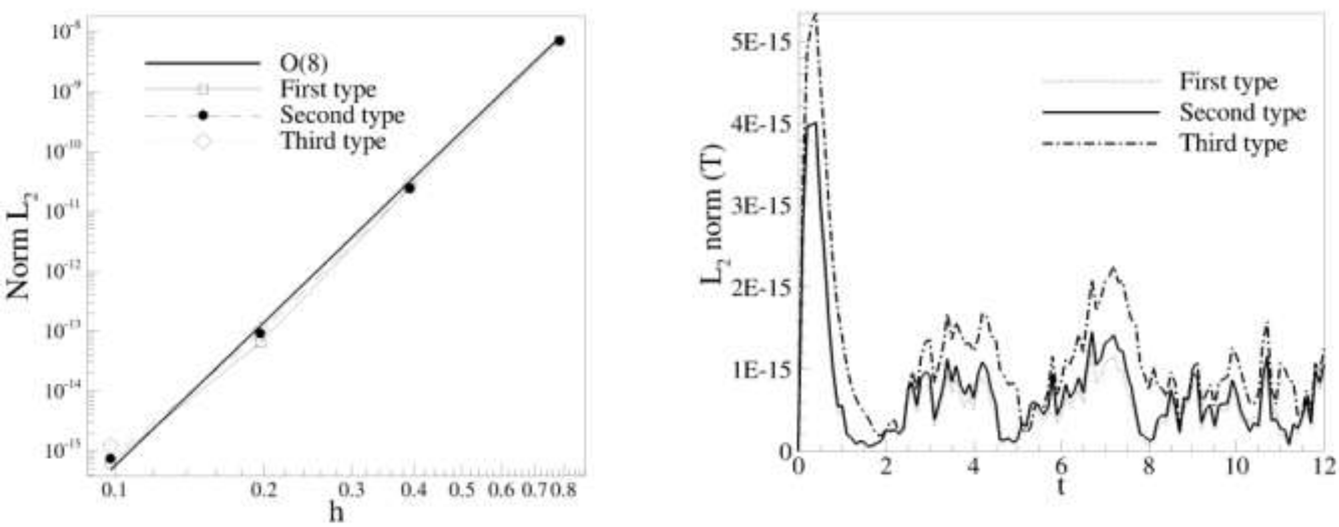

Figure 2. Green-Taylor problem, with thermal effect and with immersed boundary for first, second and third type thermal boundary condition (a) Rate of convergence and (b) The errors (L_2) of the temperature for N_xxN_y=64x64 collocations nodes

4. CONCLUSION

A new kind of immersed boundary methodology for fluids flows with thermal effects was proposed. Mathematical model for thermal boundary conditions of first, second and third types were proposed. They were implemented in a pseudo-spectral numerical code. The implementation was verified using a synthetized analytical solution for the Navier-Stokes and energy equations. The simulations that are presented in the present paper show that the proposed methodology is very accurate, at least for the simulated problem.

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6. REFERENCES

BOTELLA, O.; PEYRET, R. “Benchmark spectral results on the lid-driven cavity flow”. *Computer & Fluids*, Vol. 27, 1998, pp. 421-433.

CANUTO, C.; HUSSAINI, M.Y.; QUARTERONI, A.; ZANG, T.A. “Spectral methods: evolution to complex geometries and applications to fluid dynamics”, Spriger Verlag, 2007.

FERZIGER, J.H.; PERIC, M. “Computational Methods for Fluid Dynamics”, Springer, 1996.

GOLDSTEIN, D.; ADACHI, T.; SAKATA, H. “Modeling a no-slip flow with an external force field”, *Journal of Computational Physics*, Vol. 105, 1993.

JEONG, H.K.; YOON, H.S.; HA, M.Y.; TSUTAHARA, M. “An immersed boundary-thermal lattice Boltzmann method using an equilibrium internal energy density approach for the simulation of flows with heat transfer”, *Journal of Computational Physics*, Vol. 229, 2010, pp. 2526–2543.

JUNGWOO, K.; HAECHEON, C. “An Immersed-Boundary Finite-Volume Method for Simulation of Heat Transfer in Complex Geometries”, *KSME International Journal*, Vol. 18, 2004, pp. 1026-1035.

LIMA E SILVA, A.L.; SILVEIRA-NETO, A.; DAMASCENO, J. “Numerical simulation of two dimensional flows over a circular cylinder using the immersed boundary method”, *Journal of Computational Physics*, Vol. 189, 2003, pp. 351–370.

MARIANO, F.P.; MOREIRA, L.Q.; SILVERIA-NETO, A.; SILVA, C.B.; PEREIRA, J.C.F. “A new incompressible navier-stokes solver combining Fourier pseudo-spectral and immersed boundary method”, *Computer Modeling in Engineering Science*, Vol. 59, 2010, pp. 181–216.

MITTAL, R.; IACCARINO, G. “Immersed boundary methods”, *Annual Review Fluid Mechanics*, Vol. 37, 2005, pp. 239–261.

NAKAHASHI, K.; ITO, Y.; TOGASHI, F. “Some challenges of realistic flow simulations by unstructured grid cfd”, *International Journal for Numerical Methods in Fluids*, Vol. 43, 2003, pp. 769–783.

PAN, D. “An immersed boundary method on unstructured Cartesian meshes for incompressible flows with heat transfer”, *Numerical Heat Transfer, Part B*, Vol. 49, 2006, 277–297.

PESKIN, C.S. Flow patterns around heart valves: a numerical method, Vol. 10, 1972, pp. 252–271.

WANG, Z.; FAN, J.; LUO, K; CEN, K. “Immersed boundary method for the simulation of flows with heat transfer”, *International Journal of Heat and Mass Transfer*, Vol. 52, 2009, pp. 4510–4518.

YOUNG, D.L.; JAN, Y.J.; CHIU, C.L. “A novel immersed boundary procedure for flow and heat simulations with moving boundary”, *Computers & Fluids*, Vol. 38, 2009, pp. 1145–1159.

7. RESPONSIBILITY NOTICE