

STRUCTURAL RELIABILITY ANALYSIS COMBINING ANT COLONY OPTIMIZATION AND FINITE ELEMENT MODELING

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Abstract: *Traditional structural design techniques use deterministic information of the problem, such as geometry, material properties, and loads. However, uncertainties regarding the analytical models, variability of material properties and fluctuations of loads can contribute to the possibility that the structure does not perform as intended. To address this concern, analysis methods have been developed to deal with the statistical nature of the input information. In this context, Reliability Analysis (RA) intends to find the best compromise between cost and safety and to supply guidelines for carrying out reliable and cost-effective projects, accounting for the statistical variability of the system properties and loads. A set of methods are readily applicable in the case where the limit state function is available in explicit analytical form. In the opposite case, it is necessary to evaluate it by using numerical models like those based on the Finite Element Method. In this work, it is presented a RA methodology that integrates a nature-inspired optimization method, namely Ant Colony Optimization, and Finite Element analysis. Numerical applications in statics and dynamics are performed in order to check the accuracy and efficiency of the present methodology.*

Key-words: Reliability Analysis, Ant Colony Optimization, Finite Element Analysis.

1. INTRODUCTION

Traditional structural design procedures utilize exclusively deterministic information of the problem. Suitable geometry, material properties and loads are assumed, and an analysis is then performed to provide a detailed behavior of the structure. However, fluctuations of loads, variability of material properties and uncertainties regarding the analytical models can contribute to the probability that the structure does not perform as intended. In this context, analysis methods have been developed to deal with the statistical nature of input information. Over the last ten years there has been an increasing trend for analyzing structures using probabilistic information of loads, geometry, material properties, and boundary conditions. As the structures are becoming more complex and the performance requirements are becoming more ambitious, the need for analyzing the influence of uncertainties and computation of probabilities of events has been growing.

Reliability Analysis (RA) can be used for the calculation of the probability of failure determined by a limit state for structural members or structures at any time during their service life. The reliability analysis intends to find the best relation between cost and safety and to supply guidelines for carrying out reliable and cost-effective projects.

Hasofer and Lind (1974) introduced a notion of reliability index, which can be used to calculate the structure's reliability and the failure probability. Basically, the reliability index can be computed by taking the ratio of the mean and standard deviation of a performance function. To obtain the reliability index, it is needed to solve a constrained optimization problem.

Structural time invariant reliability assessment consists in modeling every uncertain design variable as random variables. After that, a failure criterion is defined by a limit state function or performance function that defines the failure domain in the space defined by those variables. To assess the structure's reliability, it is needed to know the joint probability density function (PDF) of the random variables. The failure probability will then be obtained by integrating the PDF over the failure domain, which is a very difficult task in practical situations involving a large number of random variables (Haldar and Mahadevan, 2000). For engineering designers, it is not only important to obtain only the failure probability but also to know the design point of the structure, or in other words, the values of the design parameters.

In general, probabilistic analysis methods use probability functions to represent the design variables. This way, the solutions found through these analyses must also be represented by statistical parameters. Actually even Monte Carlo (MC) simulation, which is often referred to as the “exact” solution. However, for large-scale high fidelity models, the computational effort renders Monte Carlo simulation virtually impractical for use. Many “efficient” methods have been devised as alternatives to MC simulation. These methods include the first and second order reliability method: FORM and SORM respectively (Der Kiureghian and De Stefano, 1991), the advanced mean value family of methods: AMV (Wu et al., 1990) and the response surface method: RSM (Faravelli, 1989). These methods replace the original deterministic model with a computationally efficient analytical model in order to speed up the analysis.

Regarding optimization, it is well known that the solution of reliability analysis problems by using classical methods is a difficult task due to the existence of local minima in the design space. This aspect has motivated the authors of this paper to explore a modified approach based on FORM and SORM for the determination of reliability index and design points, based on the so-called Ant Colony Optimization (ACO). In this paper, it is proposed an algorithm that is able to solve the global search optimization in reliability problems by using ACO. This algorithm is not competing with existing methods, but it is introduced because of its ability to solve global optimization problem efficiently. The analysis methodology integrates a set of reliability analysis tools developed under MATLAB[®] and finite element analysis using the commercial software ANSYS[®]. Numerical applications in statics and the dynamics are performed in order to check the accuracy and efficiency of the suggested algorithms.

2. RELIABILITY ASSESMENT

2.1 General Concepts

The need for taking into account the uncertainties of the design parameters led Freudenthal (1956) to create the concept of risk-based design, which constitutes the basis of the RA. Since the design parameters are considered as random variables, the satisfactory performance of a system can not be absolutely guaranteed. However, it can be expressed in terms of the probability of a certain failure criterion to be satisfied. In the Engineering terminology, this probability is called reliability. The counterpart of the reliability is the fault probability (also known as risk probability). Thus, reliability is defined as the probability related to a perfect operation of a system (within the bounds specified by the design) during a pre-defined period in normal operation conditions.

The first procedure of the RA of a structure is to define the design variables X_i and a performance function expressed as $Z = g(X_1, X_2, \dots, X_n)$. The failure surface or limit state function is defined by the condition $Z = 0$. This surface defines the limit between the safe region ($Z > 0$) and unsafe region ($Z < 0$), of the design space in which the failure occurs. The failure probability is calculated as:

$$P_f = \int \dots \int_{g(\cdot) < 0} f_X(x_1, x_2, \dots, x_n) dx_1 dx_2 \dots dx_n \quad (1)$$

where $f_X(x_1, x_2, \dots, x_n)$ is the joint probability density function (PDF) of the design variables. Equation (1) is the fundamental expression of the RA. In general, the number of random variables is high and these variables do not appear explicitly in the performance function. In these cases, it is almost impossible to obtain the joint PDF. In addition, due to the correlation among the design variables, the evaluation of the Equation (1) is extremely difficult.

In the two last decades, intensive research has been carried out to provide methods to approximate this integral. The main approaches to solve this equation are direct integration of PDF over the failure domain, analytical approximations such as the first order and second order reliability methods, mean value method, most probable point of failure, response surface method, Monte Carlo simulations and others. Some of these methods are detailed in the following section.

The recent survey of the state of the art on structural reliability analysis presented by Rackwitz (2001) gives a good insight of most of these methods.

2.2 First Order and Second Order Reliability Methods

FORM is more adequate to be used when the limit state function is a linear function of uncorrelated normal variables or when the nonlinear limit state function can be represented by a linear approximation in terms of equivalent normal variables. In this case, the random variables are transformed to reduced variables in a reduced coordinate system. For estimating the reliability index based on FORM one can use the algorithm suggested by Rackwitz and Fiessler (1978) in which the limit state function does not need to be solved because a Newton-Raphson type recursive algorithm is introduced to find the design point. Particularly, this method is useful when a limit state function is implicit, that is, when it can not be expressed as an explicit function of the random variables. This algorithm has been widely used in the literature (Haldar and Mahadevan, 2000).

SORM estimates the probability of failure by using a nonlinear approximation of the limit state function (or a linear limit state function with correlated non-normal variables) by a second order representation. The curvatures of the limit state function are approximated by the second-order derivatives with respect to the original variables. Thus, SORM improves FORM by including additional information about the curvature of the limit state function through of a curvature parameter. SORM was explored by Fiessler et al. (1979) using quadratic approximations. In this work was used a simple closed-form solution for the computation of failure probability using a second-order approach given by Breitung (1984) based on the theory of asymptotic approximation.

It is important notice that the design point vector of FORM and SORM is the same, knowing that the most probable point of failure of both methods is the same. Additionally, SORM uses as initial value the reliability index value estimated through FORM. Zhao and Ono (1999) and Rojas et al. (2006) give more details of FORM and SORM.

3. RELIABILITY INDEX ESTIMATION AS A GENERAL OPTIMIZATION PROBLEM

In deterministic design optimization, the optimization problem is generally formulated in the physical space of the design variables and consists in minimizing or maximizing an objective function subject to geometrical, physical or functional constraints in the form:

$$\min f(\{y\}) \quad (2)$$

subject to $g_k(\{y\}) \leq 0$, where $\{y\}$ designates the vector of deterministic design variables.

In RA, where the problem involves random variables $\{x\}$, the deterministic optimal solution isn't considered exact solution of the optimum design. In this case, the failure surface is given by $G(\{x\}, \{y\}) = 0$. This surface defines the limit between the safe region $G(\{x\}, \{y\}) > 0$ and unsafe region $G(\{x\}, \{y\}) < 0$ of the design space and the failure probability is $P_f = \text{prob}[G(\{x\}, \{y\}) \leq 0]$.

The reliability index β is introduced as a measure of the reliability level of the system and is estimated in the so-called reduced coordinate system, where the random variables $\{u\}$ are statistically independent with zero mean and unit standard deviation. Thus a pseudo-probabilistic transformation $\{u\} = T[\{x\}, \{y\}]$ must be defined for transforming the original space into the reduced space. The reliability index is defined as the minimum distance between the origin of the reduced space and hyper surface representing the limit state function $H(\{u\}, \{y\})$. Hence, it is possible to find the most provable point or design point by solving a constrained optimization problem that is:

$$\beta = \min \sqrt{\sum_{i=1}^n u_i^2} \quad (3)$$

subject to:

$$H(\{u\}, \{y\}) = 0 \quad (4)$$

where:

- $u_i = (x_i - \mu_i) / \sigma_i$ for normal variables;
- $u_i = (\ln(x_i) - \lambda_i) / \zeta_i$ for lognormal variables.

with: $\lambda_i = \ln \mu_i$, $\zeta_i = \sqrt{\ln(1 + \delta_i^2)}$, $\delta_i = \sigma_i / \mu_i$, μ_i is the mean and σ_i is the standard deviation of random variable x_i

The solution of the optimization problem given in Equation (3) by using classical gradient-based optimization methods is a difficult task due to the existence of local minima in the design space, computation of the gradient (partial derivatives) and computational effort.

Liu e Der Kiureghian (1991) compared different algorithms on the basis of four criteria: generality, robustness, efficiency and capacity. They recommended three algorithms for structural reliability evaluations: the Sequential Quadratic Programming, the modified Rackwitz-Fiessler approach and the gradient projection method.

Traditional methods as FORM and SORM require an initial guess of the solution (reliability index and random variables) and it is not possible to assure global convergence. These aspects have motivated the authors of this paper to explore an alternative approach for estimation of reliability index, based on heuristic method namely Ant Colony Optimization.

4. ANT COLONY OPTIMIZATION

Even though most of general-purpose optimization software used in industrial applications makes use of gradient-based algorithms (Venter and Sobieski, 2002), nowadays nature-inspired probabilistic search have attracted attention from the research community. In spite of the heavy computational effort, when compared to gradient-based techniques, these methods have several advantages, such as the ease to code, the efficiency in making use of parallel computing architectures, the ability to overcome numerical convergence difficulties and the capability of dealing with discrete and continuous variables simultaneously.

ACO is inspired in the behavior of ants and their communication scheme by using pheromone trails (Dorigo, 1992). A moving ant lays some pheromone on the ground, thus marking the path. The collective behavior that emerges from the participating agents is a form of positive feedback in such a way that the more the ants follow a trail, the more attractive that trail will become for being followed.

When searching for food, real ants start moving randomly, and upon finding food they return to their colony while laying down pheromone trails (Socha, 2004). This means that if other ants find such a path, they return and reinforce it. However, over time the pheromone trail starts to evaporate, thus reducing its attraction strength. When a short and a long path are compared, it is easy to see that a short path gets marched over faster and thus the pheromone density remains high. Thus, if one ant finds a short path (from the optimization point of view, it means a good solution) when marching from the colony to a food source, other ants are more likely to follow that path, and positive feedback eventually encourages all the ants in following the same path.

The idea behind ACO is to mimic this behavior by using artificial ants. The outline of a basic ACO algorithm is presented in Figure 1.

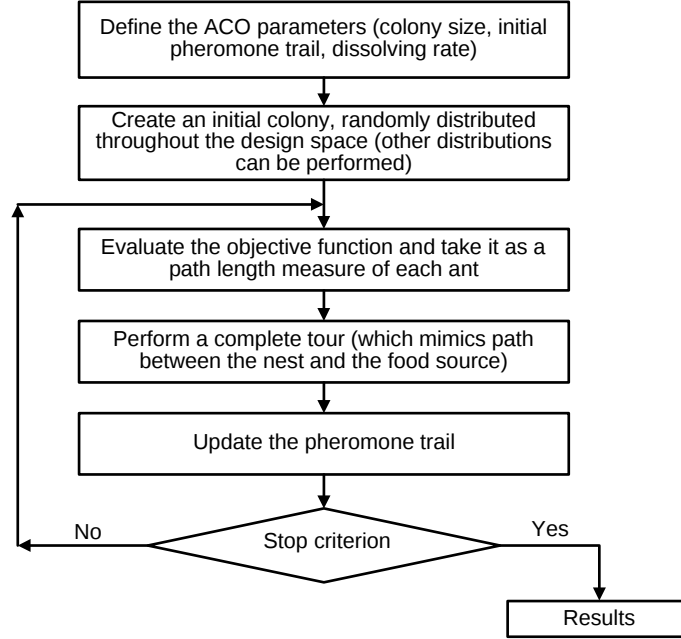


Figure 1: ACO basic algorithm.

The first point that has to be taken into account is how to model the pheromone communication scheme. According to Pourtakdoust and Nobahari (2004), for continuous model implementation, this can be done by using a normal probability distribution function (PDF), as follows:

$$pheromone(x) = e^{-\frac{(x-x_{min})^2}{2\sigma^2}} \quad (5)$$

where x_{min} is the best point found within the design space and σ is an index related to the ants aggregation around the current minimum.

In Figure 1, “To perform a complete tour,” means to update the values of each design variable for all ants of the colony. Or, more precisely, it is the process in which, for a given iteration, each ant sets the values for the trial solution based on the probability distribution specified by Equation (5). Computationally, this can be achieved through a random number generator based on a normal PDF that plays the role of a variable transition (update) rule to choose the next design variable value associated with each ant. From Equation (5), it can be noticed that each variable uses a different random number generator together with its respective PDF.

Finally, pheromone distribution over the design space is updated by collecting the information acquired throughout the optimization steps. Since the pheromone is modeled by Equation (5), it is necessary only to update x_{min} and σ by:

$$\sigma = std(colony) \quad (6)$$

where $Std(Colony)$ makes use of the colony of ants (candidate solutions) to return a vector containing the standard deviation for each design variable.

About the pheromone scheme, it is possible to see that the accumulation of pheromone increases in the area of the candidate to the optimum. This approach (also called positive update) reinforces the probability of the choices that lead to good solutions. However, to avoid premature convergence, negative update procedures are not discarded (Socha, 2004).

In this work, a simple method to perform negative update is used, which consists in dissolving the pheromone. The idea of this scheme is to spread the amount of pheromone by changing the current standard deviation (for each variable) according to the following equation:

$$\sigma_{new} = \gamma \sigma_{old} \quad (7)$$

where $\gamma > 1$ is the dissolving rate.

To initialize the algorithm:

- x_{min} is randomly chosen within the design space using a uniform PDF;
- σ is taken as being 3 times greater than the length of the search interval

Differently from what occurs with Genetic Algorithms (GA) (Michalewicz, 1994; Haupt and Haupt, 2004) and Particle Swarm Optimization (PSO) (Kennedy and Eberhart, 1995), which have a set of parameters to be defined by the user, ACO has one single special parameter to be chosen, namely the dissolving rate.

A comparison between what happens in nature and the counterparts in the ACO algorithm can be viewed in Table 1:

Table 1: Nature versus ACO.

Nature	ACO
Possible paths between the nest and food	Set of possible solution (vector of design variables)
Shortest path	Optimal solution
Pheromone communication in action	Optimization procedure

When solving an optimization problem, one must keep in mind that it will be always necessary to run more than once the optimization procedure. In the case of using classical methods, this is done to avoid local minima by starting from different initial designs. In the case of using nature-inspired methods, one has just to run the algorithm each time with a different seed for the random number generator. At then end, the engineer can compare all results obtained and make decisions about which will be chosen as the final design. Usually, the candidates are either the mean or the best result of the set.

5. NUMERICAL EXAMPLES

The present application is concerned with the use of dynamic vibration absorbers (DVAs) in a dome structure. The objective is to study the design of the DVA in four different scenarios of reliability analysis. These scenarios vary from the case of a single limit state function to the case of multiple limit state functions.

Dynamic vibration absorbers (DVAs) are systems constituted by mass, spring and damping elements, which are coupled to a mechanical system (named primary structure) in order to attenuate the vibrations in a given frequency range.

The classical procedure for tuning the DVA, i.e., to define a convenient set of values of the DVA parameters (mass, spring and damping values), is based on the existence of the so-called fixed points of the FRFs (Den Hartog, 1956).

Figure 2 shows details about the finite element (FE) model of a dome structure with a DVA. Figure 2-(a) illustrates the oblique view with the boundary and load conditions. Figure 2-(b) illustrates the target mode shape ($f = 32.2$ Hz). Figure 2-(c) gives details about the cross section of the beams. Finally, Figure 2-(d) illustrates the scheme of the DVA.

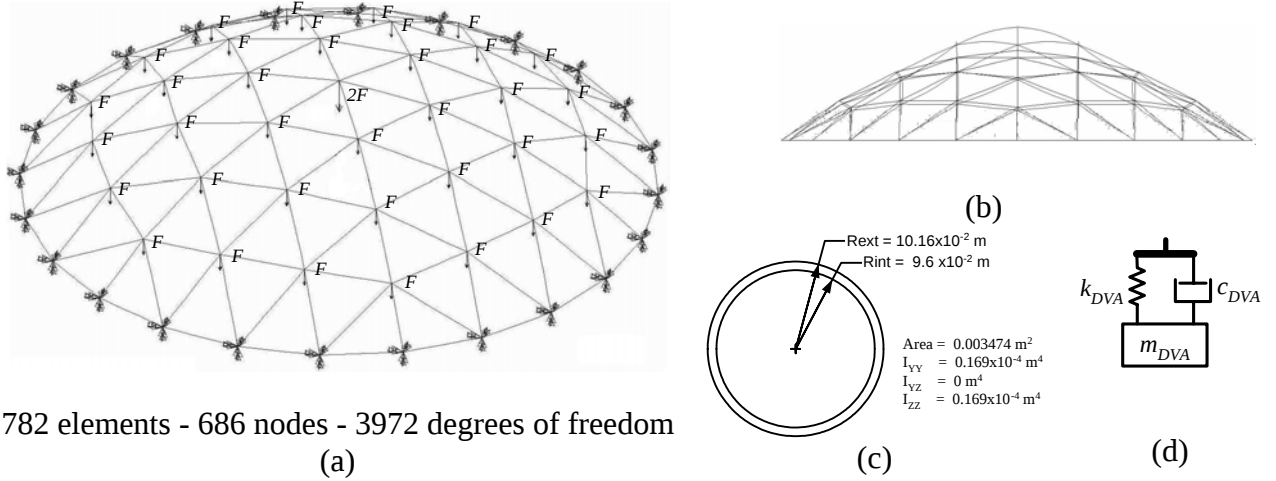


Figure 2: Finite element model of dome.

First of all, the deterministic design of the DVA for the dome structure is treated as an optimization problem in which the goal is to reduce the vibration amplitudes of the first mode. As suggested by Steffen and Rade (2000; 2001) this optimization problem is defined as the minimization of the objective function given by:

$$J(m_{DVA}, c_{DVA}, k_{DVA}) = \max_{26 \leq f \leq 36} \{H(f)\} \quad (8)$$

where $H(f)$ is the amplitude of a given frequency response function of the system with the DVA.

Following this approach, the optimal values for the DVA parameters were found to be $m_{DVA} = 72.96$ kg, $k_{DVA} = 2406493.62$ N/m and $c_{DVA} = 6250.24$ N.s/m.

Back to the reliability problem, it was considered as random variables: m_{DVA} , k_{DVA} , c_{DVA} and F . The interest in including the dead load F as a random variable is related to the fact that it is expected that the stress-stiffening effect can have some influence on the dynamic behavior of the structural system. This effect has been investigated by Rojas (2004).

Table 3 summarizes the design parameters and their statistical moments.

Table 3: Random variables and statistic parameters of DVA.

Random variable	Distribution	Mean	Standard deviation
m_{DVA} (Kg)	Normal	72.96	7.296
k_{DVA} (N/m)	Normal	2406493.62	240649.362
c_{DVA} (N.s/m)	Normal	6250.24	625.024
F (N)	Normal	40000	400

5.1 First Scenario

The first scenario is dedicated to the study of reliability analysis based on a single state limit function. Under this circumstance, it is possible to check the possibility of using the Heuristic Based Reliability Method (HBRM) approach to generate an initial guess to FORM and SORM.

The limit state function is defined as:

$$G_1(F, m_{DVA}, c_{DVA}, k_{DVA}) = 1 - \frac{|f_3 - f_3^{dome}|}{\Delta f} \quad (9)$$

where:

- f_3 is the third natural frequency of the whole structure (dome + DVA),
- f_3^{dome} is the third natural frequency of the dome, and
- $\Delta f = 5$ Hz is the frequency band of interesting.

It is easy to see that this function measures the tuning of the DVA for the 3th mode.

Table 4 and others shows the results obtained from 20 runs of HBRM. In these tables: Min. β is the less reliable design, Max. β is the most reliable design, μ_i is the average of the 20 runs, σ_i is the standard deviation of the 20 runs, and δ_i is the coefficient of variation of the 20 runs.

It can be observed that the standard deviations for each design variable are always small when compared to the mean values. It means that the ACO was capable to reach solutions that are close in the design space.

Table 4: Results of the first scenario.

	F [N]	m_{DVA} (Kg)	k_{DVA} (N/m)	c_{DVA} (N.s/m)	β	P_f (%)	R_{level} (%)
Min. β	39681.78	71.78	2337996.69	5995.30	0.5292	29.83	70.17
Max. β	46205.01	75.07	2324807.72	6511.53	1.6674	4.77	95.23
μ_i	40558.45	72.10	2435668.64	6295.67	0.9532	17.82	82.18
σ_i	2269.95	3.49	95362.35	320.81			
δ_i	0.06	0.05	0.04	0.05			

The best result was obtained in the Experiment #14, with a reliability level of 95.23%, which is considered satisfactory as a final design. The worst result was obtained in the Experiment #8 with a reliability level of 70.17%, which is not a good final design. Following the proposed strategy, the values of the design variables obtained in the Experiment #8 were used to feed a cascade-type approach with FORM and SORM. Table 5 shows the results. It can be seen that for both FORM and SORM the results are the same and there is a significant improvement when compared with the initial guess of the Experiment #8.

Table 5: Results of FORM and SORM.

	F [N]	m_{DVA} (Kg)	k_{DVA} (N/m)	c_{DVA} (N.s/m)	β	P_f (%)	R_{level} (%)
FORM	39681.78	85.01	1585039.72	5995.30	3.7109	0.0103	99.9897
SORM	39681.78	85.01	1585039.72	5995.30	3.7109	0.0103	99.9897

5.2 Second Scenario

Here it is added a second limit state function. According to what was discussed in the previous sections, the reliability problem is treated as a constrained optimization problem. It is important to notice that FORM and SORM are not capable to deal with this type of problem.

The second limit state function is written as:

$$G_2(F, m_{DVA}, c_{DVA}, k_{DVA}) = 1 - \frac{\delta_y}{\delta_y^{\lim}} \quad (10)$$

where:

- δ_y is the displacement in the vertical direction of the DVA attachment point.
- $\delta_y^{\lim}$ is the limit assumed for the displacement in the vertical direction of the DVA attachment point.

According to the literature, this value is given by $\delta_y^{\lim}(cm) = L / 200$, where L is diameter of the dome. Differently from G_1 , this function tries to ensure that the maximum displacement be lesser than the pre-defined limit.

Table 6 shows the results obtained from 20 runs of HBRM. Similarly to the previous case, the standard deviations for each design variable are small when compared to the mean values.

The best result was obtained in the Experiment #15, with a reliability level of 90.65%, which is not a satisfactory result but can be considered as a final design. The worst result was obtained in the Experiment #17 with a reliability level of 68.15%, which is not a good final design.

Table 6: Results of the second scenario.

	F [N]	m_{DVA} (Kg)	k_{DVA} (N/m)	c_{DVA} (N.s/m)	β	P_f (%)	R_{level} (%)
Min. β	38970.60	72.60	2490490.97	6362.26	0.4719	31.85	68.15
Max. β	36157.08	78.93	2481119.65	6393.75	1.3197	9.35	90.65
μ_i	39953.01	72.66	2413646.73	6369.26	0.9970	16.39	83.61
σ_i	2469.73	3.82	119889.67	235.92			
δ_i	0.06	0.05	0.05	0.04			

5.3 Third Scenario

In this scenario, a third function is added to the constrained optimization problem:

$$G_3(F, m_{DVA}, c_{DVA}, k_{DVA}) = 1 - \frac{F_{\max}}{F_y^{\lim}} \quad (11)$$

where:

- $F_{\max}$ is the maximum reaction force on the dome structure, and
- $F_y^{\lim}$ is the assumed limit for the maximum reaction force. According to the literature, this value is given by $F_y^{\lim} = f_{yk} A$, where $f_{yk} = 250$ MPa and A is cross section area.

This function tries to ensure that the maximum reaction force be lesser than the pre-defined limit.

Table 7 shows the results obtained from 20 runs of HBRM. Once again, the standard deviations for each design variable are small when compared to the corresponding mean values.

Table 7: Results of the third scenario.

	F [N]	m_{DVA} (Kg)	k_{DVA} (N/m)	c_{DVA} (N.s/m)	β	P_f (%)	R_{level} (%)
Min. β	40322.51	70.42	2372503.89	6434.26	0.4838	31.43	68.57
Max. β	34221.59	73.34	2382159.46	6616.36	1.5630	5.90	94.10
μ_i	40242.74	74.10	2403407.09	6127.73	0.98	17.57	82.43
σ_i	2504.88	3.81	70244.92	334.29			
δ_i	0.06	0.05	0.03	0.05			

The Experiment #7 was the best result obtained, with a reliability level of 94.10%, which is a satisfactory result considered as a final design. The worst result was obtained in the Experiment #18 with a reliability level of 68.57%, which is not a good final design.

5.4 Fourth Scenario

Here, a fourth function is added to the constrained optimization problem:

$$G_4(F, m_{DVA}, c_{DVA}, k_{DVA}) = 1 - \frac{F}{F^{crit}} \quad (12)$$

where F^{crit} is the buckling load of the dome structure.

This function is particularly interesting since F , which is not a DVA parameter, is explicitly used in the formulation.

Table 8 shows the results obtained from 20 runs of HBRM. Again, the standard deviations for each design variable are small when compared with the mean values.

Table 8: Results of the fourth scenario.

	F [N]	m_{DVA} (Kg)	k_{DVA} (N/m)	c_{DVA} (N.s/m)	β	P_f (%)	R_{level} (%)
Min. β	38925.15	72.18	2387927.58	6139.25	0.3482	36.39	63.61
Max. β	38114.12	69.99	2644614.33	6883.82	1.5474	6.09	93.91
μ_i	40233.96	71.33	2404621.49	6197.09	1.1405	13.60	86.40
σ_i	2299.97	3.25	156146.44	412.96			
δ_i	0.06	0.05	0.06	0.07			

In the Experiment #1 was obtained the best result, with a reliability level of 93.91%, which is a satisfactory result considered as a final design. The worst result was obtained in the Experiment #10 with a reliability level of 63.61%, which is not a good final design.

6. CONCLUSIONS

In this work is proposed a new reliability analysis methodology which integrates a nature-inspired optimization method named Ant Colony Optimization. This methodology was so-called HBRM and was used to estimate the design point and reliability level to dynamic and static parameters using different limit state functions. There were four probabilistic variables, three of them related to the design of a DVA and one related to the load condition. In the applications it was used the RA tools of HBRM integrated with finite element analysis. In most of the cases, the best results in different scenarios can be considered as final design, mean and standard deviation of the results was considered satisfactory. Taking the performance of HBRM into account, it can be concluded that the presented methodology is able to handle limit state functions based on numerical models and probabilistic variables related to both physical or geometrical parameters, as well as loads. The results obtained encourage the authors to improve this methodology for use in complex problems and obtain best results.

7. ACKNOWLEDGMENTS

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