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METAMODELS FOR MULTIOBJECTIVE OPTIMIZATION OF VISCOELASTIC DAMPED STRUCTURES

Antonio Marcos Gonçalves de Lima
Federal University of Uberlandia – School of Mechanical Engineering,Campus Santa Monica – P.O. Box 593, CEP. 38400-902, Uberlandia-MG, Brasil
amglima@mecanica.ufu.br

Domingos Alves Rade
Federal University of Uberlandia – School of Mechanical Engineering,Campus Santa Monica – P.O. Box 593, CEP 38400-902, Uberlandia-MG, Brasil
domingos@ufu.br

Noureddine Bouhaddi
FEMTO-ST Institute, UMR 6174, R. Chaléat Applied Mechanics Laboratory, University of Franche Comté, 24 Chemin de l'Epitaphe 25000, Besançon-France.
noureddine.bouhaddi@univ-fcomte.fr

Abstract: *The use of viscoelastic materials is an efficient strategy to reduce the vibration levels of mechanical systems. However, the exploitation of an optimal and/or robust design of the viscoelastic damped systems is a difficult task: laws of behaviour, complexity of FE models, etc, making very expensive any optimization procedure. In this work, the strategy proposed is a multiobjective optimization methodology combining metamodels: robust condensed models and artificial neural networks approach (ANNs).*

Keywords: *multiobjective optimization, robust condensation, artificial neural networks, viscoelastic damping*

1. INTRODUCTION

In the context of sound and passive vibration control of mechanical vibrations, the use of viscoelastic materials has been regarded as a convenient strategy in many types of industrial applications where the materials can be applied either as discrete or surface devices (free or constrained layer treatments and viscoelastic mounts). Much of the knowledge available to date is compiled in the books Nashif *et al.* (1985) and Mead (1998), and in some review papers Rao (2001) and Samali and Kwok (1995). To enable efficient analysis and design of viscoelastic dampers as applied to real-world complex structures, the multi-objective optimization approach is an important step in the design phases (pre-project and/or project steps) (Lima *et al.,* 2003). Unfortunately, the large number of cost functions evaluation employed in optimization (reanalysis procedure) and the complexity of the finite element (FE) models for the industrial applications (large number of degrees of freedom), conducts to long reanalysis and sometimes prohibitive due to the large calculation costs (computational effort) to obtain the optimal solutions. Moreover, the non-linear modeling behavior particularly involved for the typical dependency of the viscoelastic properties with respect to operational and environmental parameters, mainly temperature and vibration frequency, is observed (Ait Brik *et al.,* 2004). To circumvent this drawback, one strategy proposed is the use of MOEAs combining metamodels (Ait Brik *et al.*, 2004; Soteris, 2004; Keys and Rees, 2002) such as artificial neural networks.

The present work discusses some of the crucial issues of the modelling of viscoelastic systems and the coupling procedure between evolutionary algorithms and metamodels. The paper is organized as follows: introductory comments are first presented regarding some aspects related with the viscoelastic effects and its incorporation into the FE models. Next, some comments are dedicated to the modelling procedure of the multilayer sandwich plates. Some discusses are dedicated to the Non Dominated Sorting Genetic Algorithm (NSGA) incorporating artificial neural networks (ANN) as approximation function of the responses of the damped system.

In this work, the frequency response functions (FRFs) of the system containing viscoelastic damper were analysed. Numerical optimization of the geometrical parameters and the temperature of the viscoelastic material, with the aim of augmenting the damping effectiveness of the surface treatment, are presented.

2. VISCOELASTIC BEHAVIOR INCORPORATED INTO FE MODELS

Consider the following FE equations of motion of a viscoelastic structure containing N degrees-of-freedom:

$$\begin{aligned} \left([K(\omega,T)]-\omega^2[M]\right)\{q\}=[Z(\omega,T)]\{q\}=[b]\{u\} \\ \{y\}=[c]\{q\} \end{aligned} \tag{1}$$

where $[M],[K(\omega,T)] \in R^{NxN}$ are the mass (symmetric, positive-definite) and stiffness (symmetric, nonnegative-definite) matrices. $\{q\}\in R^N$ and $\{f\}=[b]\{u\}\in R^N$ are the vectors of displacement and external loads, and $\{y\}$ is the vector of response. It is assumed that the structure contains both elastic and viscoelastic elements, so that the stiffness matrix can be decomposed as follows:

$$[K(\omega,T)]=[K_e]+[K_v(\omega,T)] \tag{2}$$

where $[K_e]$ is the stiffness matrix corresponding to the purely elastic substructure and $[K_v(\omega,T)]$ is the stiffness matrix associated with the viscoelastic substructure. The inclusion of the frequency-dependent behaviour of the viscoelastic material can be made by using the so-called *elastic-viscoelastic correspondence principle* (Christensen, 1982), according to which, for a given temperature, $[K_v(\omega,T)]$ can be first generated for specific types of finite elements (rods, beam, plates, etc.) assuming that the longitudinal modulus $E(\omega,T)$ and/or the shear modulus $G(\omega,T)$ (according to the stress-state) are frequency-independent. Then, one of the two *moduli* can be factored-out of the stiffness matrix, $[K_v(\omega,T)]=G(\omega,T)[\overline{K}_v]$, witch can be combining with equations (1) and (2) to produce the following complex dynamic stiffness matrix:

$$[Z(\omega,T)]=\left([K_e]+G(\omega,T)[\overline{K}_v]-\omega^2[M]\right) \tag{3}$$

After the construction of the finite element global matrices, the frequency-temperature dependent properties of the viscoelastic material can be introduced according to a particular viscoelastic model adopted (Lima *et al.,* 2003). By combining the equations (1) and (3), the complex FRF matrix can be expressed as follows:

$$[H(\omega,T)]=[c][Z(\omega,T)]^{-1}[b] \tag{4}$$

For industrial applications, it is not useful and impossible to use directly the complete model expressed by the relation (4), because it leads to prohibitive computing times generated by the inversion of the complex dynamic stiffness matrix for each value of excitation frequency (Balmès, 2005; Balmès and Germès, 2002). Moreover, if this generation intervenes during the optimization procedure, it leads to practically not exploitable. One then uses reduction model method in order to reduce the number of degrees of freedom. By assumption that the exact responses can be approached by solutions in the reduced space, $\{q\}=[B]\{q_r\}$, the transfer function (4) can be written under the following form:

$$[H_c(\omega,T)]=[c][Z_c(\omega,T)]^{-1}[b] \tag{5}$$

where $[Z_c(\omega,T)]=\left([B]^T[K_e][B]+G(\omega,T)[B]^T[\overline{K}_v][B]-\omega^2[B]^T[M][B]\right)$, $[T]\in R^{N\times NR}$ is the Ritz base and $\{q_r\}\in R^{NR}$ with $NR<<N$ the number of reduced modes.

The reduced system is then solved frequency by frequency, in a direct way. For the systems treated with viscoelastic material, the base of reduction poses the problem in the frequency domain, because of the non-constant stiffness matrix. Much of the knowledge available to date is compiled in some papers (Balmès, 2005; Balmès and Germès, 2002). In this work, one fixes (frequency-temperature independent) the base of reduction and one determines the response of the system by using the standard approximation of Ritz-Galerkin. For that, one uses the tangent stiffness concept as starting point to determine the base of reduction (Balmès and Germès, 2002), having the advantage of being real and easier to be reversed, related to the static behaviour of the viscoelastic material, represented in figure 1.

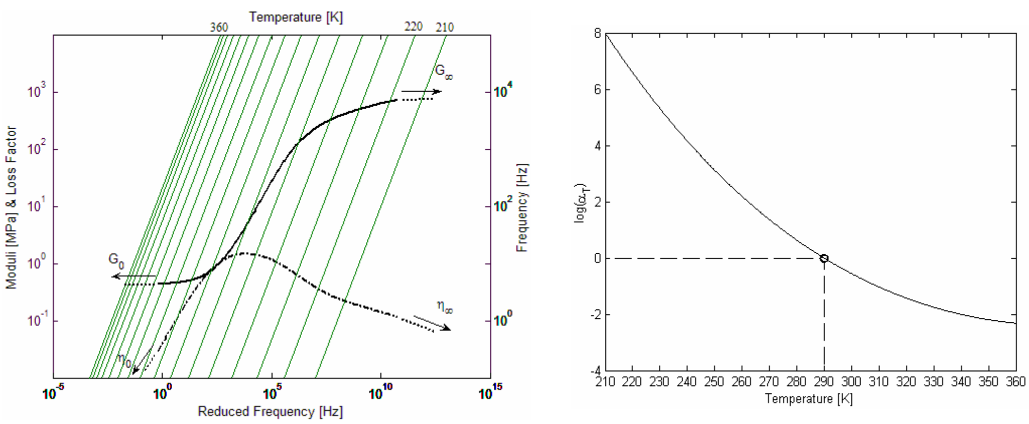


Figure 1: Master and shift factor curves for the 3M™ ISD112.

This figure was computed by the following equations representing, respectively, the *complex modulus* and *shift factor* as functions of temperature, T, and reduced frequency, ω_r, for the 3M ISD112™ viscoelastic material (which is considered in the numerical applications that follows), as provided by Drake and Soovere (1984):

$$\begin{aligned}&G(\omega_r)=B_1+B_2/\left(1+B_5(i\omega_r/B_3)\right)^{-B_6}+(i\omega_r/B_3)^{-B_4}\\&log(\alpha_T)=a\left(\frac{1}{T}-\frac{1}{T_r}\right)+2.303\left(\frac{2a}{T_r}-b\right)log\left(\frac{T}{T_r}\right)+\left(\frac{b}{T_r}-\frac{a}{T_r^2}-S_{AZ}\right)(T-T_r)\end{aligned} \tag{6}$$

Details of fully expressions are given by Drake and Soovere (1984). As showed in figure 1, in the low frequency, one prolongs the curves by the asymptotes, and the extrapolation curves give the real values G_0 and $\eta_0=0$. The tangent stiffness matrix can thus be calculated as $[K_0]=[K_e]+G_0[\overline{K}_v]$, and by the resolution of the associated eigenvalue problem, one can obtain the nominal base of reduction $[T_0]=\{\phi\}$ containing the first free NR retained modes of the static damped system, by taking into account the tangent elastic matrix (static behaviour of viscoelastic material). It is necessary to use the residues based on the static displacements associated with imposed loadings, $[R]=[K_0]^{-1}[b]$, which should be completed by a correction for the imaginary part of complex dynamic stiffness matrix, $[R_1]=[K_0]^{-1}[\overline{K}_v]\{\phi\}$, to take into account the viscoelastic effects (damping) (Balmès and Germès, 2002). The complete base of reduction for the viscoelastic models can then be written as:

$$[B]=[[T_0]\quad[R]\quad[R_1]] \tag{7}$$

The modal base represented by the expression above is real and frequency-temperature independent with corrections of first orders. For higher orders, it is enough to use the iterative procedure based on the residues (Balmès and Germès, 2002), or residues of order 2 (Masson *et al.*, 2003).

3. A THREE-LAYER SANDWICH PLATE FINITE ELEMENT

In this section the formulation of a three-layer sandwich plate FE is summarized, based on the original development made by Khatua and Cheung (1973). Figure 2 depicts a rectangular plate FE formed by an elastic base-plate (1), a viscoelastic core (2) and an elastic constraining layer (3), whose dimensions in directions x and y are denoted by a and b. Space discretization is made by considering 4 nodes and 7 dofs per node, representing the nodal longitudinal displacements of the base-plate middle plane in directions x and y (denoted by u_1 and v_1), the nodal longitudinal displacements of the constraining layer middle plane in directions x and y (denoted by u_3 and v_3), the transverse displacement, w, and the cross-section rotations of the layers (1) and (3) about axes x and y, denoted by θ_x and θ_y.

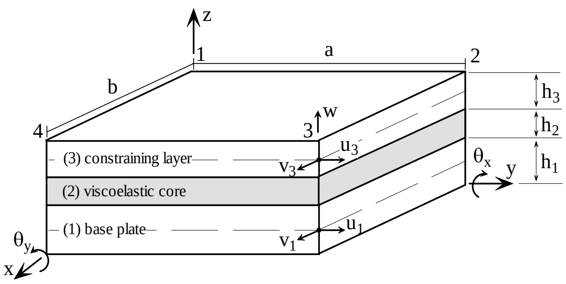


Figure 2: Three-layer sandwich plate finite element

In the development of the theory, the normal stresses/strains in direction z are neglected for all the three layers. The elastic layers (1) and (3) are modeled according to Kirchhoff's theory, and for the viscoelastic core, the Mindlin's theory is adopted. By imposing kinematic relations that enforce the continuity of displacements along the interfaces between the layers, the following expressions are found for the longitudinal displacements of the viscoelastic core middle plane, in directions x and y, in terms of the previously defined nodal coordinates:

$$u_2=\frac{1}{2}\left[u_1+u_3+\frac{h_1-h_3}{2}\frac{\partial w}{\partial x}\right],\ v_2=\frac{1}{2}\left[v_1+v_3+\frac{h_1-h_3}{2}\frac{\partial w}{\partial y}\right] \tag{8}$$

where the longitudinal and transverse displacements are interpolated as follows:

$$\begin{aligned}
&u_1=u_1(x,y)=a_1+a_2x+a_3y+a_4xy \quad u_3=u_3(x,y)=a_9+a_{10}x+a_{11}y+a_{12}xy\\
&v_1=v_1(x,y)=a_5+a_6x+a_7y+a_8xy \quad v_3=v_3(x,y)=a_{13}+a_{14}x+a_{15}y+a_{16}xy\\
&w=w(x,y)=b_1+b_2x+b_3y+b_4x^2+b_5xy+b_6y^2+b_7x^3+b_8x^2y+b_9xy^2+\dots\\
&\qquad\qquad b_{10}y^3+b_{11}x^3y+b_{12}xy^3
\end{aligned} \tag{9}$$

Based on the hypotheses of the stress-states assumed for each layer, the stress-strain relations can be obtained, and the strain and kinetic energies of the sandwich plate finite element are formulated. Details are given by Khatua and Cheung (1973) and Lima *et al.* (2005). The Lagrange equations are then used, considering the nodal displacements and rotations as generalized coordinates, to obtain the element stiffness and mass matrices. After assembling, the global stiffness matrices of elastic and viscoelastic substructures can be expressed as follows:

$$[K_e] = \left[K^{(1)}\right] + \left[K^{(3)}\right]; \qquad [K_v(\omega,T)] = G(\omega,T)\left[\overline{K}^{(2)}\right] \tag{10}$$

4. EVOLUTIONARY ALGORITHMS (EAs)

Genetic algorithms (GAs) and evolutionary algorithms (EAs) are widely used to deal with the engineering problems due to their general numerical setting and their robustness optimal solutions. Much of the knowledge available to date is compiled in some papers (Cheng and Dan, 2004; Srinivas and Deb, 1993). Many designs of engineering problems involve simultaneous optimisation of multiples objectives, where the goal is to find the best design solution, which corresponds to the minimum or maximum value of the objective functions. On the contrary, in a multi-objective optimization with conflicting objectives, there is no single optimal solution, and the interaction among different objectives gives rise to a set of compromised solutions, classically known as the Pareto optimal solutions (Das and Dennis, 1998). Since none of these Pareto-optimal solutions can be identified as better than others without any further consideration, and the interest is to find as many Pareto-optimal solutions as possible (Ait Brik *et al.,* 2004). A multi-objective problem includes a set of k parameters (decision variables) and a set of n objective functions ($n \geq 2$), summarized as follows:

$$\begin{cases} min\,imize \quad F(x) = (f_1(x), f_2(x), \ldots, f_n(x)) \\ \qquad\qquad\quad x \in C \end{cases} \tag{11}$$

where $x = (x_1, x_2, \ldots, x_k)$ is a k design variables, $C \in R^k$ is the decision space, and $\{F(x)/x \in C\}$ is the criteria space. For a practical design problem, F is non-linear, multi-modal and not necessary analytical.

Although simplistic from a biologist's viewpoint, these algorithms are sufficiently complex to provide robust and powerful adaptive search mechanisms, and the algorithm used to solve this problem is the non-dominated sorting genetic algorithm (NSGA) developed by Srinivas and Deb (Srinivas and Deb, 1993). The NSGA differs from classical multi-objective genetic algorithm in the way that a selection operator is used, as can be showed in figure 3. After the computation of Pareto ranking representing the whole of population, a same dummy fitness value is provisionally assigned to the members of the first front of Pareto. This fitness defines an equal reproductive pressure for these individuals, and with the aim in maintaining the diversity of the population, a sharing technique is applied and the dummy fitness is recomputed accordingly. The fitness is shared when an individual has at least one close neighbour, by multiplying its fitness by the factor, $m_i = \sum_{j=1}^{N} sh(d)$, where the sharing function $sh(d)$ is proposed by Srinivas and Deb (1993).

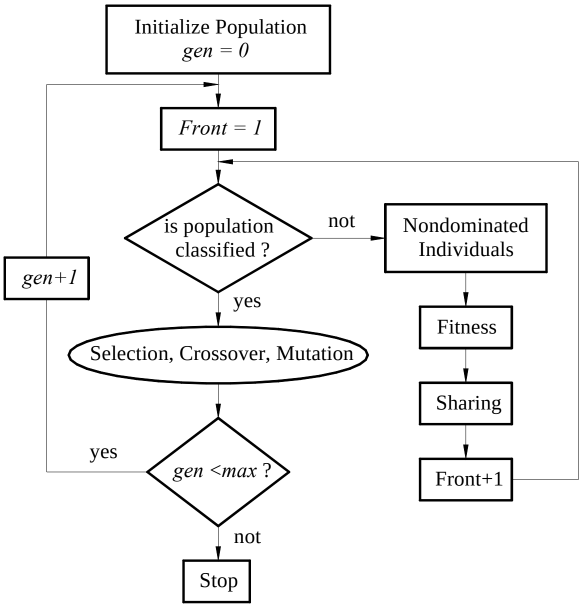


Figure 3: Non-dominated sorting genetic algorithm

5.1. NSGA-ANN COUPLING METHOD

Because of the NSGA is a stochastic iterative method which require several evaluations of the exact solutions, the cost of calculation are of primary importance, and very expensive to use in optimization of the large viscoelastic models. It is thus necessary to combine the NSGA with approximations functions methods in order to reduce the computing time of cost functions during the optimization process. In the literature one can find several works dedicated to the coupling of genetic algorithms with metamodels such as neural networks and response surface methodology (RSM) (Soteris, 2004; Keys and Rees, 2002). The differences between these approaches are reliably in their applicability, but in generally, the two approaches are used for the approximations of multimodal and nonlinear functions. However, the use of NN as approximations of problems composed by several numbers of variables is more interesting, due to the fable computing time.

The interest on the NNs is to approximate the responses of the system during the optimization process, in order to avoid the exact evaluation. The NN used, is the so called multilayer perceptron (MLP) constructed by one hidden layer (composed by two layers) and one output layer, as illustrated in figure 4.

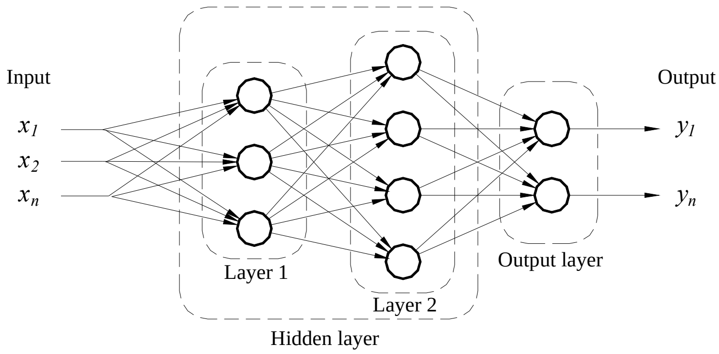


Figure 4: Multilayer perceptron (MLP).

The MLP are built with exact evaluations of the reduced damped system. Then, NSGA is used to explore the good solutions in the decision space. Once that NSGA produces a new solution, the MLP is used to determine its value of fitness so that NSGA continues its process of research until the stop criterion is satisfied. A re-initialization of the MLP is necessary to correct the quadratic error generated by the NNs. Figure 5 illustrates NSGA-MLP process.

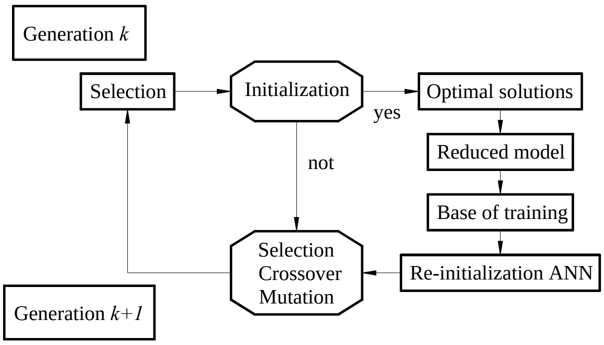
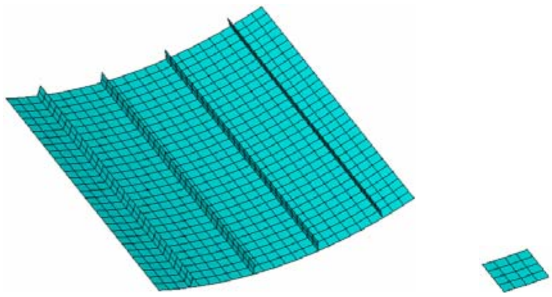


Figure 5: NSGA-PMC coupling process

6. NUMERICAL APPLICATION

A numerical application illustrating the optimization methodology described previously is presented, to evaluate the optimum design of the viscoelastic treatment applied to the free stiffened panel showed in figure 6. The panel (commercially steel material) is composed by 4 stringers having the geometric dimensions [mm]: internal radius, 938, length, 720, arc-length, 680, thicknesses of the panel and the stringers, 1.5, 0.75, and the height of stringers, 30.



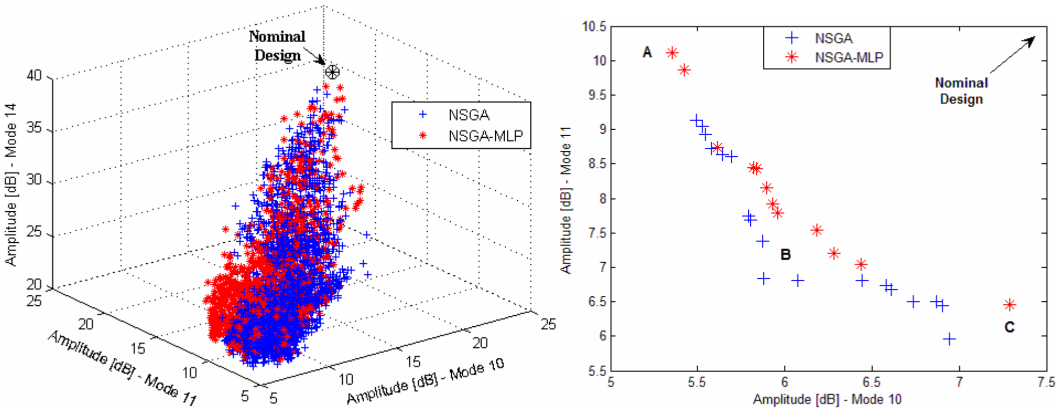
(a) (b)

Figure 6: FE model of the stiffened panel (a) and the viscoelastic path (b).

The FE model without treatment is composed by 928 finite elements of plate having 6840 degrees of freedom, and the surface treatment is composed by 15 viscoelastic paths, in witch each path is composed by 16 three-layer sandwich plate FE, developed in Section 3 (figure 6b). The material properties of the elastic layers are the same of the panel, and for the viscoelastic core, one uses the modulus function presented in figure 1 for the ISD112 3M ($\rho = 950$ Kg/m^3). The design parameters and the corresponding variations are: the thicknesses of the viscoelastic layer, ($h_2 = 0.254mm ; \pm 70\%$), the constraining layer ($h_3 = 0.5mm ; \pm 50\%$); the temperature of the viscoelastic material ($T = 25^\circ C ; \pm 15\%$) and the positions of each path. The cost functions are the amplitudes of modes 10 (M10), 11 (M11) and 14 (M14) of the stiffened damped panel, with the aim of augmenting the damping performance for these modes. The parameters of NSGA are defined as: the population (30), probabilities of selection (0.25), crossover (0.25) and mutation (0.25).

To evaluate the NSGA-MLP, one proposes to use 2 hidden layers and 20 neurones per layer. The objective functions (amplitudes of FRFs of viscoelastic system) are calculated based on the reduced model of the stiffness damped panel, in witch the base of reduction is composed by 121 eigenvectors (60 related to the static model; 1 related to the static residue; and 60 related to the damping effects).

In figure 7a one compares the optimal solutions obtained for the two approaches, and figure 7b presents the first front of Pareto for the amplitudes of modes 10 and 11, demonstrating a good agreement between the two clouds of optimal solutions obtained by the two approaches.

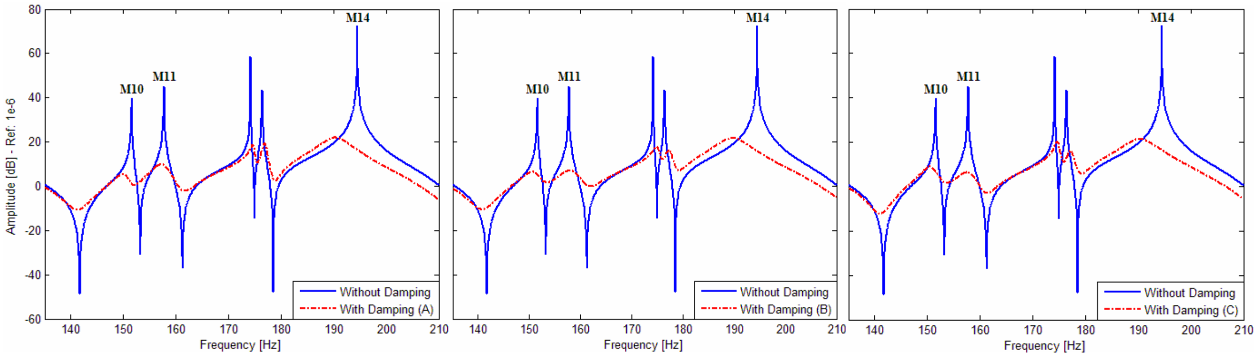


(a) (b)

Figure 7: NSGA and NSGA-MLP solutions (a); First front of Pareto (b).

The total number of generation to find the optimal solutions is equal to 100, which means that the exact number of evaluations is equal to 3000. For the NSGA-MLP, the MLP is updated at each 20 generations after the generation five, with the total number of evaluation equal to 270. The CPU computed time for the two optimization approaches are: 2016 min for NSGA and 180 min for NSGA-MLP coupling process. This comparison demonstrates that the NSGA-MLP coupling procedure, allows a significant reduction of the evaluations (ratio of 20).

From the optimal solutions generated by the points *A*, *B* and *C* (figure 7b), one can compares the amplitudes of the FRFs of system with and without damping related to the force/displacement applied on point *P* (figure 9).



(a) (b) (c)

Figure 8: Amplitudes of FRFs for the optimal design solutions *A*, *B* and *C*.

By comparing the results (figures 7b and 8a) one can conclude that for the point *A*, the optimal solutions are found as augmenting the damping performance on M11 instead of M10. On the contraire, figures 7b and 8c demonstrates that for the point *C*, the optimal solutions are found as augmenting the damping performance on M10 instead of M11. For the point *B*, the solutions are found with the aim of augmenting the damping of both M10 and M11.

As illustration, figure 9 shows the optimal positions of each viscoelastic path for the optimal point *B*, and the optimal values of the viscoelastic and constraining layers thicknesses for each path are given in table I. For this optimal point, the optimal value of temperature of the viscoelastic material is about *22.5ºC*.

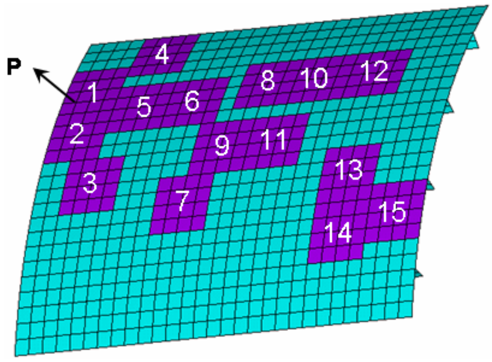


Figure 9: Optimal positions of the surface treatments for the optimal point *B*.

Table 1: Optimal values of design parameters of the viscoelastic paths (point *B*)

paths 1,4,7,10,13 [m]		***paths 2,5,8,11,14 [m]***		***paths 3,6,9,12,15 [m]***	
h_2	h_3	h_2	h_3	h_2	h_3
1.58x10^{-5}	1.25 x10^{-3}	1.65x10^{-5}	0.97 x10^{-3}	1.54x10^{-5}	1.25x10^{-3}
1.54x10^{-5}	1.25 x10^{-3}	1.54x10^{-5}	1.25 x10^{-3}	15.4x10^{-5}	0.40 x10^{-3}
1.54x10^{-5}	1.05 x10^{-3}	1.54x10^{-5}	1.25 x10^{-3}	1.54x10^{-5}	1.17 x10^{-3}
1.54x10^{-5}	1.20 x10^{-3}	2.45x10^{-5}	1.25 x10^{-3}	1.54x10^{-5}	1.25 x10^{-3}
1.54x10^{-5}	0.95 x10^{-3}	1.54x10^{-5}	1.25 x10^{-3}	1.54x10^{-5}	1.25 x10^{-3}

8. CONCLUSIONS

In this paper, the multi-objective optimization algorithms combined with approximation functions for the systems treated with surface (passive constraining damping layers) viscoelastic damping device was investigated. The originality of this optimization methodology, consists in introduce a MLP during the optimization process in order to approximate the cost functions of the viscoelastic models, using a back propagation algorithm, normally employed to train the MLP.

For very large viscoelastic models, the optimization procedures based on the frequency response functions is not viable, because of the inversion of the dynamic stiffness at each frequency point for a fixed temperature of the viscoelastic material. Thus, the classically reduced model approach adapted for the viscoelastic systems based on the concept of tangent stiffness matrix is essential.

The two approaches, multi-objective optimization incorporating the Sharing technique (NSGA) and a multi-layer perceptron (MLP) as approximation function, are seamlessly combined into a NSGA-MLP approach, which was successfully applied to a real engineering problem containing the commonly used viscoelastic damping treatments.

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REFERENCES

Nashif A., Jones D., Handerson J., Vibration Damping, J.W, 1985.

Mead D.J., Passive Vibration Control, Wiley, Canada, 1998.

Rao M.D., Recent Applications of Viscoelastic Damping for Noise Control in Automobiles and Commercial Airplanes, USA Symposium on Emerging Trends in Vibration and Noise Engineering, India, 2001.

Samali B., Kwok K.C.S., Use of Viscoelastic Dampers in Reducing Wind- and Earthquake-induced Motion of Building Structures, Engineering Structures, Vol. 17, No. 9, 1995, pp. 639-654.

Lima A.M.G., Stoppa M.H., Rade D.A., Finite Element Modeling of Structures Incorporating Viscoelastic Materials, Proceedings to XXl IMAC, Hyatt Orlando, Florida, 2003.

Lima A.M.G., Rade D.A., Modelling of Structures Supported on Viscoelastic Mounts Using FRF Substructuring, Proceedings of the 12th Intr. Congress on Sound and Vibration, ICSV12, Lisbon, Portugal, 2005.

Ait Brik B., Bouhaddi N., Cogan S., Neural Networks and Evolutionary Algorithm for Solving Multi-objective Optimization Problems, Proceedings of the Seventh International Conference on Computational Structures Technology, Lisbon, Portugal, 2004.

Soteris K.A., Optimization of Solar Systems using Artificial Neural Networks and Genetic Algorithms, Applied Energy, 2004, pp. 383-405.

Keys A.C., Rees L.P., Greenwood A.G., Performance Measures for Selection of Metamodels to be used in Simulation Optimization, Decis. Sci., Vol. 33, 2002, pp. 31-57.

Christensen R.M., Theory of Viscoelasticity: An Introduction, Academic Press, Inc., New York, 2nd Edition, 1982.

Lima A.M.G., Stoppa M.H., Rade D.A., Finite Element Modeling of Structures Incorporating Viscoelastic Materials, Proceedings to XXl IMAC, Hiatt Orlando, Florida, 2003.

Balmès E., Modes and Regular Shapes. How to Extend Component Mode Synthesis Theory, Proceedings of Xl DINAME, Ouro Preto, Brazil, 2005.

Balmès E., Germès S., Tools for Viscoelastic Damping Treatment Design: Application to an Automotive Floor Panel, Proceedings of the 28th Inter. Seminar on Modal Analysis (ISMA), Leuven, Belgium, 2002.

Drake M.L., Soovere J., A Design Guide for Damping of Aerospace Structures, AFWAL Vibration Damping Workshop, Vol. 3, 1984.

Masson G., Bouhaddi N., Cogan S., Laurant S., Component Mode Synthesis Method Adapted to Optimization of Structural Dynamics Behaviour, Proceedings of XXl IMAC, Hyatt Orlando, Florida, 2003.

Khatua T.P., Cheung Y.K., Bending and Vibration of Multilayer Sandwich Beams and Plates, International Journal for Numerical Methods in Engineering, Vol. 6, 1973, pp.11-24.

Lima A.M.G., Stoppa M.H., Rade D.A., Steffen Jr. V., Sensitivity Analysis of Viscoelastic Structures, Proceedings of the Xl DINAME, Brazil, 2005.

Cheng Y.F., Dan L., Multi-objective Optimization Augmented with a Robustness Function, 22nd International Modal Analysis Conference, IMAC XXll, Dearborn, USA, 2004.

Srivinas N., Deb K., Multi-objective using Nondominated Sorting in Genetic Algorithms, Technical Report, Department of Mechanical Engineering, Institute of Technology, India, 1993.

Das I., Dennis J.E., Normal-boundary Intersection: A New Method for Generating the Pareto Surface in Multicriteria Optimization Problems, SIAMJ, Optimization, 1998, pp. 631-657.