INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – May 16-20, 2016

THE NON-AXISYMMETRIC MAGNETIC SEPARATRIX IN FUSION PLASMAS

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Abstract: The magneto-hydrodynamic (MHD) equilibrium of fusion plasmas is nominally axisymmetric, presenting nested toroidal magnetic surfaces enclosed by a homoclinic separatrix. In more general situations the axial symmetry is broken by internal MHD instabilities or by the external application of 3D fields used to mitigate instabilities. In such cases, the magnetic separatrix can be redefined in terms of the perturbed magnetic saddle, leading to homoclinic lobes. In this work we present a numerical method to calculate high-resolution invariant manifolds in the Poincaré section, leading to the magnetic surface delimiting the confined field-lines for realistic tokamak discharges.

keywords: Fluidodynamics, Plasma and Turbulence, Modeling, Numerical Simulation and Optimization, Nonlinear Dynamics in Thermal and Fluid Sciences, Analysis and Control of Nonlinear Dynamical Systems with Practical Applications.

1. INTRODUCTION

In tokamak devices, an ideally axisymmetric magnetic field is created to confine a high-temperature plasma in a toroidal volume. Formally, from the divergence-free nature of the magnetic field, its streamlines can be obtained from the equations of Hamilton. Consequently, the axial symmetry must lead to the conservation of the magnetic flux along the field lines, which in turn are integrable and span well defined toroidal surfaces. In modern tokamak discharges, a magnetic saddle is created at the plasma edge and the plasma becomes limited by a homoclinic magnetic surface. This separatrix is, however, structurally unstable in three-dimensions, consequently, any arbitrary small departure from axisymmetry will break the magnetic separatrix into a pair of invariant manifolds converging or diverging from the perturbed magnetic saddle.

Experimental evidence of these invariant manifolds have been observed in numerous tokamak laboratories, by tangential imaging of single-null discharges in MAST [1] and by heat deposition patterns on the divertor tiles of DIII-D [2], JET [3], Asdex [4] and others. In a first approximation, the magnetic field lines close to the manifolds drive the charged particles to collide with the divertor tiles in coherent structures delimited by the manifold intersection with the material surfaces. Calculation of this structures by means of laminar plots evidence the intricate patterns produced by the invariant manifolds in suitable Poincaré sections [5, 6], and, consequently, the precise determination of these curves is an important factor to determine the features of the plasma edge subjected to non-axisymmetric magnetic perturbations. In this work, we introduce an efficient, adaptive method to calculate well-resolved invariant manifolds with arbitrary extension at low computational cost. The method is used to calculate the manifolds for a realistic plasma discharge reconstruction, leading to a high quality representation of the plasma edge at arbitrary scale factors with a significant reduction on the required computations.

2. CALCULATION METHOD

Assume that we solve the simplified hydromagnetic equilibrium for a tokamak discharge [7], satisfying

$$\vec{j} \times \vec{B} = \nabla p, \tag{1}$$

where $p(\vec{r})$ is the kinetic pressure, $\vec{B}(\vec{r})$ is the magnetic field and $\vec{j} = \mu_0^{-1} \nabla \times \vec{B}$ is the current density inside the plasma. The magnetic field is usually thought to be separable in the form

$$\vec{B}(\vec{r}) = \vec{B}_0(R, z) + \vec{B}_1(R, z, \phi), \tag{2}$$

where $\{R, z, \phi\}$ are cylindrical coordinates and $|\vec{B}_1| << |\vec{B}_0|$ are the asymmetric and symmetric parts of the magnetic field. $|\vec{B}_0|$ is the solution to the Grad Shafranov equation [7], which describes the axisymmetric confined plasma and $|\vec{B}_1|$ contains the field due to external and internal asymmetric currents due to the experimental setup and/or plasma response and instabilities. The magnetic field lines can be traced by solving numerically

$$\frac{dR}{d\phi} = R\frac{B_R(R,z,\phi)}{B_\phi(R,z,\phi)}\,,\; \frac{dz}{d\phi} = R\frac{B_z(R,z,\phi)}{B_\phi(R,z,\phi)}, \tag{3}$$

for given initial conditions.

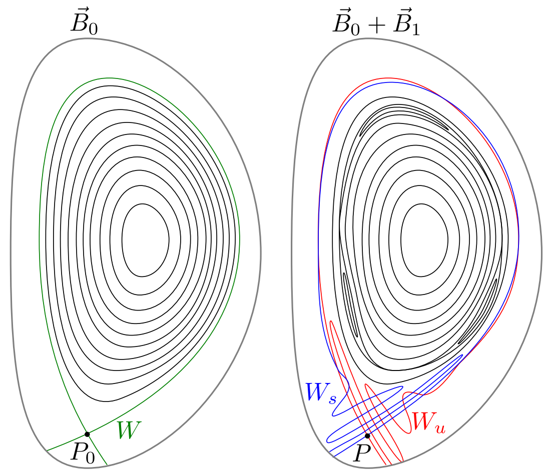

Figure 1 – Cartoon of the Poincaré section of the invariant manifolds for a single-null axisymmetric equilibrium (left) and a near-axisymmetric one (right). In the perturbed case the separatrix W splits into the stable W_s, and unstable W_u manifolds of the perturbed saddle orbit passing through P.

In Fig. 1 we show the usual magnetic topology of a single null discharge in a Poincaré plane for the axisymmetric and non-axisymmetric situations. In the axisymmetric case the field lines span nested toroidal surfaces everywhere inside the plasma delimited by the homoclinic separatrix W. When the perturbation is applied the field lines converging to, and diverging from, the saddle orbit at P, span different and mutually intersecting magnetic surfaces W_s and W_u at the plasma edge.

The first step to trace these invariant surfaces or manifolds is to locate the closed field line passing through P. For this, consider the Poincaré map $M(R_0, z_0) = (R, z)$, such that (R, z) is the point obtained by following a field line starting at (R_0, z_0) and completing one toroidal cycle. The point $P = (R^*, z^*)$ corresponding to the intersection of the closed field line with the Poincaré plane is then, a fixed point of the map, defined by $M(R^*, z^*) = (R^*, z^*)$. To find P we use the Broyden's method [8], which, ideally, requires one single calculation of the Jacobian of M by following three close field lines. If convergence is lost a new calculation of the Jacobian is performed, and the process repeats until the desired precision is obtained (typically, we require the field line to close within an error of $10^{-10}m$).

Given the fixed point P, the local behavior of the manifolds can be approximated using the eigenvectors of the jacobian matrix

$$JM_{i,j}(P) = \left.\frac{\partial M_i}{\partial x_j}\right|_{(R,z)=(R^*,z^*)}, \tag{4}$$

which are aligned with the invariant manifolds in a small neighborhood of the fixed point in the Poincaré section. For the unstable manifold, we define an elementary straight segment $U_0 = \overline{AA_1}$, where that $A_1 = M(A)$, satisfying that P, A and A_1 rest in an approximately straight line parallel to the eigenvector $\vec{v}_+$ with eigenvalue $\lambda_+ > 1$. This guarantees that both A and A_1 are in the linear region of P and is safe to say that the points in U_0 belong to W_u. The finite collection of segments $\{U_n\}_{n=0}^N$, such that $U_n = M^n(U_0)$, is a finite representation of the unstable manifold. An analogous procedure is performed for the elementary segments $\{S_n\}_{n=0}^N$ representation of the stable manifold W_s, but we use instead the inverse Poincaré map M^{-1} obtained by following the field lines in the oposite direction. Accordingly, the first segment S_0 must be aligned with the eigenvector corresponding to $\lambda_- < 1$.

The numerical integration method used to implement the Poincaré map is a Cash-Karp [9] routine with adaptive stepsize. This method was found to preserve almost exactly the magnetic flux in the axisymmetric case (with only bounded fluctuations), and provides an efficient way to approximate the field lines.

Our method for filling the segments is a flow generalization of a similar approach used for planar maps [10] where the new initial conditions are introduced only at the Poincaré plane. In our case, these are generated dinamically at any toroidal position when the field lines overcome the ressolution conditions (to be published).

3. RESULTS

To illustrate the use of this method we calculate the invariant manifolds for a Grad-Shafranov equilibrium reconstruction of the DIII-D single-null discharge $\#158826$. The magnetic perturbation in this case is caused by a set of 6 external non-axisymmetric coils with alternating currents of $\pm 2.5kA$, intended to mimic the effect of external magnetic perturbation coils (Fig. 2). This currents produce a dominant radial oscillation of the field lines near the plasma edge.

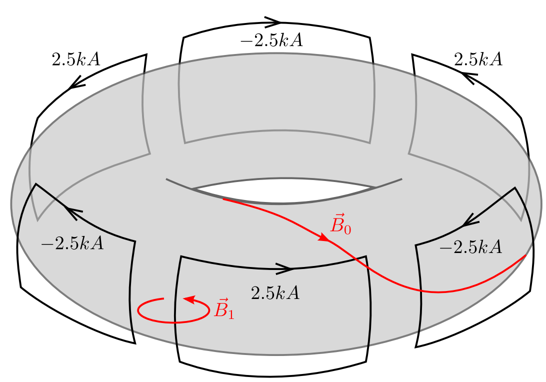

Figure 2 – Toroidal magnetic surface and perturbation coils. The helical field line oscillates radially under the influence of the external coils and the axial symmetry is lost.

Following the scheme described in the previous section, we build elementary segments $\{S_0, U_0\}$ and integrate the field lines forward for the unstable manifold and backwards for the stable. In Fig. 3 we show the Poincaré section of the invariant manifolds for this discharge and a detail of the saddle point region in the forms of a scatter-plot and a continuous line. The scatter plot shows that the points are well spaced in the manifold, but are sufficient to resolve the curve when joined, reducing the amount of points required to represent the invariant.

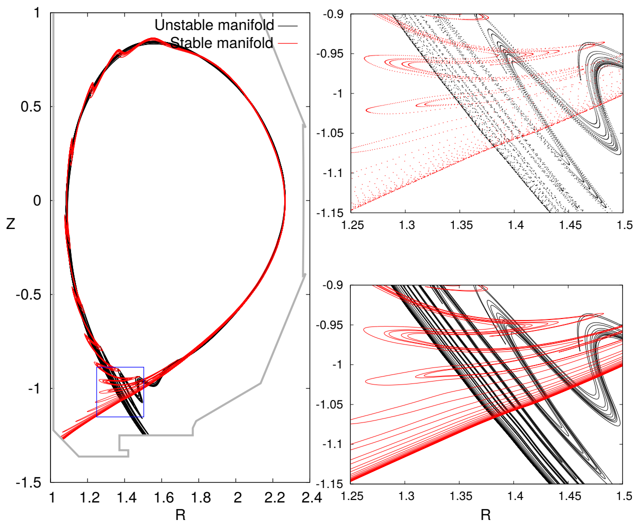

Figure 3 – Stable and unstable manifolds for a realistic MHD equilibrium subjected to $n = 3$ external magnetic perturbation. The manifolds stop where they meet the tokamak chamber (gray) where they leave a *magnetic footprint*. The saddle point detail (top-right) shows the scatter plot of points resulting from the refinement process and the continuous plot (down-right) shows the manifold resulting from joining the points.

The stable manifold is represented with 12 elementary segments and the unstable with 17. The expected homoclinic lobes develop in the high-field side for the stable manifold and the low-field side for the unstable. As the lobes get stretched towards the interior of the manifold they complete more poloidal cycles and become thinner (to preserve the toroidal magnetic flux) and return near the saddle where they are folded again. This process leads to the horseshoe mechanism that proliferates the unstable periodic orbits in the chaotic region at the plasma edge. An important signature of the perturbation nature is imprinted in the lobes that develop in groups of three per each segment, due to the mentioned field line oscillations near the perturbing loops. This method leads to an ordered sequence of points, so that to resolve the manifold we can join them, instead of requiring a large number of intermediate points. This makes it efficient and compeling for the manifold visualization.

An expected feature of the magnetic lobes is a fine structure near the saddle leading to a corresponding striation of the heat deposition patterns at the tokamak chamber. To check for this detail we magnify the strike point region in the low-field side. In Fig. 4 we show the striation resolved by this manifold tracing algorithm without requiring any additional calculations.

4. CONCLUSIONS

In this work we have introduced an efficient method for calculating, high-resolution invariant manifolds of conservative vector fields. It was implemented as a plasma edge calculator for non-axisymmetric tokamak discharges, but its principles can be used for dissipative systems as well. The obtained manifolds are well resolved with no clustering of field lines nor over-resolved regions and the resultant manifolds require less than 10% of the calculations needed by usual approaches based on the integration of a large number of initial conditions near the saddle point.

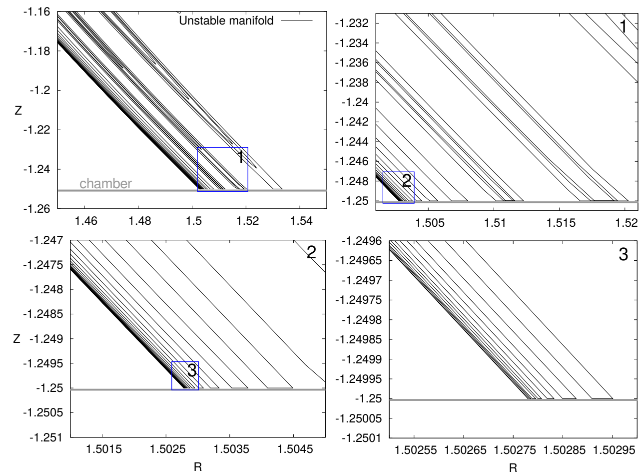

Figure 4 – Intersection of the unstable manifold with the tokamak chamber causing the striation of the strike points. The striation is resolved to a sub-millimeter scale by the present calculation method without additional computations.

5. ACKNOWLEDGMENTS

This work has been supported by Brazilian scientific agencies CAPES, CNPq and the São Paulo Research Foundation (FAPESP) under grants 2011/19269-11, 2012/18073-1 and 2014/03899-7. The authors wish to thank Dr. T. Evans and the General Atomics for providing access to MHD equilibrium reconstructions and valuable insight.

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