INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – May 16-20, 2016

VARIATIONAL ITERATION METHOD IN THE TIME FRACTIONAL BURGERS EQUATION.

A.R. Gómez Plata[1] *and E. Capelas de Oliveira*[2]

[1]Military University New Granada, Cajicá, Colombia, adrian.gomez@unimillitar.edu.co
[2]State University of Campinas, Campinas, Brazil, capelas@ime.unicamp.br

Abstract: The variational iteration method(VIM) is a analysis tool efficient for approximate non-linear fractional differential equations. Recently differents investigators are used this method in your works and we study the Lagrange multipliers of the variational iteration method for the time fractional Burgers equation and apply those in differents particular cases. In this conference we present approximations of the solutions for a particular case of the time fractional Burgers equation(BF), with the use of the variational iteration method, the Caputo derivate for $0 < \alpha \leq 1$, after make an comparation with the Adomian descomposition method(ADM).

keywords: Nonlinear Fractional Dynamics and Applications, Analysis and Control of Nonlinear Dynamical Systems with Practical Applications, Nonlinear Dynamics in Thermal and Fluid Sciences, variational iteration method, Fractional Burgers equation and Caputo derivative.

1. VIM, ADM IN THE BF

We study the VIM and ADM and your approximations for the BF. For this, we considered two intervals different in the variable x for the BF thus

$$\begin{cases} {}_c\mathsf{D}_t^{\alpha} u + u u_x - u_{xx} = 0, \quad t > 0, \quad 0 < \alpha \leq 1, \\ u(x,0) = \dfrac{2e^x}{1+e^x}, \qquad 0 \leq x \leq 1, \qquad 0 < x \leq 5. \end{cases} \tag{1}$$

First recording the approximation for the solution of the BF with ADM, where your Adomian polynomials for non-linear term uu_x and iteration formula is calculated for an initial condition arbitrary in [1]. We adjust yourself for your case in [2] and get

$$u(x,t) = \tfrac{2e^x}{1+e^x} - \left[gg' + g'' \right] \tfrac{t^{\alpha}}{\Gamma(\alpha+1)}$$
$$+ \left[2g(g')^2 + g^2 g'' + 4g'g'' + 2gg''' + g^{(4)} \right] \frac{t^{2\alpha}}{\Gamma(2\alpha+1)}. \tag{2}$$

On the other hand we have the second approximation of the VIM, where the Lagrange multipliers and correction functional iteration is give in [2] for this case by

$$u_2(x,t) = g - \left[gg' + g'' \right] \tfrac{t^{\alpha}}{\Gamma(\alpha+1)}$$
$$+ \left(2g(g')^2 + g^2 g'' + 2gg^3 + 4g'g'' + g^{(4)} \right) \tfrac{t^{2\alpha}}{\Gamma(1+2\alpha)}$$
$$-(gg' + g'') \left((g')^2 + gg'' + g^{(3)} \right) \frac{\Gamma(1+2\alpha)}{\Gamma^2(1+\alpha)} \, \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} \tag{3}$$

The exact solution is know when $\alpha = 1$ [1] by

$$u(x,t) = \frac{2\exp(x-t)}{1+\exp(x-t)}. \tag{4}$$

2. TABLES AND FIGURES

We can observe that the approximation of ADM, Eq.(2) and VIM, Eq.(3), Figures 1,2, respectively, have an approximation nearby for the exact solution, Eq.(4), Figure 3, when $0 < x \leq 1$. A comparation real is found in the Table 1, where is observe that the VIM converge more quickly for the exact solution. Nevertheless an analysis of the figures when

Table 1 – Aproximaciones del VIM contra ADM

x	t	ADM	VIM	Exacta
0.1	0.1	0.999959	0.999963	1.00000
0.1	0.5	0.797468	0.797986	0.802625
0.1	0.9	0.59099	0.59401	0.620051
0.5	0.1	1.19734	1.19736	1.19738
0.5	0.5	0.995526	0.99778	1.00000
0.5	0.9	0.775291	0.788438	0.802625
0.9	0.1	1.37993	1.37996	1.37995
0.9	0.5	1.19472	1.19769	1.19738
0.9	0.9	0.981771	0.999089	1.00000

$0 < x \leq 5$, evidence that the VIM, Figure 4 is best approximation for the exact solution, Figure 5, in comparation with the ADM approximation, Figure 6.

References

[1] S. Momami, "Non-perturbative analytical solutions of the space-and time-fractional Burgers equation", Chaos, Solitons and Fractals, Vol. 28, No.4, pp. 930-937, May 2006.

[2] A.R. Gómez Plata "Non-linear fractional differential equations ", PhD thesis in applied mathematics, Imecc-State University of Campinas, 2016.

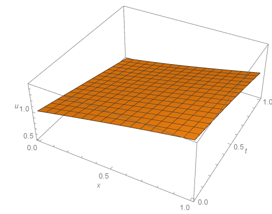

Figure 1 – Approximation ADM for BF, with $0 < x \leq 1$,$\alpha = 1.0$.

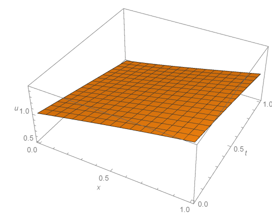

Figure 2 – Approximation VIM for BF, with $0 < x \leq 1$,$\alpha = 1.0$.

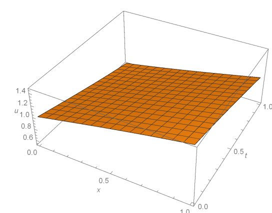

Figure 3 – Exact solution for BF, with $0 < x \leq 1$,$\alpha = 1.0$.

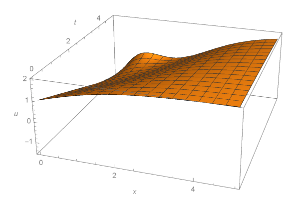

Figure 4 – Approximation VIM for BF, with $0 < x \leq 5$,$\alpha = 1.0$.

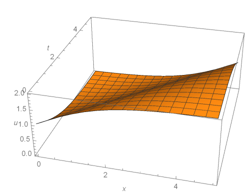

Figure 5 – Exact solution for BF, with $0 < x \leq 5$,$\alpha = 1.0$.

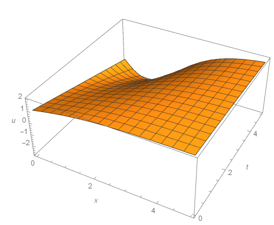

Figure 6 – Approximation ADM for BF, with $0 < x \leq 5$,$\alpha = 1.0$.