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MODELING THE ATMOSPHERIC TURBULENCE WITH INTERMITTENCY

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Abstract: The intermittency feature is considered to derive a generalize Kolmogorv's law on turbulence in the inertial subrange. The intermittency is modeled by a multifractal approach. A formulation for atmospheric eddy diffusivities is developed using the generalized Kolmogorov's spectral equation for turbulent kinetic energy and the Taylor's statistical theory on turbulence.

keywords: Intermittency, multifractal approach, Taylor's statistical theory of turbulence, eddy diffusivity, atmospheric dynamics.

1. INTRODUCTION

Expressions for eddy diffusivities considering the intermittency have developed to atmospheric turbulence [3, 4]. Those results represent the first attempt to incorporate the intermittency concept into a parameterization of the turbulence. However, those parameterizations are not dimensionally consistent. Here, new expressions for the eddy diffusivities are presented for Convective Boundary Layer (CBL) and Stable Boundary Layer (SBL).

The Taylor's statistical theory for turbulence and a generalized Kolmogorov's spectral equation for kinectic energy are employed. The intermittency is modeled by a multifractal approach.

2. MULTIFRACTALITY AND INTERMITTENCY

Some experiments have shown a desagreement with the Kolmogorov's theory on turbulence, and the concept of intermittency has been introduced to explain the observations [7]. Parisi and Frisch [10] employed a multifractal formulation to describe the intermittency.

The Kolmogorov's theory models turbulent fluxes from the statistics of the velocity gradient $v_r(x) = v(x+r) - v(x)$, for several scales r. The structure function $S_p(r)$ for inertial subrange ($\eta \ll r \ll L$) can be formulated by a multifractal approach [7]

$$\frac{S_p(r)}{v_0^p} \equiv \frac{\langle v_r^p \rangle}{v_0^p} \sim \int_I d\mu(b) \, \left(\frac{r}{L}\right)^{pb+3-D_F(b)} \quad , \qquad (1)$$

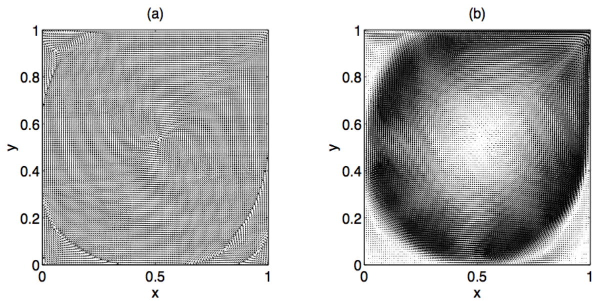
where L and η are the integral and Kolmogorov's scales, $\langle \cdot \rangle$ is the mean value, and $D_F(b)$ is the fractal dimension. The above expression will dominate by the smallest exponent, and $\zeta_p = \inf \{pb + 3 - D_F(b)\}$ is the exponent for scaling the structure function, with $b \in (b_{\min}, \, b_{\max})$. The fractal dimension in Eq. (1) can be calculated using experimental or numerical data [7]. However, analytical expressions for $D_F(b)$ can obtained (see: [1, 2, 13, 14] from the Tsallis' non-extensive thermostatistics [17].

An spectral representation for the second order of structure function (1) can be written as:

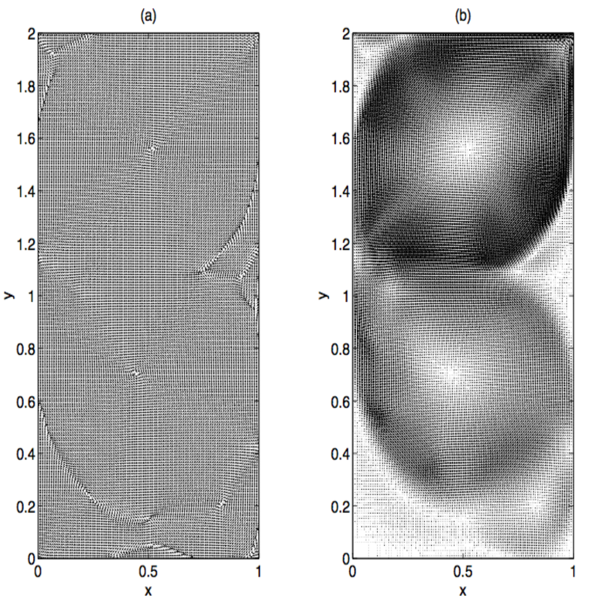
$$E(k) = c_2 \, \varepsilon^{2/3} \, k^{-5/3} \, (k/k_\eta)^{2/3-\zeta_2} \qquad (2)$$

where k_η is the wave number of a characteristic length of the system. The exponent $\nu \equiv \zeta_2 - 2/3$ can be understood as the intermittency exponent [1]. The wave number k_η represents the smallest geometrical size of the physical system considered.

Equation (2) is the spectral *generalized Kolmogorov's law* for inertial subrange. The geometrical feature into physi-



(a)



(b)

Figure 1 – Eddies simulated in the 2D driven cavity flow for different aspect ratio ($A = h/\ell$): (a) $A = 1$; (b) $A = 2$. Normalized (left) and non-normalized (right) velocity field are shown for each aspect ratio.

cal system is taking into account only if the intermittency exponent $\nu \neq 0$. Such geometrical fature could be the grid size producing turbulence [1]. An idea of how this works is shown in Figure 1, obtained from flow simulation in a 2D cavity [11, 12] for two different aspect ratios $A = \text{height}/\text{length}\ (h/\ell)$. Clearly, one eddy with diameter ℓ appears at $A = 1$, but 2 big eddies are formed for $A = 2$; showing the influence of the geometry on the flow. If the smallest geometrical scale $\ell \to \infty$ there will not be eddies formation, therefore the turbulence will not take place.

3. EDDY DIFFUSIVITY WITH INTERMITTENCY

The turbulence is a permanent feature in the atmospheric dynamics. The Taylor's statistical diffusion theory [16] can be applied to derive asymptotic expressions for Planetary Boundary Layer (PBL) for modeling the turbulence with first order closure scheme, where the Reynolds fluxes are parameterized as a product of a eddy diffusivity and the gradient of an property. Taking the large travel times ($t \to \infty$), the eddy diffusivity in a PBL can be given by [5, 6]

$$K_{\alpha\alpha} = \frac{\sigma_i^2\, \beta_i\, F_i(0)}{4}\ , \qquad \text{with} \left\{ \begin{array}{l} \alpha = x,\ y,\ z\ ; \\ i = u,\ v,\ w\ ; \end{array} \right. \tag{3}$$

where $\sigma_i^2 = \int_0^\infty S_i(n)\, dn$ is the velocity variances; $\beta_i = (\sqrt{\pi}\, U)/(4\, \sigma_i)$ is the ratio between Langrangian and Eulerian time-scales; $F_i(n) = S_i(n)/\sigma_i^2$ is the normalized Eulerian spectrum of energy.

The spectra had a central role in the Taylor's theory, since all parameters of the turbulence depend on it. A formula for spectra can be derived considering a general model, as suggested by Sorbjan [15] (see also [5, 6]):

$$\frac{n\, S_i^E(n)}{u_*^2} = \frac{A\, f^{m_3}}{(1 + B\, f^{m_1})^{m_2}}\ . \tag{4}$$

Constants A, B are determined by matching the experimental spectral peak with the maximum of the spectral model, and also fitting this model with generalized Kolmogorov's energy spectra (2). In this manner, a generalized expression for the energy spectra is obtained. However, different exponents are used to match the spectral model (4) with experimental data, for CBL: $m_1 = m_3 = 1$, and $m_2 = 1 + \zeta_2$, for SBL: $m_1 = 1 + \zeta_2$, and $m_2 = m_3 = 1$ [9].

Therefore, generalized expressions for the normalized energy spectrum for CBL and SBL are given respectively by:

$$F_i(n) = \frac{1}{(f_m)_i} \left(\frac{z}{U}\right) \left[1 + \frac{f}{\zeta_2\, (f_m)_i}\right]^{-(1+\zeta_2)} \tag{5}$$

$$F_i(n) = \frac{c_\zeta}{(f_m)_{n,i}} \left(\frac{z}{\rho_i U}\right) \zeta_2^{-1/(1+\zeta_2)}\, F_{\rm SBL} \tag{6}$$

$$F_{\rm SBL} = \left[1 + \frac{1}{\zeta_2\, (f_m)_{n,i}^{1+\zeta_2}} \left(\frac{f}{\rho_i}\right)^{1+\zeta_2}\right]^{-1}$$

where $c_\zeta = [\sin(\pi/(1+\zeta_2))]/[\pi/(1+\zeta_2)]$; $n = k\,U/2\pi$ is the relation between the wavelength k and the frequency n, U is the wind speed; $f = nz/U$ is the nondimensional frequency, z is the height above the surface; and $\rho_i = (f_m)_i\, (f_m)_{n,i}^{-1} = [z/(\lambda_m)_i]\, (f_m)_{n,i}^{-1}$ is a stability function, being $(f_m)_i$ the maximum value of the spectra, $(f_m)_{n,i}$ the maximum frequency for neutral stratification, and $(\lambda_m)_i$ is the value of the wavelength at the spectral peak.

From Eqs. (3), (5) and (6), formulations are derived for eddy diffusivities for Convective and Stable boundary layers (CBL, SBL)

$$K_{\alpha\alpha} = \frac{\sqrt{\pi}}{16}\, \frac{\sigma_i\, z}{(f_m)_i} \qquad \text{for CBL}\ ; \tag{7}$$

$$K_{\alpha\alpha} = \frac{\sqrt{\pi}}{25}\, \frac{\sigma_i\, z}{(f_m)_i} \qquad \text{for SBL}\ . \tag{8}$$

Therefore, using expressions for $(f_m)_i$ [15] formulas can be derived for eddy diffusivities. For example, the vertical eddy diffusivity for CBL is expressed as

$$\begin{aligned}\frac{K_{zz}}{w_* \, h} &= \beta^* \left(\frac{h}{\ell_\eta}\right)^{\zeta_2/2-1/3} \Psi^{1/3} \, \xi_h \\ \xi_h &= \left[1 - e^{-4(z/h)} - 3 \times 10^{-4} \, e^{8(z/h)}\right]^{1+\zeta_2/2}\end{aligned}$$

where $\beta^* = \sqrt{\pi \, c^*}/16 \, \left(1.8^{1+\zeta_2/2}\right)$, $\Psi = \epsilon \, z_i/w_*^3$ is the nondimensional molecular dissipation rate function, w_* and h are the velocity scaling [6] and the PBL height, respectively. For SBL, the vertical eddy diffusivity is written as

$$\frac{K_{zz}}{u_* \, h} = \frac{0.32 \, (1 - z/h)^{\alpha_1/2} \, (z/h)}{1 + 3.7 \, (z/\Lambda)} \left(\frac{z}{\ell_\eta \, \rho_w}\right)^{-1/3+\zeta_2/2} \tag{9}$$

being u_* the friction velocity on the ground, and α_1 is an experimental constant. The local Monin-Obukhov length Λ can be written as [8]

$$\frac{\Lambda}{L_{MO}} = \left(1 - \frac{z}{h}\right)^{3 \, \alpha_1/2 - \alpha_2} \tag{10}$$

with L_{MO} the Monin-Obukhov length for entire PBL $\alpha_1 = 3/2$ and $\alpha_2 = 1$ for fully developed SBL [8]. The parameter ℓ_η represents a characteristic length of the physical system, associated to the wavelength k_η in Eq. (2).

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