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COMPLEXITY METRIC APPLIED TO DISCRETE EVENTS SYSTEMS

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Abstract: This paper proposes the application of a complexity metric to Discrete Events Systems. Two case studies are presented: i) a distribution center problem and ii) a hospital emergency center problem, modeled in terms of entities, resources, and queues. From the simulation of scenarios with different combinations of the number of resources, the complexity is calculated based on the system's active connections. The obtained results show the relationship between complexity, the number of resources and the queue waiting time. Complexity behavior is intrinsically related to the use of more sensitive resource systems..

keywords: Complex Systems, Discrete Events Systems, Modeling, Numerical Simulation, Optimization.

1. INTRODUCTION

Radical changes in the structure and dynamics of the of the human civilization are marks of the complexity increase. In response to the challenges arising from the relationship with the environment, the system's separate parts, whose behavior are independent and partially correlated, unite to perform complex collective actions [1].

The system's behavior in time can be investigated in order to verify regularities, relations, hierarchy, among other features. In many systems, is found a high variability in its parameters, a characteristic that defines them as complex for Per Bak [3]. Simulations using Discrete Event System Theory, contribute to check how the modeled systems behave when some event occurs.

In physical, biological and social systems out of balance, the system's behavior is preserved by the flux of energy, where entities require resources from the environment for their survival [1]. Similarly, Discrete Event Systems are modeled in terms of entities, resources, and queues. Queues, when present, usually indicate entities' demand for resources [7].

In order to define how complex a given system is, various metrics have been developed in relation to the degree of: i) difficulty of description, ii) difficulty of conception or iii) organization. These metrics are based on the systems' dimension, entropy, hierarchy, organization, among other criteria [6]. When calculating complexity, one usually desires to obtain numerical values that serve to compare different configurations of the same system or similar systems. Therefore, the complexity measure is dimensionless [8].

To measure the systems complexity, Lemes [10] fits the entropy metric in information exchange defined by Shannon [9] based on Statistical Mechanics studies. The proposed methodology in this paper is based on Lemes' modeling [10] to measure the complexity of real systems, however focusing on system connections.

The complexity metric is applied to two systems: i) a Distribution Center and ii) a Hospital Emergency Center. The Center Distribution problem consists of a delivery logistics, modeled in terms of entities (requests), queues and resources (docks, trucks, chargers) [7]. The Hospital Emergency Center problem consists of the initial care to patients (entities) with health problems, performed by doctors, nurses and examination employees (resources), possibly generating queues during the process steps of: i) medical appointment,

ii) medication and iii) exams.

The purpose of this work is to apply a complexity metric based on connections to Discrete Event Systems. Section 2 shows the proposed methodology, followed by Section 3, where the results are showed. Finally, Section 4 presents some conclusions on the topic.

2. PROPOSED METHODOLOGY

2.1. Complexity Metric

Based on Lemes' modeling [10], the proposed method measures real systems' complexities, considering the active connections at a certain time t. These connections are expressed by the matrix of relationship M among entities, resources, and queues, which are used to calculate the complexity $\gamma(s)$ of any system by expressions (1) and (2).

$$\gamma(s) = -\sum_{i=1}^{\rho} p(c)_i \cdot \, log_2 \, p(c)_i \ , \tag{1}$$

$$p(c)_i = \frac{1}{e \cdot (n_r + n_q)} \ , \tag{2}$$

where e is the number of entities that each resource can attend, n_r is the number of resources, n_q is the number of queues, $p(c)_i$ is the probability of occurrence of connection i and ρ is the number of active connections at the moment t, expressed by (3).

$$\rho = \sum_{j=1}^{k} n_{c_j} \cdot n_{e_j} \ , \tag{3}$$

where k is the number of entity states, n_{c_j} is the number of active connections per entity at j^{th} state and n_{e_j} is the number of entities at j^{th} state.

2.2. Distribution Center Model

The Distribution Center receives requests to transport to the recipient. So the delivery time t_d of each request matches the sum of: i) waiting time in queue t_{r_q}, ii) loading time t_{r_l} and iii) transportation time between the Distribution Center and the receiver t_{r_d}. For this case study, the delivery time t_d is considered as the performance metric.

The complexity measure is calculated by expression (1). The number of active connections ρ of the Distribution Center is mapped based on states: i) queued, ii) loading and iii) traveling. The number of active connections is determined by (3), and each request in queue adds one connection to the system (connected to the next request), each request being loaded adds three connections (one to the dock, one to the truck and one to a loaders team) and each request being transported adds one system connection (to the truck).

The likelihood of a connection in the Distribution Center Problem is given by the expression (2). Considering that each entity (request) can be attended by any system resource, the number of entities e that each resource can attend is equal to the total number of requests in the system at the time t.

Considering a configuration of 5 requests with 1 dock, 2 trucks and 1 group of loaders, at some moment t of the system, a relationship matrix M can be created, expressed by (4), where the columns represent the requests, R_1 to R_5, and the lines represent the queue (Q) and the resources: dock (D), trucks (T_1 and T_2) and group of loaders (G), respectively.

$$M = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

In (4), it is verified that the request R_1 is being transported by truck T_2, the request R_2 is being loaded by group of loaders G to truck T_1 parked at dock D and requests R_3, R_4 and R_5 are waiting in queue. It is emphasized that the number of active connections can be computed from matrix M.

2.3. Hospital Emergency Center Model

The Hospital Emergency Center consists of the processes of medical care and basic procedures until the patient hospitalization or discharging from the hospital. The patients are modeled as system entities and the doctors, nurses and examination employees are modeled as available resources.

During the initial care process, queues can be formed by the following steps: i) medical appointment, ii) medication and iii) exams. In this case study, the time t_p between a patient's arrival and their discharge or hospitalization is adopted as the performance measure.

The complexity measure is calculated by expression (1). The number of active connections ρ of Hospital Emergency Center are mapped based on states i) waiting in queue, ii) on medical consultation, iii) receiving medication and iv) doing exams. The number of active connections is determined by (3) and each patient adds just one system connection, no matter which state they are in. Patients in queue connect to the patients in front of them, patients in medical consultation connect to one doctor, patients receiving medication connect to one nurse, and patients doing exams connect to one examination employee.

The connection likelihood in the Hospital Emergency Center is given by expression (2), where the number of entities e that each resource can attend is equal to the total number of patients in the system at the moment t, considering that each entity (patient) can be attended by any system resource. Therefore, e is the sum of the entities (patients) in each state. The number of resources n_r is equal to the sum of the number of doctors, nurses, and examination employees. The number of queues n_q is equal to four, i.e, i) queue for medical consultation on arrival Q_1, ii) queue for medication Q_2, iii) queue for examination Q_3 and iv) queue for medical consultation on return after basic procedures Q_4.

Considering a configuration of 7 patients, 2 doctors, 2 nurses and 1 examination employee, the relationships are represented in matrix M (5). Each column represents each patient, from P_1 to P_7, and the lines represent the queues (from Q_1 to Q_4) and the resources: doctors (C_1 e C_2), nurses (N_1 e N_2) and examination employee ($E1$), in this order.

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

In (5), it is verified that patient P_1 is doing exam with the employee E_1, patients P_2 and P_6 are receiving medication by nurses N_1 and N_2, patient P_3 is waiting in queue Q_3 to be examined, P_4 e P_5 are consulting with doctors C_1 and C_2, and P_7 is waiting in queue Q_1 to get a medical consultation. The number of active connections in the system can be computed from matrix M.

3. RESULTS

3.1. Case Study 1: Distribution Center

Distribution Center model, presented in Section 2.2, was simulated with the following dynamics: 1) the arrival of requests occur at time intervals t_1; 2) loading of each truck lasts a time interval t_2; 3) when a truck leaves for delivery, the dock and the group of loaders are released for new loading; 4) transporting each request to the destination lasts a time interval t_3; 5) after delivery the empty truck returns to the distribution center at a time t_4.

Time t_1 was generated from an exponential distribution, with a mean rate of 120 minutes. Time t_2 was generated from a normal distribution with mean of 100 minutes and standard deviation of 30 minutes. Times t_3 and t_4 were generated following a uniform distribution within the range of 120 to 240 minutes.

The number of docks, trucks and groups of loaders used in the simulation ranged between 1 to 10, 1 to 15 and 1 to 10, respectively, therefore featuring $1,500$ different scenarios. The simulation was performed with 180 days for each scenario, considering 24 daily hours of operation and every truck being loaded with only one request at a time.

Figure 1 shows that the complexity $\gamma(d)$ (in red) has oscillations that follow the shape of the truck utilization curve (in green), i.e. the scenarios that contain peaks of truck utilization coincide with the scenarios that present peaks of complexity. However, the peaks of complexity in the first 700 scenarios range from 0.5 to 1.0 while in the following scenarios they range from 0.3 to 0.5, due to the high amount of available resources and therefore fewer queues.

It can be observed from Figure 1 that the utilization of the truck resources (in green) does not change with the increase in the number of docks (in blue) or groups of loaders (in gray).

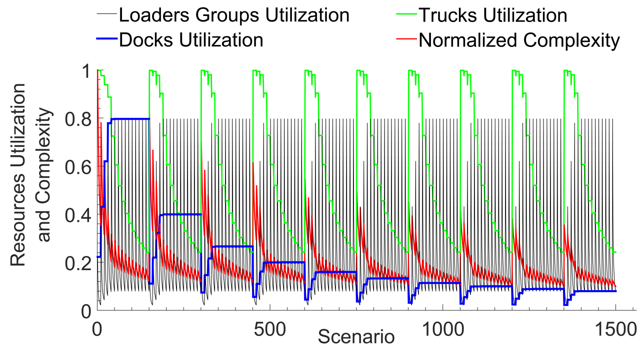

Figure 1 – Relation between the use of resources and complexity for the Distribution Center

3.2. Case Study 2: Hospital Emergency Center

The model of the Hospital Emergency Center presented in Section 2.3 was simulated with the following procedure flow: 1) the patients arrive at the Emergency Center in time intervals t_5; 2) patients wait in queue Q_1 for medical assistance; 3) patients get the medical appointment for a time interval t_6; 4) after the appointment, 1% of patients are released and 99% are forwarded to 5) perform basic procedures, wherein 6) 40% of patients wait in queue Q_2 for a period of time t_7 in order to receive medication and 7) 60% of patients wait in queue Q_3 for exams that last time t_8; 8) after medication or exams, 10% of patients are released, 40%, perform new tests or take more medication, returning to step 5, and 50%, wait in queue Q_4 to return to the doctor, while a new assistance is conducted for a time interval t_9; 9) after a new medical appointment, 40% of patients are directed to further exams/medication, returning to step 5, and the other 60% leave the system, 40% being hospitalized and 20% being released.

Time t_5 was generated from an exponential distribution with an average rate of 5 minutes. Times t_6, t_7, t_8 and t_9 were generated from normal distributions with means of 15, 20, 15 and 10 minutes, respectively, and standard deviation of 5 minutes.

The number of doctors, examination employees and nurses used in the simulation ranged from 3 to 8, 2 to 10 and 5 to 15, respectively, featuring therefore 594 different scenarios. The simulation was performed for 180 days for each scenario, considering 24 daily hours of operation. Time in minutes spent in the Emergency Center t_p and complexity $\gamma(p)$ of the system were calculated in the 594 simulated scenarios.

The smallest complexity obtained from all scenarios, $\gamma(p) = 0.2407$, occurred with the maximum number of resources, i.e., 8 doctors, 15 nurses and 10 examination employees. The largest complexity, $\gamma(p) = 0.8105$, occurred when there was the minimum number of resources, i.e., 3 doctors, 5 nurses and 2 exam employees, accordingly more patients waiting in queues. Figure 2 shows that the smallest complexity is obtained at the last scenario and the largest complexity, at the first scenario.

Figure 2 presents the percentage values of resource utilization and the measure of normalized complexity for all simulated scenarios. It can be observed that in the 200 initial scenarios, where there were less than 5 doctors, the utilization of this resource had values between 0.9 and 1. The complexity values were lower when there were more resources, and the peaks of complexity occured whenever the number of doctors was increased, but there was a minimum number of nurses and examination employees, increasing the size of the queues Q_2 and Q_3.

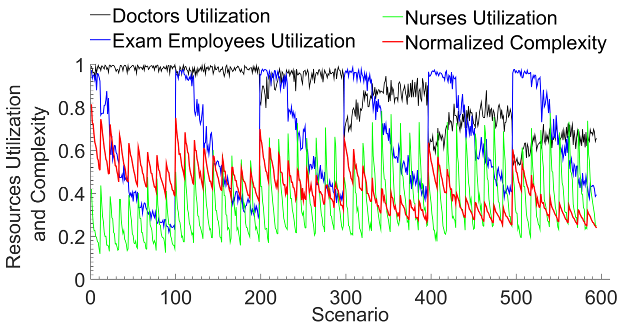

Figure 2 – Relation between resource utilization and complexity for the Hospital Emergency Center

In the scenario with the smallest complexity, the average time t_p that patients were in Hospital Emergency Center was approximately 14.9, 58.7, 71.13 and 74.07 minutes for those who left: 1) without examination or medication, 2) after being examined and/or taking medication, 3) after a new medical appointment and 4) for hospitalization, respectively. For the scenario with the largest complexity, the times were 774.68, 1032.20, 1191.25 and 1128.28 minutes, respectively, due the queuing time.

4. CONCLUSION

This paper has presented a methodology for the calculation of systems' complexity based on existing active connections between its elements. In the problem of the Distribution Center, the measure of complexity associated with the resource utilization rate pointed the amount of trucks as the most sensitive parameter of the system. In the problem of the Hospital Emergency Center, it was found that the complexity decreases as the availability of resources increases, because with more resources patient care occurs with less time, resulting in less queuing.

References

[1] BAR-YAM, Y. Complexity rising: From human beings to human civilization, a complexity profile. Cambridge: NECSI Report, 1997.

[2] E. Rechtin and M. W. Maier, *The art of systems architecting*. CRC Press, 2010.

[3] P. Bak, *How Nature Works: the science of self-organized criticality*. Springer New York, 1996.

[4] J. H. Holland, *Complexity: A very short introduction*. Oxford University Press, 2014.

[5] H. A. Simon, "The architecture of complexity," *Proceedings of the American Philosophical Society*, vol. 106, no. 6, pp. 467–482, 1962.

[6] S. Lloyd, "Measures of complexity: a nonexhaustive list," *IEEE Control Systems Magazine*, vol. 21, no. 4, pp. 7–8, 2001.

[7] L. Chwif and A. C. Medina, *Modeling and simulation of discrete events (in portuguese)*. Campus-Elsevier, 4 ed., 2014.

[8] D. W. Repperger, R. G. Roberts, and C. G. Koepke, "Quantitative measurements of system complexity," Aug. 8 2012. US Patent 8,244,503 B1.

[9] C. E. Shannon, "A mathematical theory of communication," *The Bell System Technical Journal*, vol. 27, pp. 379–423, 623–656, 1948.

[10] M. J. R. Lemes, *Complexity, coupling and criticality (C^2A) as risk factors in system design (in portuguese)*. PhD thesis, University of São Paulo, 2012.