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MULTIOBJECTIVE OPTIMIZATION APPLICATION IN DOE PROBLEMS WITH MULTIPLE RESPONSES

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Abstract: In many areas, from Engineering to Economics, problems present themselves as multiobjective, which makes a decision-making process complex. Generally, these are conflicting objectives, and optimization techniques are necessary to achieve better results. This paper applies agglutination methods in classical problems of design of experiments with multiple responses. A theoretical review was made, and a new method was developed, using Compromise Programming and Goal Programming, with results comparison and analysis. The new proposal presented better results when compared to the traditional approach, qualifying this procedure as an alternative in multiple responses optimization.

Keywords: Epidemiology and Mathematical Models, Modeling, Numerical Simulation and Optimization, Nonlinear Systems and Neural Dynamics, Goal Programming, Compromise Programming.

1. INTRODUCTION

Since 1980, the searching for methods that can assure quality with scarce resources by considering not only the target value, but also the variance, has grown in relevance. A major part of it has been directed to alternative methods to a efficient determination of optimal arrays in order to achieve a given target [1]. Given so, the decision-making process for improvement of industrial processes is an example of such a situation, since there are several variables involved, independent and dependent ones, which are associated to the same process.

Nevertheless, optimization techniques that could be used to do achieve such improvements presents a gap between theory and practice, since these techniques are not usually applied to real life problems in which they would have great impact. This occurs mainly because the majority of real life problems are multiobjective, and also the input data for these aren't know in advance. For modelling and optimizing such problems there are two most common methods: experimentation and optimization [2].

Experimentation is done, most frequently, by Design of Experiments – DOE, that uses mathematical models from the process analyzed. Optimization aims to find the best solutions for multiresponse problems and there can be two approaches: agglutination and priorization [3].

Along all the tools used in DOE, the Response Surface Methodology – RSM is efficiently applied in optimization and evaluation of meaningfulness of several factors affected by complex interactions, and by doing so has the objective of finding the optimal conditions in a given process [4]. Besides, explain that the application of DOE aims to reduce the amount of experiments in modelling and optimization and, between the diverse agglutination methods that are most commonly used in DOE problems, the Desirability agglutination function has great prominence.

The authors of [5], although, discuss that by using the agglutination method called Compromise Programming – CP it is possible to achieve better results when comparing to the Desirability method, for multiple responses experimental problems. In addition to these two, there is a third method called Goal Programming – GP.

Being the Desirability method the most used in companied nowadays and presented in softwares such as Minitab®, few is available about the application of another multiobjective optimization methods, what means there is a gap between the practical applications and the state-of-the-art theory already available [6]. Given this, it brings the question: can the optimization methods CP and GP achieve better results when compared to the traditional method (desirability)?

2. PURPOSE

The general objective of this work is to propose new strategies for the optimization of multiple response (multiobjective) DOE problems.

The specific objectives are:

- To compare the results obtained by the methods Desirability, Compromise Programming – CP and Goal Programming – GP.
- To compare, for the same model/method, the results obtained by the Generalized Reduced Gradient – GRG algorithm and by the Optquest's software metaheuristics (which combines tabu search, scatter search and neural networks in one inexact algorithm).

This work is limited to the study of classic models and only a few from recent papers, being all the models nonlinear, deterministic and multiobjective [7].

3. METHODS

The research was done following the steps presented in Figure 1, which are further explained.

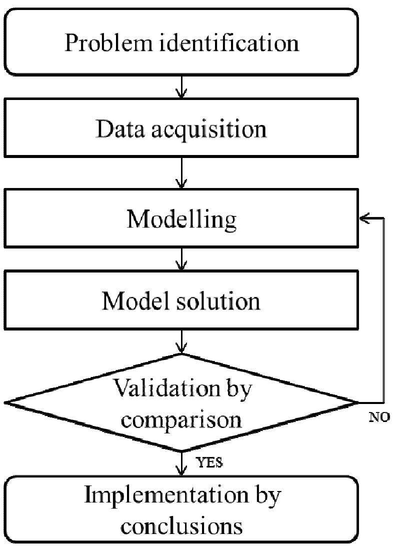

Figure 1 – Research steps

Research steps:

1. Data acquisition for the tests – it will be used information from the works of Derringer e Suich (1980) [8] and Khuri e Conlon (1981) [9], related to real life experiments.
2. Modelling – the modelling will be made using the softwares MS-Excel® and Crystal Ball®. The implemented models will be determined by crossing the two models obtained from the problems concerned in the papers mentioned above with each of the optimization methods: Desirability, CP e GP. All models are nonlinear, with differences between each other, like amount of objective functions and different scales.
3. Model solution – the models developed in step 2 will be solved using the GRG algorithm (MS-Excel® software) and Crystal Ball® Optquest software metaheuristic.
4. Results comparison – the comparative performance evaluation between multiobjective optimization methods and algorithms will be made by calculating the Average Percentage Deviation – APD [10].
5. Conclusions about the models and algorithms – step 4 allow analyzing how close to the target value of each objective function each of the models was able to get. By doing so, it will be possible to discover which method is more efficient in achieving the goal.

4. RESULTS AND DISCUSSION

For solving the CP models Oracle Crystal Ball® Optquest software was used (establishing a limit of 20.000 runs, for which the data processing mean time was ca. 105 seconds), test version, besides the GRG algorithm from MS-Excel® software, using a PC Intel Core i7-3612QM 2,10 GHZ with 8 GB RAM and MS-Windows 64 bits operational system.

Table 1 shows the results obtained by CP function compared with the ones from desirability function Derringer and Suich, considering all answers have the same relevance (weights for each are 25%). Tables 2 and 3 do comparisons between the same functions, but with different weights, being $\alpha_1 = 0{,}3$; $\alpha_2 = 0{,}25$; $\alpha_3 = 0{,}25$; $\alpha_4 = 0{,}20$ for Table 2 and $\alpha_1 = 0{,}15$; $\alpha_2 = 0{,}22$; $\alpha_3 = 0{,}27$; $\alpha_4 = 0{,}35$ for Table 3. For all these Tables, the variables values were kept the same, in order to analyze the impact that changing the weights would have in CP results. The analysis of the deviation variables influence (d_i for Desirability, d_i^+ e d_i^- for GP), as also the α and β weights applied to the GP deviation variables was not done at this moment. For the same reason, occurs the repetition of the values from Desirability column in Tables 4 and 5, while in GP values column (GRG algorithm) there is a difference between them, because it was applied a different value for weight β_i.

Tables 4 and 5 presents the CP results for the Khuri and Conlon Desirability application. To calculate the Average Percentage Deviation – APD, in order to compare the results from CP and GP with the ones from Desirability, we used Equation 1 as follows:

$$DPM = \left(\sum_{i=1}^{p} \left| R_i - T_i \right| \Big/ T_i \right) \Big/ p \qquad (1)$$

Where R_i is the final value of the optimization for the answer i, T_i the target value associated with the answer i and p the number of analyzed answers.

Table 1. CP and GP (no weights applied) outputs comparison with Derringer and Suich Desirability approach – CP applies equal weights: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 25\%$.

Variables	Optimization Results				Targets	Answers' APD			
	***Desirability Function* (GRG)**	**CP (OptQuest)**	**CP (GRG)**	**GP (GRG)**		***Desirability Function***	**CP (OptQuest)**	**CP (GRG)**	**GP (GRG)**
x_1	-0,05	-0,672	-0,685	-0,702	-	-	-	-	-
x_2	0,145	0,772	0,940	0,935	-	-	-	-	-
x_3	-0,868	0,630	0,626	0,644	-	-	-	-	-
Y_1	129,50	142,38	144,18	143,79	170,00	0,238235	0,1625	0,1519	0,1542
Y_2	1.300,00	1.301,14	1301,1	1300,00	1.300,00	0	0,0009	0,0008	0,0000
Y_3	465,70	497,19	497,02	500,00	500,00	0,0686	0,0056	0,0060	0,0000
Y_4	68,00	74,09	75,00	75,00	67,50	0,007407	0,0976	0,1111	0,1111
DPM	-	-	-		-	**7,86%**	**6,665%**	**6,745%**	**6,633%**

Table 2. CP and GP (no weights applied) outputs comparison with Derringer and Suich Desirability approach – CP has applies the following weights: $\alpha_1 = 0,3$; $\alpha_2 = 0,25$; $\alpha_3 = 0,25$; $\alpha_4 = 0,20$.

Variables	Optimization Results				Targets	Answers' APD			
	***Desirability Function* (GRG)**	**CP (OptQuest)**	**CP (GRG)**	**GP (GRG)**		***Desirability Function***	**CP (OptQuest)**	**CP (GRG)**	**GP (GRG)**
x_1	-0,05	-0,683	-0,682	-0,702	-	-	-	-	-
x_2	0,145	0,941	0,941	0,935	-	-	-	-	-
x_3	-0,868	0,623	0,622	0,644	-	-	-	-	-
Y_1	129,50	144,23	144,25	143,79	170,00	0,238235	0,1516	0,1514	0,1542
Y_2	1.300,00	1301,28	1301,31	1300,00	1.300,00	0	0,0010	0,0010	0,0000
Y_3	465,70	496,61	496,45	500,00	500,00	0,0686	0,0068	0,0071	0,0000
Y_4	68,00	75,00	75,00	75,00	67,50	0,007407	0,1111	0,1111	0,1111
DPM	-	-	-		-	**7,86%**	**6,761%**	**6,767%**	**6,633%**

Table 3. CP and GP (no weights applied) outputs comparison with Derringer and Suich Desirability approach – CP has applies the following weights: $\alpha_1 = 0,15$; $\alpha_2 = 0,22$; $\alpha_3 = 0,28$; $\alpha_4 = 0,35$.

Variables	Optimization Results				Targets	Answers' APD			
	***Desirability Function* (GRG)**	**CP (OptQuest)**	**CP (GRG)**	**GP (GRG)**		***Desirability Function***	**CP (OptQuest)**	**CP (GRG)**	**GP (GRG)**
x_1	-0,05	-0,685	-0,693	-0,702	-	-	-	-	-
x_2	0,145	0,837	0,938	0,935	-	-	-	-	-
x_3	-0,868	0,636	0,634	0,644	-	-	-	-	-
Y_1	129,50	142,92	144,01	143,79	170,00	0,238235	0,1593	0,1529	0,1542
Y_2	1.300,0	1300,92	1300,74	1300,0	1.300,00	0	0,0007	0,0006	0,0000
Y_3	465,70	498,39	498,38	500,00	500,00	0,0686	0,0032	0,0032	0,0000
Y_4	68,00	74,45	75,00	75,00	67,50	0,007407	0,1030	0,1111	0,1111
DPM	-	-	-		-	**7,86%**	**6,655%**	**6,696%**	**6,633%**

Table 4. CP and GP (no weights applied) outputs comparison with Khuri and Colon Desirability approach – CP has applies the following weights: $\alpha_1 = 0,36$; $\alpha_2 = 0,02$; $\alpha_3 = 0,02$; $\alpha_4 = 0,60$.

Variables	Optimization Results				Targets	Answers' APD			
	***Desirability Function* (GRG)**	**CP (OptQuest)**	**CP (GRG)**	**GP (GRG)**		***Desirability Function***	**CP (OptQuest)**	**CP (GRG)**	**GP (GRG)**
x_1	-0,219	-0,362	-0,689	-1,000	-	-	-	-	-
x_2	-1,000	-1,000	-1,000	-1,000	-	-	-	-	-
Y_1	2,14	2,25	2,49	2,67	2,68	0,2015	0,1594	0,0723	0,0022
Y_2	0,61	0,61	0,58	0,54	0,69	0,1159	0,1197	0,1571	0,2203
Y_3	1,82	1,84	1,87	1,86	1,69	0,0769	0,0896	0,1043	0,1000
Y_4	0,37	0,34	0,28	0,23	0,71	0,4789	0,5141	0,6002	0,6746
DPM	-	-	-	-	-	**21,83%**	**22,07%**	**23,35%**	**24,93%**

Table 5. CP and GP (applied a weight of 4,0 linked to the deviation variable associated with a "better than the target" achievement) outputs comparison with Khuri and Colon Desirability approach – CP has applies the following weights: $\alpha_1 = 0{,}46$; $\alpha_2 = 0{,}02$; $\alpha_3 = 0{,}02$; $\alpha_4 = 0{,}5$.

Variables	Optimization Results				Targets	Answers' APD			
	Desirability Function (GRG)	CP (OptQuest)	CP (GRG)	GP (GRG)		*Desirability Function*	CP (OptQuest)	CP (GRG)	GP (GRG)
x_1	-0,219	-0,429	-0,782	-1,000	-	-	-	-	-
x_2	-1,000	-1,000	-1,000	-1,000	-	-	-	-	-
Y_1	2,14	2,30	2,55	1,6718	2,68	0,2015	0,1405	0,0501	0,3762
Y_2	0,61	0,60	0,57	0,5970	0,69	0,1159	0,1249	0,1732	0,1348
Y_3	1,82	1,85	1,87	1,6900	1,69	0,0769	0,0942	0,1049	0,0000
Y_4	0,37	0,33	0,27	0,4848	0,71	0,4789	0,5323	0,6232	0,3172
DPM	-	-	-	-	-	**21,83%**	**22,30%**	**23,78%**	**20,70%**

5. CONCLUSION

It was possible to notice that the results achieved using CP and GP presented improvement when comparing to the Desirability function classic results. For Derringer and Suich case, it is possible to verify that the outputs obtained using GRG algorithm, besides its improvement, were not better than the results achieved by using the Optquest's metaheuristic. This happens because the GRG has a chance of getting stuck in a local optimality, which means it may not achieve the global optimality.

Either way, the GP method is prominent, once it presented the better results for all cases, even by using GRG. For Khuri e Conlon cases, similar results are observed, and GP-GRG method presents the lower APD, followed by CP-Optquest. Also, for GP, the application of weights generated a good trade-off relationship between objectives. This way, it is up to the manager to assign these weights, aiming a more balanced solution.

The GP achieves better results by including constraints that are flexible, associsated to the objectives, and also deviation variables related to achieving a "higher than" or "lower than|" the target output, that will compose a new objective function, allowing that not all constraints are considered fixed, and that is what differs GP from classical optimization. This characteristic allows that real life situations, where the RHS (resources available) used are distinct from the ones determined before, and can only be treated by deviation variables incorporated to GP modelling.

Therefore, it concluded that this work reached its objectives by proposing a new GP-GRG method and comparing the performance from the classical Desirability function cases with the ones from CP, OptQuest and GRG, and GP GRG.

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