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PARAMETRIC DYNAMICS OF AN EULER-BERNOULLI BEAM

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Abstract: In this contribution we investigate the dynamics of vibration of non uniform Euler-Bernoulli Bean. The bean cross area is a linear function of x. The time dynamics of normal modes is similar of parametric oscillator, we use the approximated analytical solutions compare with two different numerical approaches.

keywords: Chaos and Global Nonlinear Dynamics, Modeling, Numerical Simulation and Optimization, Nonlinear Dynamics of Systems with Infinite Dimension.

1. INTRODUCTION

Mechanical Vibrations have a huge application scope, from Mechanics to Biology, also, any advance in this area can produce a reflex in other interdisciplinary applications of it, this justify the existence of a study area related to it. In this contribution, we investigate the dynamics of a non uniform beam. Non uniform beams also have been extensively studied in literature with lot of good results, see [1]. They have a large application range, such as mechanical projects present in aero spacing devices, machines, ships, civil structures and others. Stress tension is one of most important motivation, since it can be the source of fails and accidents [2].

The beam is non uniform and the cross area varies linearly with x, a reason between initial area and final is the parameter σ that quantifies its asymmetry. The spatial solution was obtained using the Differential Transform Method (DTM) [4], and we found a good agreement with known results available in the literature [2]. The temporal solutions where obtained by three different methods, analytical solutions for small oscillations, and numerically with four other Runge Kutta (RK) and with DTM. The solutions quality were compared and we define a range of applicability of them, this is the main result of this work.

In our model, we consider a periodic external force that induce parametrical oscillations, also we consider dissipation. In absence of dissipation, the equations are of Mathieu type and Landau solutions for small oscillations are applicable [3].

2. THE MODEL

We model the beam in terms of Bernoulli-Euler equation, i.e.:

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2\Psi}{\partial x^2}\right)+\rho A(x)\frac{\partial^2\Psi}{\partial t^2}=f(x,t). \quad (1)$$

where Ψ is the deformation measured from neutral line, L is the length of the beam, x is the coordinate in the x direction, E is the Yong modulus of the material, I (x) is the moment of inertia cross section of the beam around the y axis passing through the centroid, A(x) is the cross-sectional area, ρ is the material density and f(x, t) is an external load per unit length. We are considering the case where the solution can be written as Ψ(x,t) = v(x)u(t), the spatial normal modes (v(x)) can be found in somewhere else [5], we are considering the natural frequencies and modes as an input to temporal model.

3. PARAMETRIC DYNAMICS

We are considering a periodic harmonic external force and a dissipative force, thus the temporal part of equation (1) can be written as

$$\ddot{u}+\delta\dot{u}+[\omega^2-\varphi\cos(\theta t)]u=0 \quad (2)$$

where δ is the dissipation straight, φ is the amplitude of disturbing force and θ the frequency of disturbance; ω is the natural frequency of the beam, which was obtained in spatial resolution for beam concerned [5].

We solved numerically equation (2) using two methods, four order RK and DTM. Results are represented in figure 1.

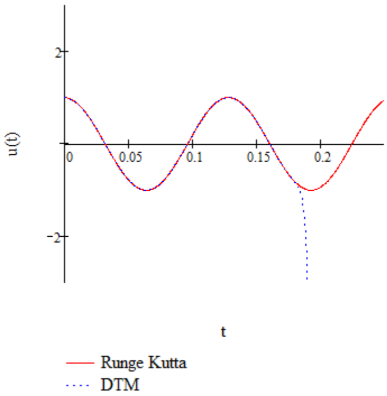

Fig 1 - u(t) solutions of eq. (1) using RK and DTM. We used δ = φ = 0,001, ω = 48,824 and θ = 2ω

As we can observe in figure 1, there is a good agreement between RK and DTM only for short times, what was expected since DTM does not work well for large range of variables. As comparing reference, we consider the non dissipative regime, thus we can identify equation (2) as a Mathieu equation, in this regime we have

$$\ddot{u} + \omega^2[1 + \eta\cos(\theta t)]u = 0 \qquad (3)$$

Equation (3) can be approximately solved using Landau method that works well for small oscillations [3]. In this regime, we compare Landau solutions with RK and DTM solutions. Some results are represented in figure 2.

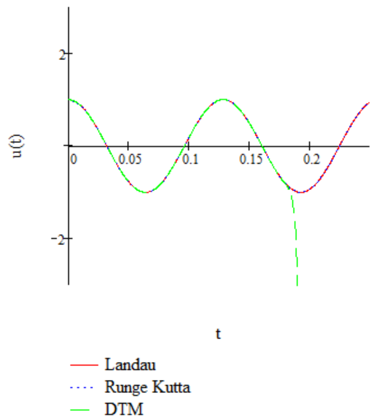

Fig 2 - u(t) solutions of eq. (1) using RK and DTM and analytical Landau result. We used δ = φ = 0,001, ω = 48,824 and θ = 2ω.

We defined the error of numerical solutions considering Landau solutions as a reference, in figure (3) we have the mean error as a time function. As we can observe, DTM works well only for short times and RK converges asymptotically to Landau solution. For large external forces, Landau solution is not applicable thus only numerical methods are reliable.

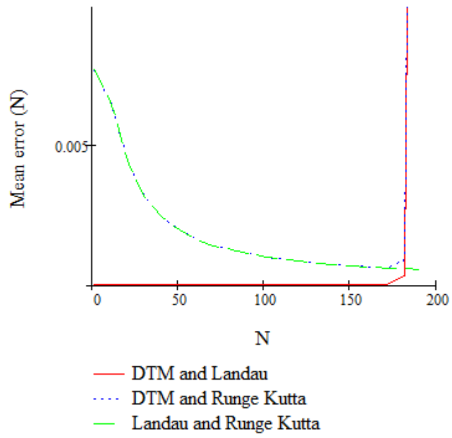

Fig 3 – Mean deviation from solutions of eq. (1), comparing DTM and Lanfdau, DTM and RK, Landau and RK. We used δ = φ = 0,001, ω = 48,824 e θ = 2ω

Finally, we investigate the case where φ = 0 and we can analytically solve equation (3). In figure (4) we compare numerical solutions with the analytical result.

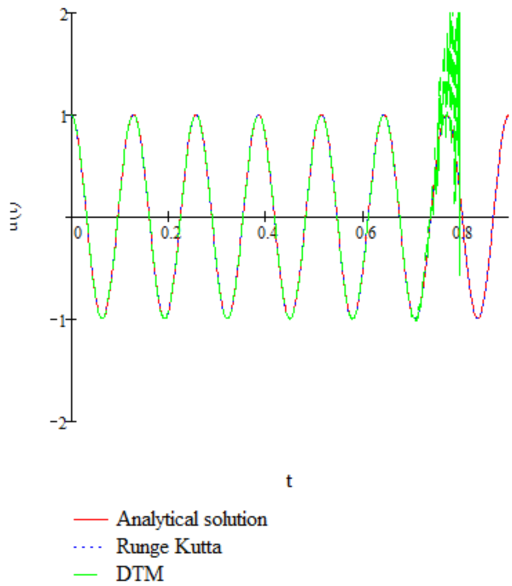

Figura 4 - - u(t) solutions of eq. (1) using RK, DTM and Landau. We used δ = φ = 0,001, ω = 48,824 e θ = 2ω

Surprisingly, DTM results fits better the analytical solutions for short time, what does not happens for longer time, as we can see in figure (4). The model dynamics is strongly dependent of the parameters, specially of **φ**. In figure (5) we show the same as figure (2) for a larger **φ,** as we can observe on it, the stability of DTM solution is lost.

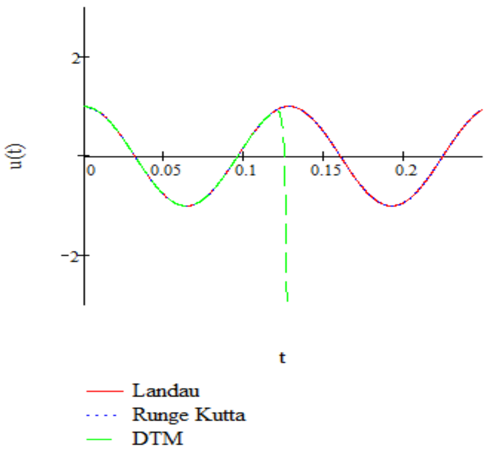

Fig 5 - u(t) solutions of eq. (1) using RK and DTM and analytical Landau result. We used δ =0,001, φ = 1,25, ω = 48,824 and θ = 2ω.

4. CONCLUSION

We have studied a model of non uniform beam in tree cases; only free oscillations, with forcing term and dissipation and without dissipation, in the last case, the time evolution equation becomes a Mathieu equation. The investigation was carried out analytically for small amplitude vibrations, and numerically by Differential Transform Method and using four order Runge Kutta. In general we can say that for short times DTM works better than RK, the opposite occurs for longer times. Also we note that, for larger oscillations DTM fails even for short time.

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