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ANALYSIS OF COUPLED DRILL-STRING VIBRATIONS USING A NONSMOOTH SYSTEM

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Abstract: A coupled four-degree of freedom nonsmooth model is adopted to describe the drill-string vibration. A parametric study is done through bifurcation diagrams. The goal is to understand how the vibration modes interact with each other and identify critical responses. A variety of operational conditions are investigated, allowing one to obtain a deep understanding of the drill-string global behavior proposing mitigation of critical responses.

Keywords: Discontinuous Dynamical Systems, Nonlinear Dynamics and Complex Systems, Chaos and Global Nonlinear Dynamics, Drilling, Non-smooth Systems.

INTRODUCTION

Drill-string dynamics are caracterized by three modes of vibration: axial, torcional and lateral. However one of them is often negleted for simplification purposes on many works. A torsinal-lateral analsysis are conducted considering the unbalance in [6] and the fluid interaction in [5] as sources of excitation. Both showed results in qualitative agreement with experimental and field observations.

An extensive study of the axial-torsional coupling is conducted in reference [4]. Bifurcation diagram analysis established an interesting approach to cover a wide range of operational conditions, defining critical responses. A compariosn between two different bit-rock interaction theories is carried on [1] for the torcional mode.

This paper deals with a torcional-axial-flexural coupled analysis considering a four degree of freedom nonsmooth model proposed by [3]. Nonsmooth nonlinearites are incorporated by bit-rock interactions and wellbore contact. The effect of unbalance is also included and the fluid forces are modeled as hydrodynamic forces following reference [3].

The coupled analysis gives a proper comprehension of the drill-string dynamics, elucidating several critical vibration responses. In this regard, it is important to highlight bit-bounce, stick-slip, and whirl.

MATHEMATICAL MODEL

The drill string dynamics is investigated considering a discrete model proposed by Christoforou and Yigit [3]. A schematic picture of the system is shown in Figure 1. This is a four-degree of freedom oscillator that couples axial-torcional-flexural dynamics. The stiffness of lateral, axial and torcional loads are respectively represented by k_r, k_a, k_t. The dissipation associated with each one of the modes are represented by c_r and c_θ as hydrodynamic dissipation during the lateral motion, c_a is the axial dissipation and c_t is the rotational dissipation.

Basically, lateral motion is restrict to the lower part of the drill-string (drill-collars) that is the only part being under compression and susceptible to deflexion [7]. The upper part of the drill-string (drill-pipes) are subjected to torsional vibrations. Drill-collars can be assumed to be rigid in torsion when compared with drill pipes [3] and [4]. The system has an eccentricity e_o and forcing characteristics are defined by bit-rock interaction F_{ob} and T_{ob}, as well as lateral contact, Fc_r and Fc_θ.

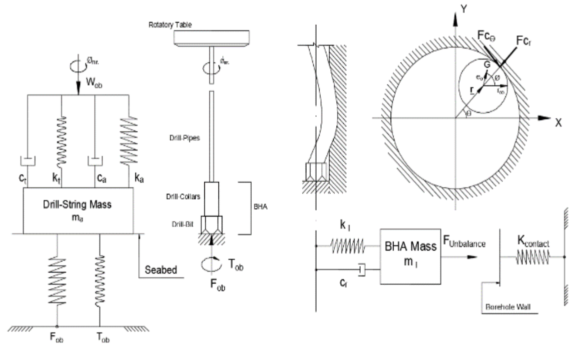

Figure 1 - Drill string model schematics.

Based on these assumptions, equations of motion are defined as follows:

$$m_l(\ddot{r} - r\dot{\theta}^2) + c_r\dot{r} + k_r r = m_l e_o[\dot{\emptyset}\cos(\emptyset - \theta) - \ddot{\emptyset}\sin(\emptyset - \theta)] - Fc_r \quad (1)$$

$$m_l(r\ddot{\theta} + 2\dot{r}\dot{\theta}) + c_\theta\dot{\theta} = m_l e_o[\dot{\emptyset}^2\sin(\emptyset - \theta) + \ddot{\emptyset}\cos(\emptyset - \theta)] - Fc_\theta \quad (2)$$

$$m_a\ddot{z} - c_a\dot{z} - k_a z = W_{ob} - F_{ob} \quad (3)$$

$$J\ddot{\emptyset} + c_t\dot{\emptyset} + k_t\emptyset + c_r\dot{r}e_o\sin(\emptyset - \theta) - c_\theta\dot{\theta}e_o\cos(\emptyset - \theta) = T_{ob} - F_r e_o\sin(\emptyset - \theta) + F_\theta\left(\frac{De_{co.}}{2} - e_o\cos(\emptyset - \theta)\right) \quad (4)$$

Wellbore contact is represented by two nonsmooth forces: radial (Fc_r) and tangential (Fc_θ). Based on [5] and [9], they are given by:

$$Fc_r = \begin{cases} k_c\left(r + \frac{De_{co.} - D_{well}}{2}\right), & \text{if } r + \frac{De_{co.}}{2} > D_{well} \\ 0, & \text{if } r + \frac{De_{co.}}{2} < D_{well} \end{cases} \quad (5)$$

$$Fc_\theta = \begin{cases} \mu Fc_r = f(V_{rel})Fc_r, & \text{if } r + \frac{De_{co.}}{2} > D_{well} \\ 0, & \text{if } r + \frac{De_{co.}}{2} < D_{well} \end{cases} \quad (6)$$

where, k_c, De_{co} and D_{well} are the formation stiffness, outer drill-collar diameter and wellbore diameter.

Tangential contact force is a nonsmooth component due to contact and friction conditions. In general, friction is a discontinuous function, however, a smoothened version can be adopted [4]. Therefore,

$$f(V_{rel}) = \frac{2}{\pi}\text{atan}(\varepsilon V_{rel})\left(\frac{\mu_{est.} - \mu_{dyn.}}{1 + \tau|V_{rel}|} + \mu_{dyn.}\right) \quad (7)$$

$$V_{rel} = \dot{\emptyset}r_{co} + \dot{\theta}r \quad (8)$$

where ε and τ are unidimensional coefficients; $\mu_{est.}$ and $\mu_{dyn.}$ are the static and dynamic friction coefficients. The relative velocity during the wellbore contact is represented by V_{rel} being a sum of two components: tangential velocities due to the spin and rotation motion.

Axial and torsional modes are coupled by forcing terms discussed on [3], [4] and [8]. Once again, nonsmooth characteristics can be observed,

$$F_{ob} = \begin{cases} k_c[z - s_o\sin(n_b\emptyset)], & \text{if } z < S_o\sin(n_b\emptyset) \\ 0, & \text{if } z > S_o\sin(n_b\emptyset) \end{cases} \quad (9)$$

$$T_{ob} \begin{cases} F_{ob}\left[\frac{2}{3}r_h f(\dot{\emptyset}) + \zeta\sqrt{\frac{D_p}{2}\delta_c}\right], & \text{if } z < S_o\sin(n_b\emptyset) \\ 0, & \text{if } z > S_o\sin(n_b\emptyset) \end{cases} \quad (10)$$

where s_o is related to irregular behavior associated with drilling and n_b a drill-bit factor, which varies to each type of bit. According to [3] and [4], n_b= 3 for tri-cone drill-bits and n_b= 1 for PDC bits. The wellbore radius is represented by r_h.

The torque on bit (T_{ob}) is a sum of the friction loss and cutting components. The smoothened function of friction is the same used on [3].

$$f(\dot{\phi}) = \frac{2}{\pi}\text{atan}(\varepsilon\dot{\phi})\left(\frac{\mu_{est.} - \mu_{dyn.}}{1 + \tau|\dot{\phi}|} + \mu_{dyn.}\right) \quad (11)$$

The cutting contribution is based on an empirical relationship where ζ is a dimensionless factor related to the force necessary to cut the rock and δ_c is the average cutting depth.

NUMERICAL SIMULATIONS

Numerical simulations are performed employing the fourth order Runge-Kutta method with adaptive time steps in order to deal with nonsmooth nonlinearities. A parametric analysis is developed identifying critical responses and their general aspects are discussed. Axial, radial and torcional motions of the drill-string are assesed by their displacement and velocity peaks. A drill-string with 0.7 m of excentricity and 0.381 m of wellbore diameter is tested under a weight on bit of 22.24 kN and angular velocity of 3 rad/s. All other parameters used can be found on [4] and their calculation in [3].

Two diagrams are plotted considering the same initial condition for all simulations and considering the previous parameter response as initial condition. Figure 2 presents first procedure while Figure 3 shows the second procedure. Each Figure presents axial, torcional and radial behavior. These diagrams present an identification related to critical behaviors: bit-bounce, stick-slip and whirl.

Figure 2 (a) presents the maximum value of axial response and showing an interval with efficient responses around 1500 m. Afterward, bit-bounce phenomenon dominates the drill-string dynamics. Moreover, at higher depths, whirl becomes relevant and shows a great influence in axial response. This is clear around 7000 m and over 8000 m.

Figure 2 (b) presents the torsional diagram indicating an abrupt decrease at 7000 m. The decrease observed on the angular velocity is a direct consequence of the coupling between torsional and lateral modes during contact due to bit-rock interaction.

The lateral diagram is presented in Figure 2 (c) showing a crescent aspect. Notte that critical phenomena dominates response at higher depths [4].

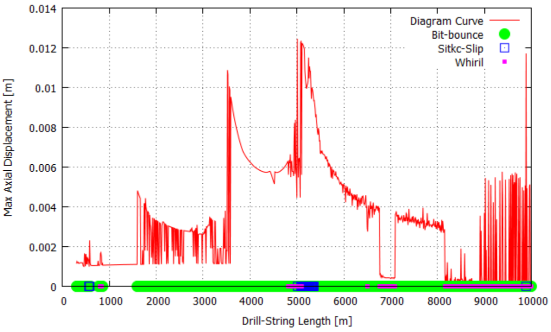

Figure 2 (a) - Axial diagram - same initial conditions.

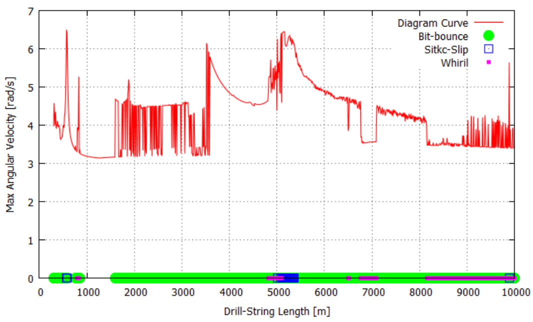

Figure 2 (b) - Torcional diagram - same initial conditions.

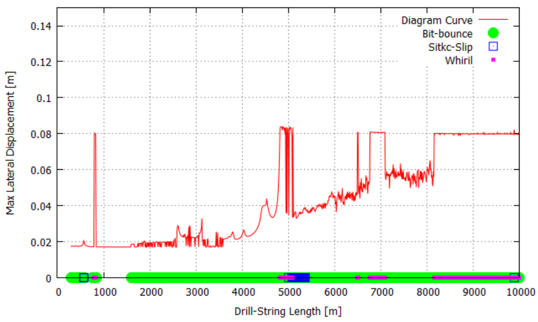

Figure 2 (c) - Lateral diagram - same initial conditions.

Figure 3 presents the same diagrams related to axial, torcional and lateral responses considering different initial conditions for each parameter. A comparison among diagrams obtained by the two procedures (the same initial condition and updated ones, Figure 2 and Figure 3) shows that the update of initial conditions tends to generate less critical responses. Moreover, Figure 2 shows that lateral contact tends to decrease axial and torcional responses. On the other hand, Figures 3 presents the opposite behavior, showing that lateral contact excites axial and torcional modes for the same configuration.

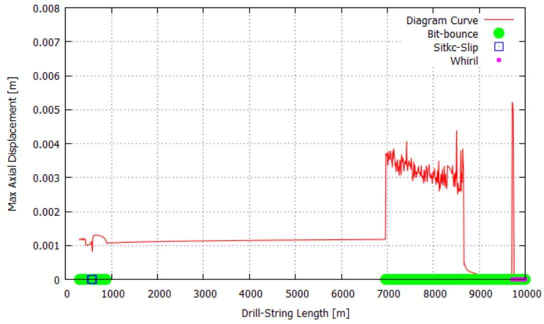

Figure 3 (a) - Axial diagram - different initial conditions.

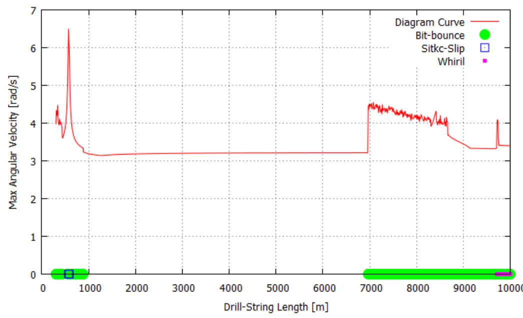

Figure 3 (b) - Torsional diagram - different initial conditions.

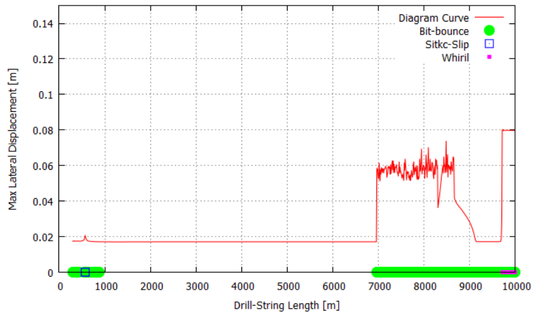

Figure 3 (c) - Lateral diagram - different initial conditions.

The influence of weight on bit and rotation on drill-string dynamics is of concern. A configuration of 5000 meters is adopted considering a rotation variation. Diagrams are plotted considering up-sweep and down-sweep responses, considering the same procedures of the previous simulations related to initial conditions (Figure 4-5). Critical behaviors are identified for each simulation (up-sweep – top; and down-sweep – bottom.

Figures 4 (a) and 4 (b) represent the axial and lateral diagrams for weight on bit of 22.24 kN. An interval without critical vibrations is observed. Approximately up to 3 rad/s bit-bounce and whirl dominates drilling dynamics. Note that whirl during up-sweep tests reduces axial amplitudes. In the same way, stick-slip during the down-sweep produces large amplitudes of the axial response. The same behavior can be observed on lateral response. Moreover, it should be highlighted that the critical responses observed for angular velocity of 3 rad/s are eliminated due to slow quasi-static variation of the angular velocity during up-sweep.

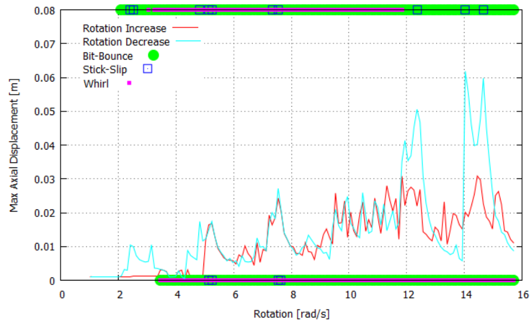

Figure 4 (a) - Axial rotation diagram – W_{ob} = 22.24 kN.

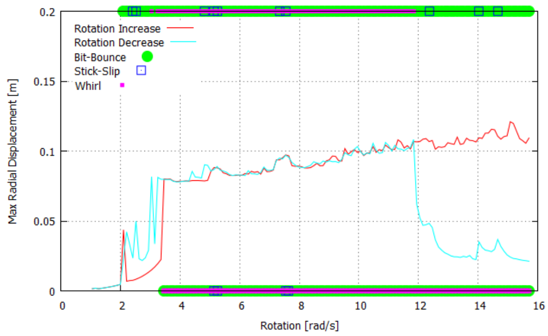

Figure 4 (b) - Lateral rotation diagram –W_{ob} = 22.24 kN.

Figures 5 (a) and 5 (b) represent the axial and lateral diagrams for a weight on bit of 66.72 kN. The same conclusions related to whirl and stick-slip are observed for this W_{ob}. Moreover, the elimination of critical behaviors due to slow quasi-static increase during up-sweep test is increase due to the increase of the W_{ob}.

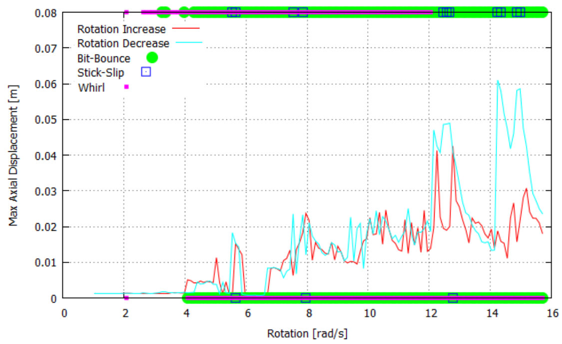

Figure 5 (a) - Axial rotation diagram – W_{ob} = 66.72 kN.

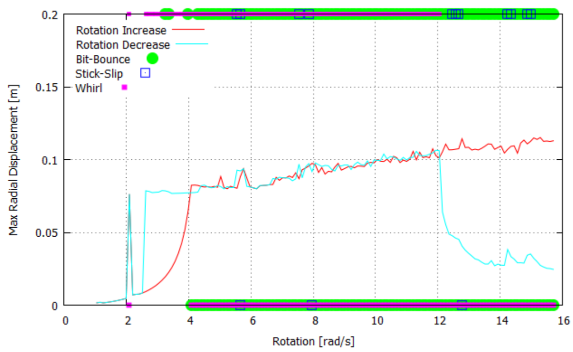

Figure 5 (b) - Lateral rotation diagram – W_{ob} = 66.72 kN.

CONCLUSIONS

Drill-string vibration is investigated considering a four-degree of freedom non-smooth model. Critical behaviors as bit-bounce, stick-slip and whirl are identified. Operational conditions are investigated from a parametric analysis, proposing alternatives to mitigate critical behaviors. The proposed model is interesting to offer a comprehension of system dynamics, evaluating critical responses.

An adequate increase of weight on bit is able to mitigate critical vibrations, whereas its indiscriminate increase can induce whirl and stick-slip. The differences between up-sweep and down-sweep responses shows that some critical behaviors can be avoided just changing the loading history. In the same way, the changes observed due to different initial conditions also points to some mitigation approach.

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