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ON THE FUNDAMENTAL CHARACTERISTICS OF COMPLEX NETWORK WITH MULTI-AGENT CONSTITUENTS

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Abstract: Multi-agent systems incorporating the concept of complex network are studied to gain insight into the dynamics that governs the collective behavior of the network. A network system along with its behavior can be defined by 3 primary network parameters; namely, Degree Distribution, Cluster Coefficient, and Average Path Length. Real-world multi-agent systems such as flocking birds, swarm fish, drone fleets, and human teams are networks whose behaviors are both nonlinear and nonstationary. Since the three network parameters defining the relationship between the nodes (agents) of the network are time-dependent, nodes consensus and network synchronization would be hard to attain. Control methodologies considered effective in enabling the consensus control of any complex network with multi-agents and time-delays, therefore, needs to address the essential aspects of nonlinearity and nonstationarity to be viable. In this presentation, a general framework for developing such a consensus control methodology is discussed which also demonstrates applicability to time-delayed complex networks. The significance of nonlinearity and non-stationarity of real-world networks such as multi-agent systems to consensus control is considered. The correlations of Degree Distribution and node number to the number of edges are explored. Two sets of experimental results are provided. Finally, a general framework for developing such a consensus control methodology is discussed.

Keywords: Complex Network, Nonlinear Dynamics and Complex Systems, Synchronization in Nonlinear System, Multi-agent Systems.

1. INTRODUCTION

The collective behaviors of all real-world network systems such as flocking birds, swarm fish, drone fleets and human teams are defined by 3 primary network parameters; namely, Degree Distribution, Cluster Coefficient, and Average Path Length [1]. The formation of flocking birds, citation relationship between scholars, and data transmission speed of WWW, for example, can be greatly affected through manipulating the 3 parameters.

An extensive literature review performed by the authors suggests that all the published formation control strategies for the consensus control or network synchronization of multi-agent systems are special cases of complex networks of one kind or the other [2-4]. In addition, these consensus control methods are all, without exceptions, based on stationary network models. However, real-world multi-agent systems whose behaviors are both nonlinear and nonstationary. In other words, the three network parameters defining the relationship between the nodes (agents) of the network are time-dependent. As a result, nodes consensus and network synchronization can be hard to attain. Moreover, the state of synchronization would be even more difficult to achieve when time-delay is considered [5-6].

While exerting consensus control to a real-world network such as a multi-agent system, it is preferable that all nodes are mutually connected with directed edges as a complete digraph. Nodes having no edges connected to the network would be unable to consent with the network. Moreover, if the amount of nodes that remain unconnected to the network is high enough, such a network tends to collapse. To achieve effective consensus control of real-world networks, the state information of each node/agent needs be acquired by all other nodes through edges/connections with the least hindrance. This implies that Degree Distribution is the primary parameter that defines the probability of a specific degree that a node can have.

On the other hand, the amount of nodes in a network affects the number of edges in the network as well. In other words, there exists a relationship between the amounts of nodes and the effectiveness of consensus control. Control methodologies considered effective in enabling the consensus control of any complex network with multi-agents and time-delay, therefore, needs to address the essential aspects of nonlinearity and non-stationarity to be viable. In this paper, a general framework for developing such a consensus control methodology is discussed which also demonstrates applicability to time-delayed complex networks.

2. PURPOSE

Network dynamics is a coupling of the individual nodes' dynamics at the local level and the system dynamics of the whole network at the global level. Both the dynamics of individual nodes and the network system are equally significant in dictating the response of the network.

An agent in a multi-agent system is a node. An edge connecting two nodes is the ability of a node to detect the state of the other connecting agent. Only when there is a directed edge between two agents, one agent is able to consent or synchronize to another agent's state. When two directed edges between two nodes are present, the nodes are able to consent to the state of each other. And the two directed edges can be viewed as one two-way connection between agents. It is not preferable that some nodes have no edges connecting them to other nodes at all. A network system would soon collapse when there are a large number of nodes unconnected to the network.

To achieve consensus control, real-world networks are required to contain as many edges as possible. Since edges are the paths through which state information are transmitted from one node to another, a network with many edges have more paths between the constituent nodes. This suggests that nodes can remain connected even when some of the edges disappeared. The state information of each node can be acquired faster by other nodes due to the high degree of each node being connected to others. Thus, it is easier for a network to hold the nodes together and achieve nodal state consensus more effectively while the network has high number of edges.

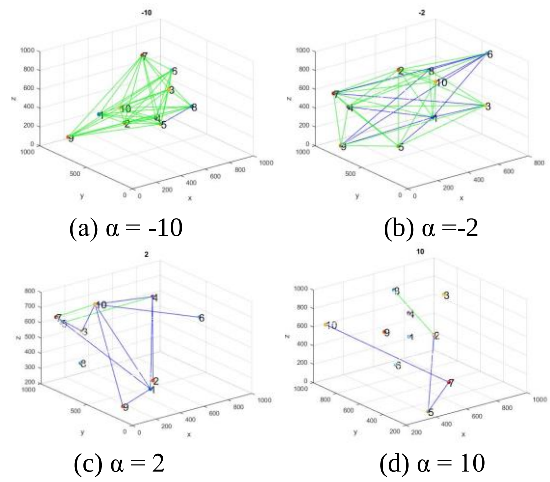
In order to better understand real-world networks, the effects of different Degree Distribution and the amount of nodes on the structural configuration of a network are investigated. Once the correlations between Degree Distribution, quantity of nodes, and network configuration are established, the effect of different degree distribution on state consensus between nodes in the network is discussed. Each node is initially at rest and randomly distributed in the 3-D space. Differences in the dynamic states (as defined by position and velocity) of each node trigger node motions. A simulation demonstrating the result of state consensus with the dynamics of each node being defined using the Lorenz attractor is also presented. In the discussion section the significance of considering real-world network systems as a dynamic system is addressed, which also serves as a guideline for achieving consensus control following from the relationship between Degree Distribution and the amount of nodes with the network configuration.

3. RESULTS and DISCUSSION

3.1. Experiment 1 - Degree Distribution

The primary objective of Experiment 1 is to study the impact of Degree Distribution on consensus control. In the experiment, n number of nodes are distributed randomly in the 3-D space and edges are constructed using the time-dependent Degree Distribution law, $P(k) = k^{-\alpha(t)}$. When α ramps up from a small number to a large one, the number of the edges varies and the structure of the network also changes. Figure 1, a 3-D plot, provides a visual aid in comprehending how Degree Distribution affects the network. The Figure shows 10 nodes with Degree Distribution that correspond to (a) α = -10, (b) α = -2, (c) α = 2, and (d) α = 10. Edges are represented by lines in the figure with the green lines indicating 2-way edges and the blue lines 1-way edges. 2-way edges allow the connected nodes to consent to each other's state. 1-way edges only allow one of the nodes on the 2 sides of the edge to consent to the state of the other node.

It is seen when α = -10 that all the nodes are connected to the network and all the edges are 2-way edges except for the one connecting node 5 and node 6. When α = -2, all the nodes remain connected to the network. However, half of the edges now have 1-way connection. When α = 2, the network becomes sparse with edges and node 3 is disconnected from the network. Finally, when α = 10, there are only 4 edges with 5 nodes being disconnected from the network.



(a) α = -10 (b) α =-2

(c) α = 2 (d) α = 10

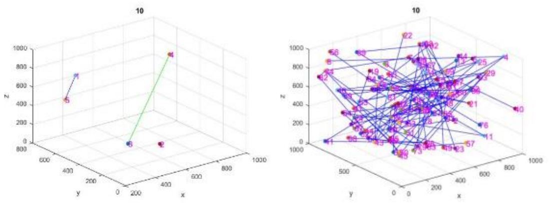
Figure 1 – Correlation of α with the edges in a network

It can be seen in Figure 1 that the edges of the 10-node network go from being dense to sparse while α increases from -10 to 10. Furthermore, when α < 0, the network demonstrates a higher probability to form a densely connected network. The observation suggests that it is desirable that α is small when performing consensus control on real-world networks such as the multi-agent network considered herein. There are more connections between agents, which in turn hold all the agents together without falling off the network. Also, with small α, the network contains more alternative connections between nodes to prevent nodes from losing connections.

3.2. Experiment 2 - Node Number

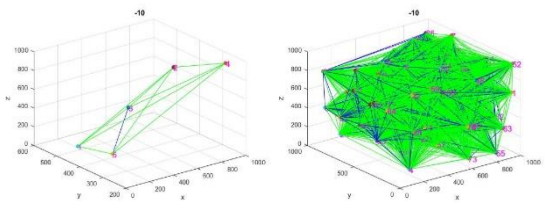
To see how the amount of nodes impact network construction, Experiment 2 sets α as a constant and examines how the amount of nodes affects the structure of the network. Figure 2 shows the network when (a) α = 10, n = 5 and (b) α = 10, n = 100 while Figure 3 gives the results corresponding to (a) α = -10, n = 5 and (b) α = -10, n=100. As Experiment 1, green lines represent 2-way edges and blue lines represent 1-way edges. In Figure 2(a), with α = -10 and only 5 nodes, there are two edges connecting two pairs of nodes as two networks with Node 2 losing connection with the rest of the nodes. In Figure 2(b), with the same α as Figure 2(a) but 95 more nodes in the network, all the 100 nodes are connected to each other with 1-way connection as a fully connected network. Figure 3(a) shows a fully connected network of 5 nodes with α = -10. Figure 3(b) shows a fully connected network of the same α as Figure 3(a) but with 100 nodes.

Figure 2 indicates that a network composed of a large amount of nodes would provide higher probability for the network to be fully connected. Even with a large α that generates fewer edges for each node, the network can still be fully connected and reach state consensus if the amount of nodes is large enough. Although the networks in Figures 3(a) and 3(b) seem to have the same property, Figure 3(b) contains countless edges in the network, thus a higher level of tolerance for loss of edges.



(a) α = 10, n = 5 (b) α = 10, n = 100

Figure 2 – Correlation of node amounts to the network α = 10



(a) α = -10, n = 5 (b) α = -10, n = 100

Figure 3 –Correlation of node amounts to the network α = -10

The results in Experiments 1 and 2 indicate the reason why real-world networks such as flocking birds, swarm fish, and social networks usually have a large number of nodes/individuals and edges/connections. First, a network requires a large number of nodes to generate enough edges to hold all the nodes together as a whole to present collective behavior even though the corresponding degree distribution is low. Second, each node can acquire state information from as many nodes as possible at a time if the α of Degree Distribution is small enough to construct the network as a complete digraph. When a network is a complete digraph or fully connected network as the one shown in Figure 3(b), there exists many alternative paths between any two nodes. Furthermore, the more paths between two nodes, the more state information is carried between the two nodes. The reason is that the nodes exist on each path are also consenting with each other's state through the path. Thus, the higher degree of distribution each node has in a network implies that each node can acquire more state information and consent to each other in a shorter amount of time. In other words, such a network has a higher efficiency of state consensus with higher tolerance to loss of edges. If nodes have the ability to construct new edges with other nodes, a network can adapt to environmental disturbance without collapsing, just as flocking birds remain as a swarm when encountering predators.

In summary, in order to achieve consensus control of real-world networks such as multi-agent systems, it is preferable to maintaining the networks with low α and a large number of nodes. The network seen in Figure 3(b) demonstrates such preferred properties ideal for performing network consensus control.

3.3. Experiment 3 – Nodes States Consensus without Nodes' Individual Dynamics

Based on the observations made in the previous experiments, the impact of the contribution of α on node states consent is studied. The result in the following illustrates how α changes nodal behaviors while executing state consensus in the network. The consensus law is as follows:

$$\dot{x}_i = v_i(t)$$
$$\dot{v}_i = u_i(t)$$
$$u_i(t) = \frac{f_i}{m_i} + \frac{c_i}{m_i}\left(\sum_1^n v_j - n v_i\right) + \frac{k_i}{m_i}\left(\sum_1^n x_j - n x_i\right) \quad (1)$$

where x_i and v_i are position and velocity of the i^{th} node, respectively. c_i, k_i, and m_i are imaginary damping coefficient, imaginary stiffness coefficient, and mass of the i^{th} node, respectively. f_i is the force reacting on the i^{th} node.

10 nodes randomly distributed in a 1,000 meter cube are generated for the study. All the nodes are considered as a point mass of 1 kg without any individual dynamics. The imaginary stiffness coefficient k is 1 (N/m) and the imaginary damping coefficient is 1 (N-s/m). The 3-D space is assumed to be an ideal environment free of friction and environmental disturbance. The length of edge/connection between nodes can reach infinitely far, and the velocity of the nodes can be very large. The consensus law is applied for 50 seconds. During the time window set for the study, the network re-constructs edges continuously following the definition of degree distribution dictated by α. In other words, the network configuration changes at all time in order to form a dynamic network. When all nodes converge to a 1 meter cubic space, the simulation terminates and the converged nodes reach state consensus. When the distance between any two nodes is greater than 20,000 meters, the simulation also terminates

and the nodes are deem as diverged.

Figure 4 through Figure 8 show the results of node state consensus with no individual node dynamics. Each figure contains four sub figures. The figure on the top right gives the trajectory of the node in the 3-D space. The sub figures on the bottom right, top left, and bottom left are the projections of the trajectory to the x-y plane, x-z plane, and y-z plan, respectively.

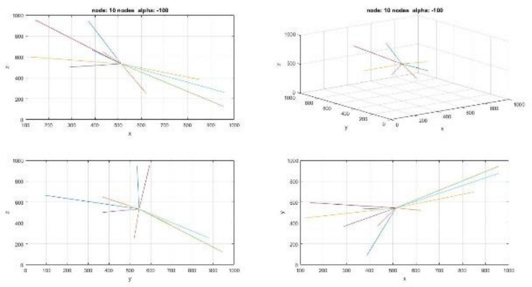


Figure 4 – State consensus of 10 nodes with α = -100

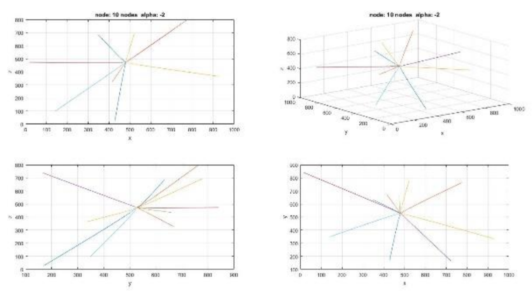


Figure 5 – State consensus of 10 nodes while α = -2

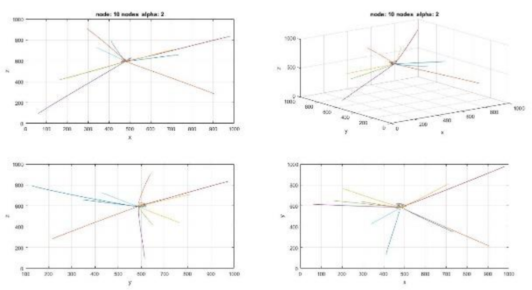


Figure 6 – State consensus of 10 nodes with α = 2

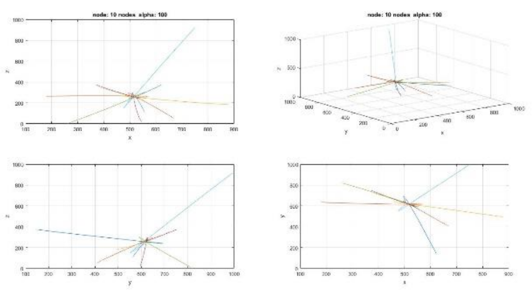


Figure 7 – State consensus of 10 nodes with α = 100

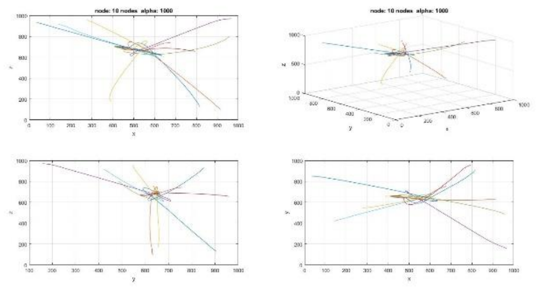


Figure 8 – State consensus of 10 nodes with α = 1000

It can be seen from the figures, the node convergence trajectories are shorter while α is small; and the trajectories are longer while α is large. In other words, the nodes come approach each other along a straight line when α is less than 0. Furthermore, the nodes converge to each other with overshoots when α is greater than 0. The larger α is, the more prominent the overshoot is. The reason is when α is small it forms a fully connected network. All nodes have the full information from all other nodes through the edges as they further converge to each other. However, when α is large, each node only has a few edges to obtain state information from other nodes. Each node can only consent to the states of a portion of nodes in the network. As a result, overshoots are generated while the nodes seek to converge.

The result suggests that α is one of the key factors for achieving consensus control effectively. A small α forms a network with more edges to pass state information between nodes and renders the nodes reaching state consensus with shorter trajectories. A large α would result large overshoots in the node consent trajectories and introduce instability to the network.

3.4. Experiment 4 - Nodes States Consensus with Nodes' Individual Dynamics

A real-world network such as flock of birds, swarm of fish, and fleet of multi agents could consist nodes with different nonlinear dynamics. Furthermore, each node may experience different environmental disturbances that are nonlinear and random. While individually displaying different nonlinear dynamics, all nodes are attempting to achieve state consensus in the dynamically changing network. Thus, this section explores the nature of state consensus while all the nodes possess their own dynamics.

3.4.1. Individual Node Dynamics

The Lorenz attractor is assigned to each node to define the individual dynamics. The Lorenz attractor is expressed as Equation (2) below:

$$\dot{x}_i = \sigma(\dot{y}_i - x_i)$$
$$\dot{y}_i = -x_i + rx_i - y_i \qquad (2)$$
$$\dot{z}_i = x_i y_i - bz_i$$

where $\sigma = 10$, b = 8/3, and r = 28.

Figure 9 shows the trajectories of 10 nodes with each node behaving nonlinearly following the Lorenz attractor as their individual dynamics. Note that the particular scenario depicted in Figure 9 is the case in which the 10 nodes are without any edges between them. Thus, the dynamics of each node will not be affected by other nodes. In other words, the nodes in Figure 9 do not belong to or form a network. They are not set to consent to each other.

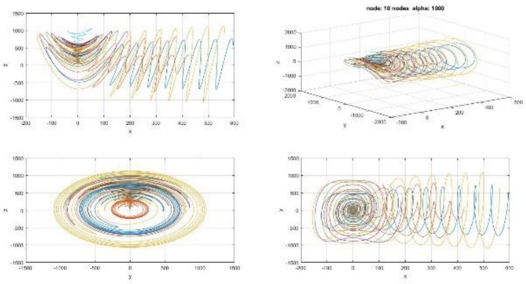


Figure 9 – Trajectories of 10 nodes with Lorenz attractor defining individual node dynamics

With the same individual dynamics as those in Figure 9, the 10 nodes seen in Figure 11 are performing state consensus in a network with the Degree Distribution defined using α = -10. Figure 10 is the snap shot of Figure 11 half way through the state consensus. The trajectories in Figure 10 shows some nodes are approaching each other in the x- direction as the result of reaching state consensus while other nodes diverge to the y-axis due to the chaotic individual dynamics. However, it is observed in Figure 11 that the diverged nodes attempt to converge with the rest of the nodes at the later stage of the time window.

It can be seen with clarity that although the individual dynamics are exactly the same for the nodes in Figure 9 through Figure 11, the behaviors of the nodes with and without edges are drastically different. The result signifies that it is essential to consider a real-world network as a dynamic system capable of evolving in both space and time. As opposed to a static network, a dynamic network constantly evolves configurationally as well as structurally, thus resulting in a very different network dynamics in space.

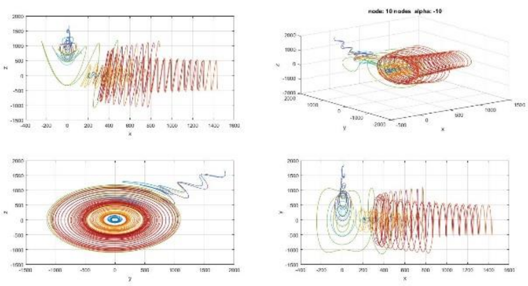


Figure 10 – State consensus of 10 nodes while α = -10

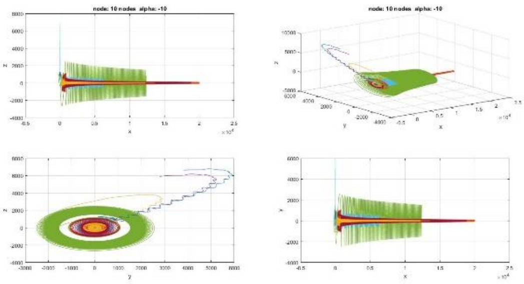


Figure 11 – State consensus of 10 nodes while α = 1,000

3.4.2. Experiment 4 – Nodes State Consensus

In this section, the state consensus results with each node in the network behaving nonlinearly are discussed. From the results in the previous sections it was concluded that it would be more efficient to achieve state consensus by coupling a small α with a large number of nodes. In the followings the state consensus results of 100 nodes with α = -100 are presented. Note that individual node dynamics following the Lorenz attractor is again considered.

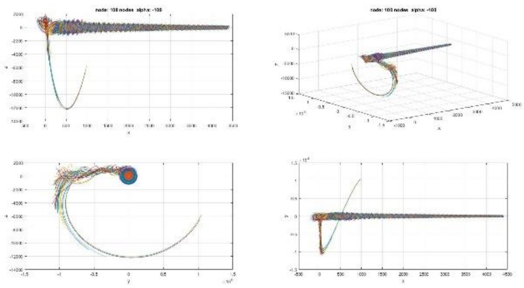


Figure 12 – State consensus of 100 nodes with α = -100

Figure 12 shows that the nodes failed to reach state consensus even when the network contain a large number of nodes with a very small α of degree distribution enabling a fully connected network to be formed. The result explicitly indicates that simply executes consensus law as defined in Equation (1) and applies controllers designed base on static network analysis will not be sufficient to achieve successful nodes states consensus. A nodes state consensus controller design viable for real-world network systems needs to consider the fact that all nodes in the network are dynamically nonlinear or even chaotic and that the time-evolving network is constantly subjected to environmental disturbance. More specifically, a viable controller treats a real world network as the coupling of the local individual nodal dynamics with the global network dynamics that governs the transmission of state information between nodes.

6. CONCLUSIONS

Real-world networks such as multi-agent systems are dynamical networks exist in highly dynamic environment. Comprehensive study of such networks requires that the individual node (local) dynamics be simultaneously considered with the network (global) dynamics.

Degree Distribution, Cluster Coefficient, and Average Path Length, the 3 primary network parameters, dictate the global network dynamics which is contanstly changing. It is mandatory that a feasible formation control strategy for network consensus control sustains at all time the network from collapsing while maintaining high level of performance of all nodes to achieve state consensus. The nonlinearity of each node at the local level can negate state consensus to a diverging path and lead to ultimate network failure. A consensus controller design should consider the coupled local and global dynamics to achieve consensus.

A network with a large number of edges can provide more alternative connections to hold the nodes together as a connected network with better efficiency to achieve node state consensus. In addition, a network with a large number of nodes also have a higher probability to provide enough edges to keep all the nodes connected with the network with a large Degree Distribution α. Moreover, a small α does not only impart a network with many edges, it also renders short nodal trajectories while performing state consensus. Thus, an ideal real world network is one defined by small α's with a large number of nodes. Such networks demonstrate rich collective behaviors and achieve state consensus with ease.

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