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FRACTAL STRUCTURES IN A MODEL FOR $\mathrm{E} \times \mathrm{B}$ DRIFT MOTION OF CHARGED PARTICLES IN MAGNETIZED PLASMAS

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Abstract: We describe the formation of fractal structures in the Poincarè surface of section for a model describing the $\mathbf{E} \times \mathbf{B}$ drift motion of charged particles interacting with two electrostatic waves, in a magnetically confined plasma. These fractal structures have important consequences for the transport properties of particles and energy.

keywords: fractal structures, drift waves, magnetically confined plasma

1. INTRODUCTION

The transport of particles occurring at the plasma edge in tokamaks has been widely investigated [1–3]. A major cause of this transport is the appearance of drift waves, which are electrostatic fluctuations produced by the density gradient in the plasma edge [4]. The objective of this work is to investigate the Hamiltonian model of drift waves proposed by W. Horton [5], to describe the transport of particles caused by these waves. This model describes the drift motion of the guide center of the particles confined in the peripheral region of the plasma in tokamak .

In general, it is used an electrostatic potential consisting of the equilibrium potential depending only on the radial coordinate and a superposition of infinite drift waves, of amplitudes, wave numbers and frequencies varied. Each of these waves propagates in the y direction (poloidal) are stationary in the x direction (radial) and are transverse to the magnetic field in the z direction (toroidal).

For just one drift wave, the system is integrable, and the trajectories are regular. For two drift waves, the equations are not integrable, and depending on the value of the amplitude of the wave the trajectories can have chaotic behavior.

The structure of this article is as follows: in Section 2 is introduced the Hamiltonian model of the drift wave, and are presented the equations of motion for one and two drift waves. In Section 3, we show some computer simulations that were obtained for the model of two drift waves. In Section 4 is devoted to conclusions.

2. DESCRIPTION OF THE HAMILTONIAN MODEL FOR DRIFT WAVES

For a charged particle in a uniform magnetic field subjected to an electric field, we have the equation of motion found from the Lorentz force:

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right), \tag{1}$$

where m is the mass, q the charge, v the speed, $\vec{E}$ the electric field and $\vec{B}$ is the magnetic field.

In the presence of the electric field perpendicular to the magnetic field, the speed of guide center is given by:

$$\vec{v}_E \equiv \frac{\vec{E} \times \vec{B}}{B^2}, \tag{2}$$

where $\vec{v}_E$ is the speed of drift of the guide center of particle due to electric and magnetic fields.

Considering a uniform magnetic field in toroidal direction, $\vec{B} = B_0 \hat{e}_z$ and the electric field perpendicular to the magnetic field in the radial direction, $\vec{E} = -\nabla\phi(x, y, t)$, the speed of drift can be written as:

$$\vec{v}_E = -\frac{1}{B_0}\nabla\phi(x, y, t) \times \hat{e}_z$$

The Hamiltonian of system is:

$$H(x, y, t) = \frac{\phi(x, y, t)}{B_0} \quad (3)$$

Thus, there is only one canonical pair and a degree of freedom, where *x* is the canonical moment and *y* is the coordinate conjugate and hence the system is integrable. We now consider a model for N drift waves where the waves propagate in the direction *y* with modulation in the direction *x*. The electric potential can be written in the form:

$$\phi(x, y, t) = \phi_0(x) + \sum_{i=1}^{N} A_i \operatorname{sen}(k_{xi}x)\cos(k_{yi}y - \omega_i t) \quad (4)$$

where $\phi_0(x)$ is the equilibrium plasma potential, A_i is the amplitude of each wave, k_{xi} e k_{yi} are wave numbers in the directions *x* and *y* respectively and ω_i is the angular frequency of the i-th wave.

Supposing a potential equilibrium linear $\phi_0(x) = E_0 x$, the Hamiltonian in form dimensionless is given by:

$$H(x, y, t) = E_0 x + \sum_{i=1}^{N} A_i \operatorname{sen}(k_{xi}x)\cos(k_{yi}y - \omega_i t) \quad (5)$$

The Hamiltonian is integrable for $N = 1$, with existence of regular trajectories. For $N > 1$, the Hamiltonian is integrable if all waves have the same phase velocity. If only one wave have phase velocity different of others, the system won't be integrable and some trajectories will be chaotic.

3. RESULTS

Considering the case of two drift waves ($N = 2$) and making a change of reference, the Hamiltonian is given by:

$$H(x, y, t) = (E_0 - u_1)x + A_1 \operatorname{sen}(k_{x1}x)\cos(k_{y1}y) + A_2 \operatorname{sen}(k_{x2}x)\cos(k_{y2}(y - ut)) \quad (6)$$

where $u_1 = \frac{\omega_1}{k_{y1}}$ is the phase velocity of the first wave and

$$u = \frac{\omega_2}{k_{y2}} - \frac{\omega_1}{k_{y1}}$$

is the phase difference between the two waves.

The equations of motion are:

$$\frac{dx}{dt} = A_1 k_{y1} \operatorname{sen}(k_{x1}x)\operatorname{sen}(k_{y1}y) + A_2 k_{y2} \operatorname{sen}(k_{x2}x)\operatorname{sen}(k_{y2}(y - ut)) \quad (7)$$

$$\frac{dy}{dt} = (E_0 - u_1) + A_1 k_{x1}\cos(k_{x1}x)\cos(k_{y1}y) + A_2 k_{y2}\cos(k_{x2}x)\cos(k_{y2}(y - ut)) \quad (8)$$

The parameters used for the numerical simulations are: $\omega_1 = 0,2$, $\omega_2 = 1,2$, $k_{x1} = k_{y1} = k_{x2} = k_{y2} = 1,0$, $u = 1,0$, $A_1 = 1,0$ and $E_0 - u_1 = 0$.

Figure 1 shows some simulations for the value of the amplitude $A_2 = 0$ and $A_2 = 1.5$. The phase spaces were obtained by the Poincarè section for a period equal to 2π. In (a) the particle trajectories are closed and there isn't transport of particles between cells which are formed by rectangles. In (b) we can see that the islands suffer bifurcations and chaotic trajectories increased dramatically. As the trajectories near the separatrixes of the islands were destroyed, the transport to the chaotic trajectories will be higher.

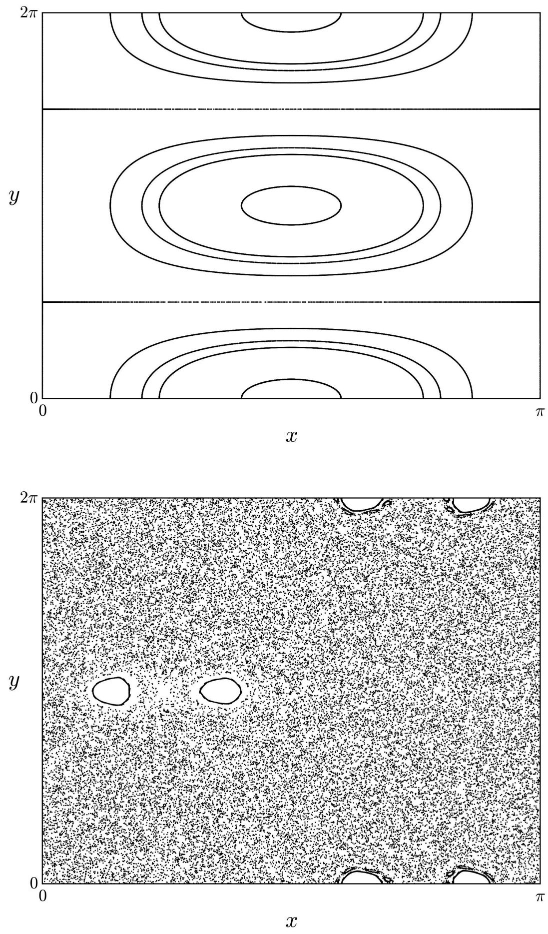

Figure 1 – Phase portraits for (a) $A_2 = 0$ and (b) $A_2 = 1.5$

As we are interested in analyzing the plasma edge, we examine the escape basins or exit basins [6]. In the model we consider the plasma edge in the wall of the tokamak at $x = 0$, and split the output into two parts, for $S_1 < \pi$ (basin in green) and $S_2 > \pi$ (basin in red). In the figure 2, we have escape basin for the value of the amplitude $A_2 = 1.5$

In order to analyze the fractal structures [7] we calculate the fractal dimension [8] of the structures present in the es-

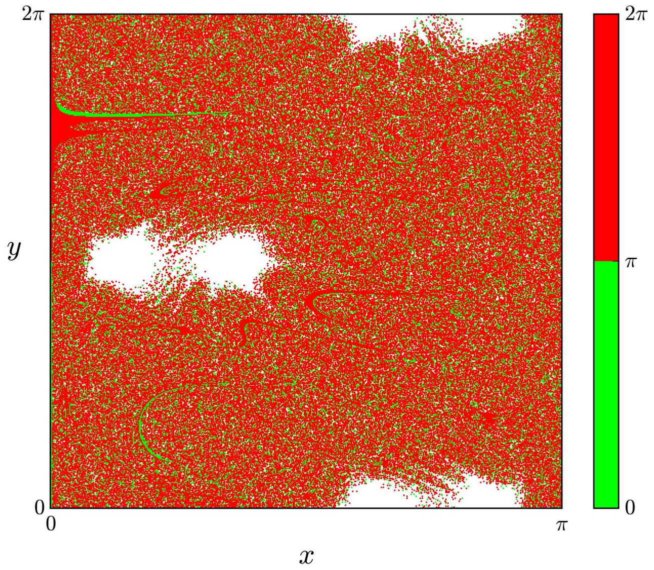

Figure 2 – Escape basin for $A_2 = 1.5$. Initial conditions that are beyond the outputs S_1 and S_2 are are colored red and green, respectively. Orbits in the white region does not escape at $x = 0$.

cape basin of figure 2. We examine the dependence of the 'uncertain fraction' of the phase portrait $f(\varepsilon)$ on the error ε. The variation o $\tilde{f}(\varepsilon)$ for this system with decreasing error ε is plotted in figure 3.

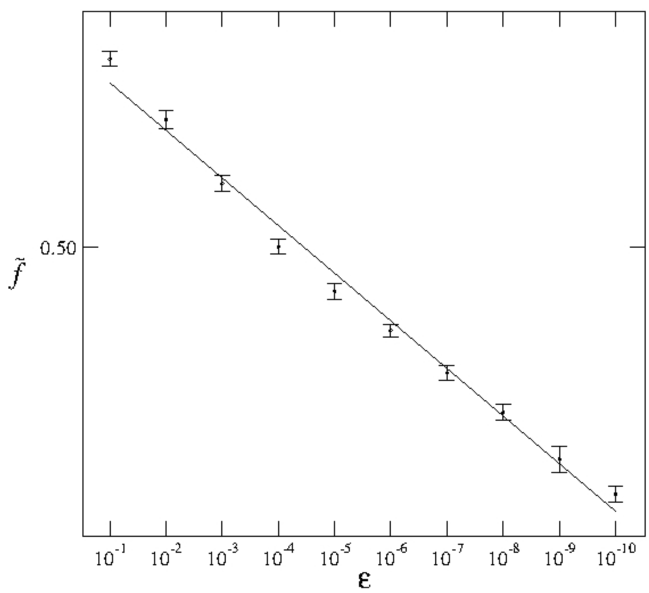

Figure 3 – Log-log plot of $\tilde{f}$ versus ε for the phase portrait shown in figure 1 (b).

Here, the log-log indicates a power law behavior with exponent 0.016 and error 0.0006 , $\tilde{f}(\varepsilon) \approx 0.58\,\varepsilon^{0.016\pm0.0006}$. As the uncertainty exponent α is related to the dimension of the basin boundary, $\alpha = D-d$, and D is the dimension of the phase space, the dimension of the basin boundary is 1.984.

4. DISCUSSION

In this work we investigate the interaction between two drift waves that arises in the edge plasma in tokamak. In the case of one drift wave, there isn't transport of particles, but when we add a second drift wave periodic orbits are deformed. In this case the particles guide centers experienced deviations in their trajectories and the particles that were contained in islands are now free to roam all over the map, thus there is transport of particles. We describe topological properties related to the exit basins, and we observe that the basins have boundaries that are fractals, and calculate the dimension these structures.

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