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UNCERTAINTY ANALYSIS OF SMART COMPOSITE MATERIALS

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Abstract: The present contribution is dedicated to the analysis of uncertainties affecting the dynamic behavior of smart composite structure by using uncertainty fuzzy analysis. The simulation results of the fuzzy analysis are confined to the frequency domain response, being generated the fuzzy structural response of the smart composite structure.

keywords: Modeling, Numerical Simulation and Optimization, Nonlinear Dynamics and Complex Systems, Stochastic Models.

1. INTRODUCTION

The recent years have seen the appearance of innovative materials, as the so-called composite materials, particularly in aerospace applications. The structures constructed by this innovated arrangement are characterized by lightness, mechanical resistance, and the possibility to be optimized for a specific working condition.

Consequently, it is necessary to develop reliable numerical models that take into account uncertain parameters, and allow the prediction of the dynamic behavior of the system under realistic conditions. The present contribution is dedicated to the analysis of uncertainties affecting the dynamic behavior of a piezoelectric actuator bonded to a composite structure forming a so-called smart composite structure. Serendipity-type finite element based on first-order shear deformation theory with rectangular shape, eight nodes, five mechanical degrees of freedom (DOF) per node and eight electrical DOF per piezoelectric layer is established for the composite structural model. With respect to the uncertain analysis, the stochastic method has been extensively used to analyze uncertain parameters~~,~~ by means of the so-called Monte Carlo simulation. However, the fuzzy theory permits to model the uncertainty as an alternative to the stochastic methods. By means of fuzzy theory it is possible to describe incomplete and inaccurate information. herefore, the inherent uncertainties are modeled with the aid of fuzzy analysis. Furthermore, the fuzzy analysis seems to be more appropriated when the stochastic process that models the uncertainties is unknown [1].

In the present contribution, the structural response of a composite cantilever beam is analyzed when some parameters are subjected to uncertainties. The aforementioned uncertain parameters are modeled as fuzzy variables. By using fuzzy uncertainty analysis, based on the α-level optimization [2,3], the uncertain fuzzy parameters are mapped onto the structural response of the composite material.

2. COMPOSITE MATERIAL MODEL

The coupling between the composite structure and the piezoelectric element is made through the Hamilton's variational principle that incorporates all energy contributions presented in the structure.

According to Chee 2006 [4], the coupling elementary matrices using Mixed Theory are the following:

$$[M^e] = \int_{V_e} \rho \left[N_u\right]^T \left[A_u\right]^T \left[A_u\right]\left[N_u\right] dV_e \tag{1}$$

$$[K^e_{uu}] = \sum_{k=1}^{nc} \int_{\xi=-1}^{+1} \int_{\eta}^{+1} \int_{z=z_k}^{z_{k+1}} \left(\left[B_u\right]^T \left[c\right]\left[B_u\right]\right) J dz d\eta d\xi \tag{2}$$

$$[K^e_{u\varphi}] = \sum_{k=1}^{nc} \int_{\xi=-1}^{+1} \int_{\eta}^{+1} \int_{z=z_k}^{z_{k+1}} \left(\left[B_u\right]^T \left[e\right]\left[B_\varphi\right]\right) J dz d\eta d\xi \tag{3}$$

$$[K^e_{\varphi\varphi}] = \sum_{k=1}^{nc} \int_{\xi=-1}^{+1} \int_{\eta}^{+1} \int_{z=z_k}^{z_{k+1}} -\left(\left[B_\varphi\right]^T \left[\chi\right]\left[B_\varphi\right]\right) J dz d\eta d\xi \tag{4}$$

where ρ is the material density, $[M^e]$ is the elementary mass matrix and $[K^e_{uu}]$ is the elementary matrix of elastic stiffness. $[K^e_{uu}]$ and $[K^e_{\varphi u}]$ are stiffness elementary matrices of electromechanical coupling and $[K^e_{\varphi\varphi}]$ is known as dielectric elementary matrix. [c], [e] and [χ] are, respectively, the elastic stiffness, piezoelectric stress and electric permissivity matrices of constant values. $[B_u]$ and $[B_\varphi]$ are input matrices. V_e is the elementary volume. J is the Jacobian of the transformation [1].

Equation (5) shows the global matrices of the model constructed through the standard procedure, in which the subscript g indicates global quantities.

$$\begin{bmatrix}[M_g] & [0]\\ [0] & [0]\end{bmatrix}\begin{Bmatrix}\{\ddot{u}_g\}\\ \{\ddot{\phi}_g\}\end{Bmatrix}+\begin{bmatrix}[K_{uu}] & [K_{u\varphi}]\\ [K_{\varphi u}] & [K_{\varphi\varphi}]\end{bmatrix}\begin{Bmatrix}\{u_g\}\\ \{\phi_g\}\end{Bmatrix}=\begin{Bmatrix}\{F_g\}\\ \{Q_g\}\end{Bmatrix} \qquad (5)$$

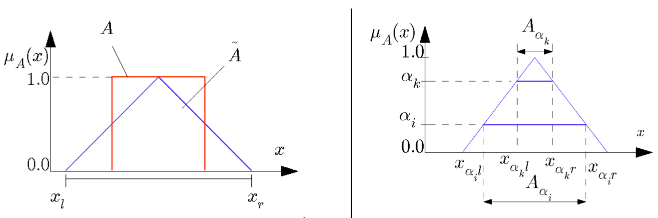
where $\{F_g\}$ and $\{Q_g\}$ are, respectively, the generalized force and nodal charge (elementary).

3. FUZZY UNCERTAINTY ANALYSIS

Several mechanical systems have been analyzed under fuzzy uncertain parameters in order to understand how the structural response varies in the presence of uncertainties. Several mechanical systems have been analyzed under fuzzy uncertain parameters: rotating machinery [3], flexible structures [1] and mass-damper-spring systems [5], among others. By using the fuzzy theory, it is possible to describe incomplete and inaccurate information. The basic concepts of the fuzzy variables are revisited next.

3.1. Fuzzy variables

Let X be an universal classical set of objects whose generic elements are denoted by x. The subset A ($A \in X$) is defined by the classical membership function μ_A: $X \rightarrow \{0,1\}$, shown in Fig. 1. Furthermore, a fuzzy set $\tilde{A}$ is defined by means of the membership function μ_A: $X \rightarrow [0,1]$, being [0,1] a continuous interval. The membership function indicates the degree of compatibility between the element x and the fuzzy set $\tilde{A}$. The closer is the value of $\mu_A(x)$ to 1, more x belongs to $\tilde{A}$.



a) Fuzzy set b) The α-levels

Figure 1. Fuzzy set and α-level representation

Thus, the fuzzy set is completely defined by (where $0 \le \mu_A \le 1$):

$$\tilde{A} = \{(x, \mu_A(x)) \mid x \in X\} \qquad (6)$$

For computational purposes, the fuzzy set $\tilde{A}$ can be represented by means of subsets that are denominated α-levels. These subsets, which correspond to real and continuous intervals, are defined by $A_{\alpha k}$ (Fig. 1), thus:

$$A_{\alpha k} = \{x \in X, \mu_A(\mathrm{x}) \ge \alpha_k\} \qquad (7)$$

The α-level subsets of $\tilde{A}$ have the property:

$$\underline{A}_{\alpha k} \subseteq \underline{A}_{\alpha i} \forall \alpha_i, \alpha_k \in (0,1] \qquad (8)$$

with $\alpha_i \le \alpha_k$. If the fuzzy set is convex for the unidimensional case, each α-level subset $A_{\alpha k}$ corresponds to the interval $[x_{\alpha kl}, x_{\alpha kr}]$, shown in Eq. (9).

$$\begin{aligned} x_{\alpha kl} &= \min\left[x \in X \mid \mu_A(x) \ge \alpha_k\right] \\ x_{\alpha kr} &= \max\left[x \in X \mid \mu_A(x) \ge \alpha_k\right] \end{aligned} \qquad (9)$$

3.2. Dynamic models with fuzzy parameters

In this work, the dynamic model describes the behavior of the rotor by means of a set of differential equations. The relationship between the inputs $\mathbf{x}$ and outputs $\mathbf{z}$ of an specific dynamic model M_f is characterized by f, which represents the set of differential equations of the model in Eq. (10).

$$M_f : \mathbf{z}(\tau) = f(\mathbf{x}) \qquad (10)$$

Therefore, the function f maps the inputs $\mathbf{x}$ onto the outputs $\mathbf{z}(\tau)$. Thus, $\mathbf{x} \rightarrow \mathbf{z}(\tau)$, where τ is the independent variable of the dynamic response that may represent time, frequency or spacial coordinates. Considering the inputs of the model as fuzzy variables $\tilde{\mathbf{x}}$ or fuzzy functions $\tilde{\mathbf{x}}(\tau)$, the dynamic response of the system corresponds to the resulting fuzzy functions $\mathbf{z}(\tau)$. These fuzzy functions result from the mapping, thus $\mathbf{x} \rightarrow \tilde{\mathbf{z}}(\tau)$.

3.3. Fuzzy dynamic analysis

The fuzzy dynamic analysis is an appropriate method to map a fuzzy input vector $\tilde{\mathbf{x}}$ onto the output $\tilde{\mathbf{z}}(\tau)$ of a numerical model by using the deterministic model given by Eq. (10). In structural analysis, the combination of uncertainties modeled as fuzzy variables with the deterministic model based on the finite element method is denominated fuzzy finite element method. The fuzzy dynamic analysis includes two stages, as shown in Fig. 2.

In the first stage, for computational purposes, the input vector that corresponds to the fuzzy parameter is discretized by means of the α-level representation, presented in the Eq. (9) and Fig. 1(b). Thus, each element of the fuzzy parameters vector $\tilde{\mathbf{x}} = (\tilde{x}_1, \ldots, \tilde{x}_n)$ is considered as an interval $X_{i\alpha k} = [x_{i\alpha kl}, x_{i\alpha kr}]$, where $\alpha_k \in (0,1]$. Consequently, the sub-space $\mathbf{X}_{\alpha k}$ is defined so that

$\mathbf{X}_{\alpha k} = (\mathbf{X}_{1\alpha k}, \ldots, \mathbf{X}_{n\alpha k})$, where $\mathbf{X}_{\alpha k} \in \mathrm{R}^n$.

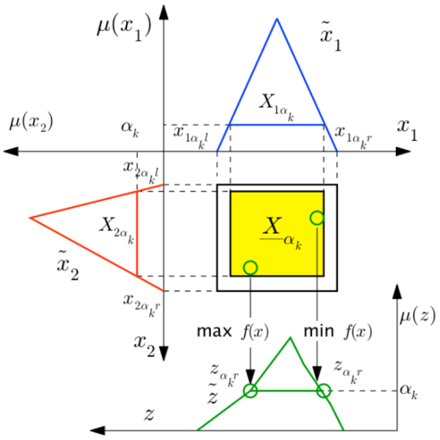


Figure 2. The *α*-Level optimization

The second stage is related to solving an optimization problem. This optimization problem consists in finding the maximum or minimum value of the output, at each evaluated τ, for the mapping model $M_f : \mathbf{z}(\tau) = f(\mathbf{x})$, thus:

$$z_{\alpha kl} = \min_{\mathbf{x} \in \mathbf{X}_{\alpha k}} f(\mathbf{x})$$
$$z_{\alpha kr} = \max_{\mathbf{x} \in \mathbf{X}_{\alpha k}} f(\mathbf{x}) \tag{11}$$

where $z_{\alpha kl}$ and $z_{\alpha kr}$ correspond to the upper and lower bounds of the interval $z_{\alpha k} = [z_{\alpha kl}, z_{\alpha kr}]$ in the α-level α_k. The set of discretized intervals $[z_{\alpha kl}, z_{\alpha kr}]$ for $\alpha_k \in (0,1]$ composes the whole fuzzy resulting variable $\tilde{z}$.

The fuzzy analysis of a transient time-domain system demands the solution of a large number of optimization problems regarding all α-level of interest for each considered time step. Each upper and lower bounds of the system analysis at a given time instant is obtained from the Differential Evolution optimization algorithm. The output value of the transient analysis at the evaluated frequency-step constitutes the objective function. The inputs to this function are the uncertain parameters described previously as fuzzy variables.

4. NUMERICAL SIMULATIONS

The studied laminated composite beam, illustrated in Fig. 3, is such that it has 306 mm length (*L*), 25.5 mm width (*b*), 275.4 mm length ($L_{(I)}$) and 1 mm thickness (*h*). The composite laminate has a total of five layers made of graphite/epoxy and oriented as θ = [45° / 0° / 45° / 0° / 45°]. The layers oriented at 0° are parallel to the *x* axis. The thicknesses of the layers are $c_z = 0.2$ mm.

The constants of elastic stiffness of the beam made of AS4/3501 carbon/epoxy composite are the following (given in GPa): $C_{11} = 173.6$; $C_{22} = C_{33} = 7.61$; $C_{12} = C_{13} = 2.48$; $C_{23} = 2.31$; $C_{44} = 1.38$; $C_{55} = C_{66} = 3.45$. The pzts are: $C_{11} = C_{22} = C_{33} = 102.23$; $C_{12} = C_{13} = C_{23} = 5.035$; $C_{44} = C_{55} = C_{66} = 2.594$. The PZT patch piezoelectric constants are (given in C/m^2) as follows: $e_{31} = -18.300$; $e_{32} = e_{33} = -9.013$. The electric permissivity are (given in F/m) given by: $\chi_{11} = \chi_{22} = \chi_{33} = 1800\varepsilon_0$. The mass densities, in kg/m^3, are 1578 for the composite laminated material.

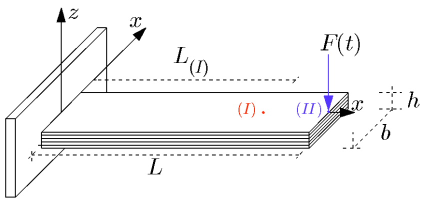


Figure 3. Composite cantiliver beam

The FE model has been derived by using a 10x1 uniform mesh. The unite impulse force was applied at point II and the time domain structure responses were captured at point (I) (see Fig. 3).

Therefore the fuzzy uncertainties are included in the orientation and thickness of the layers, thus:

$\tilde{\theta} = [45° + \tilde{\theta}_i \ / \ 0° + \tilde{\theta}_i \ / \ 45° + \tilde{\theta}_i \ / \ 0° + \tilde{\theta}_i \ / \ 45° + \tilde{\theta}_i]$ and $\tilde{c}_z = 0.2 + \tilde{c}_{zi}$

Concerning the application of the fuzzy analysis, the uncertain parameters were modeled by using fuzzy triangular numbers. Thus,

$$\tilde{\theta}_i = (\Delta\theta, 0, \Delta\theta)$$
$$\tilde{c}_{zi} = (\Delta c_z, 0, \Delta c_z)$$

As mentioned, three uncertainty scenarios were considered in the present contribution (Table 1). The first scenario was dedicated to the investigation of the influence of uncertainties only in the orientation of the composite laminated layers. The second one took into account only uncertainties in the thickness of the layers. Finally, the third scenario considered uncertainties in both the orientation and thichness of the laminated layers. In all scenarios, the FRFs at the point (I) were analyzed.

Table 1. Uncertain scenarios considered on the simulations.

Case	$\Delta\theta$ [°]	Δc_z [mm]
(a)	5	0
(b)	0	0.01
(c)	5	0.01

The Fig. 4 presents the FRF and impact response for the uncertain scenario of the case (a).

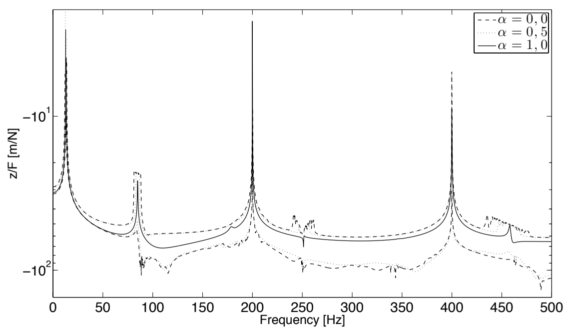


Figure 4. FRF, case (a).

In FRF presented by Fig. 4, it is possible to see a small influence of uncertainties applied in the orientation of the composite laminated layers. Only in the second mode, it can be seen the variation of the FRF. These results demonstrated that the uncertainties in the orientation of layers caused a small variation in the system behavior.

The Fig. 5 presents the obtained results for the uncertain scenario of the case (b). In terms of FRF, it is possible to see more influence of the uncertainties in the thickness of the layers. The level of dispersion increase substantially for all modes presented in the FRF. This FRF demonstrates more influence of uncertainties in the thickness than uncertainties in the orientation of layers (compared the FRFs of Fig. 5 and 4).

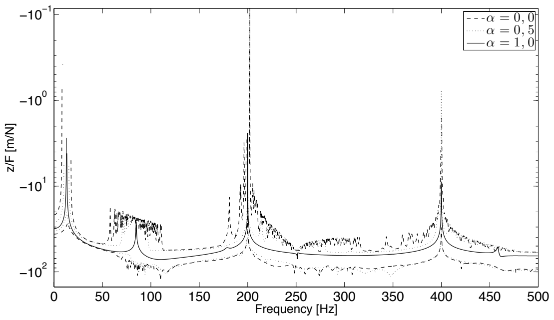


Figure 6. FRF, case (b).

Finally, the Fig. 7 presents the results for the last scenario, that considered uncertainties in both the orientation and thickness of the laminated layers. In this third scenario, it is possible to observe a combination of influnce the unceraintities. The more influence is caused by the uncertainties in the thickness of the layers, but in this third case is demonstrated that the FRF is similar to the FRF of Fig. 7, that reinforces the idea of more influence thickness than the orientation.

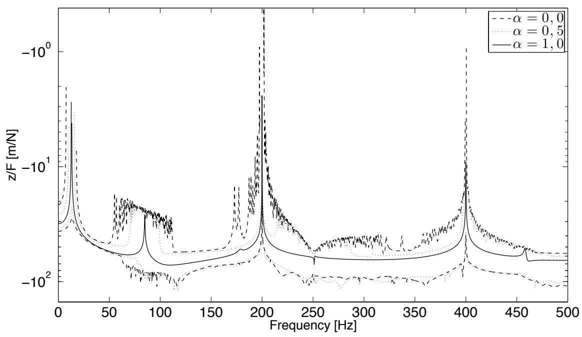


Figure 7. FRF, case (c).

3. CONCLUSION

In this paper the fuzzy uncertainty analysis was used to evaluate the structural response of smart composite materials under several uncertain scenarios. The fuzzy uncertainty analysis is an advantageous approach since it is able to predict the uncertain response produced by uncertain parameters. Based on the simulation results, the small uncertainties included on the parameters introduce a non-negligible variation on the FRF of composite materials.

Finally, the strategy proposed in this contribution shows the relevance of taking into account the uncertain parameters into the design and analysis of smart composite materials. Further studies will encompass evaluations on composite material control under uncertain parameters.

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