INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – May 16-20, 2016

THE LOCAL DYNAMIC EFFECT ON FREQUENCY SYNCHRONIZATION OF NEURONAL NETWORKS

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Abstract: Coupled bursting neurons can exhibit frequency synchronization when the coupling strength is increased above a certain value. The process that makes different frequencies converge to the same value depends on the coupling type and the local dynamic. If we compare two different models of neuronal network they can exhibit similar phase synchronization profiles, however, they might suffer frequency synchronization differently. While topology is the main focus on synchronization studies, here we show that local dynamic also plays an important rule in synchronization process.

keywords: bursting neurons, frequency synchronization, Nonlinear Systems and Neural Dynamics,Synchronization in Nonlinear Systems, Nonlinear Dynamics and Complex Systems

1. INTRODUCTION

Several studies have focus on how topology affects synchronization [1]. A simple and interesting model to study synchronization is the Kuramoto model [2]. This model consist of linear phase oscillators interacting through a senoidal coupling. For many cases this model provides analytical solutions for phase synchronization [3]. The transition observed in Kuramoto oscillators, when they change from an unsynchronized state to a partially synchronized state, is compatible with transitions in neuronal bursting oscillations [4].

In the Kuramoto model, when the coupling strength is increased above a certain value, frequency synchronization can be observed [5]. Considering oscillators synchronized in frequency, as the coupling strength decreases the frequencies splits in groups with different average frequencies. This process continues until all the oscillators recovery their natural frequencies. Approximate analytical expressions for this collective behaviour were presented at [5, 6].

While the phase synchronization in the Kuramoto model is similar to observed in neuronal models with bursting oscillations. The splitting frequency effect is rather different between them. In this work we compare two neuronal models and show their differences in the frequency behaviour.

2. LOCAL DYNAMIC

To study synchronization we will compare two models that describe bursting neurons. The Rulkov model and the Courbage-Nekorkin-Vdovin model, both models are bidimensional maps composed by a fast (x variable) and a slow variable (y variable).

2.1. Rulkv Model

The Rulkov model is described by [7]:

$$x_{n+1} = \alpha/(1+x_n^2) + y_n, \tag{1}$$
$$y_{n+1} = y_n - \eta(x_n - \sigma), \tag{2}$$

in which α, η and σ are parameters, here $\eta = 0.001$ and $\sigma = -1$ [7]. α is the bifurcation parameter, different values of α result in different dynamical behaviours. One example of the bursting activity of this model is shown in the Figure 1.

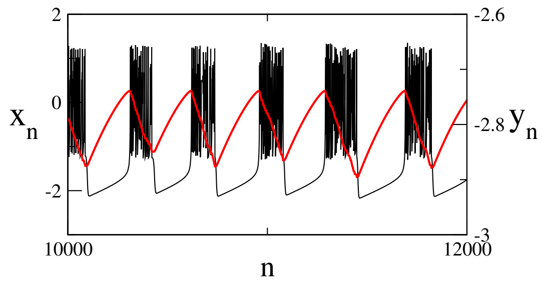

Figure 1 – Example of bursting oscillations produced by the Rulkov model. Black color represents the time serie of the x variable (left side in the graph) and red color the y variable time serie (right side in the graph). This time serie was produced after 10000 transient iterations, here $\alpha = 4.1$.

2.2. *Courbage-Nekorkin-Vdovin (CNV) model*

The CNV model is described by [8]:

$$\begin{aligned} x_{n+1} &= x_n + F(x_n) - y_n - \beta H(x_n - d), &(3)\\ y_{n+1} &= y_n - \sigma(x_n - J), &(4)\end{aligned}$$

where $\beta = 0.25$, $J = 0.13$ and $d = 0.3$ are fixed parameters, σ is a parameter associated with the bursting frequency. The functions F(x) and H(x) are of the form:

$$F(x) = \begin{cases} -m_0 x & \text{if} \quad x \leq J_{min} \\ m_1(x-a) & \text{if} \quad J_{min} < x < J_{max} \\ -m_0(x-1) & \text{if} \quad x \geq J_{max} \end{cases} \quad (5)$$

$$H(x) = \begin{cases} 1 & \text{if} \quad x \geq 0 \\ 0 & \text{if} \quad x < 0 \end{cases}, \quad (6)$$

we define

$$\begin{aligned} J_{min} &\equiv \frac{am_1}{m_0 + m_1} &(7)\\ J_{max} &\equiv \frac{m_0 + am_1}{m_0 + m_1}, &(8)\end{aligned}$$

such that $m_0, m_1 > 0$, in this paper $a = 0.2$, $m_0 = 0.4$ and $m_1 = 0.65$. One example of bursting activity produced by the CNV model is shown in the Figure 2.

2.3. *Coupled Rulkov and CNV neurons*

In this work we will consider neurons with mean field coupling, such that

$$\begin{aligned} x^{(i)}_{n+1} &= \alpha^{(i)}/(1 + (x^{(i)}_n)^2) + y^{(i)}_n + \varepsilon/N \sum_{k=1}^{N} x^{(k)}_n, &(9)\\ y^{(i)}_{n+1} &= y^{(i)}_n - \eta(x^{(i)}_n - \sigma), &(10)\end{aligned}$$

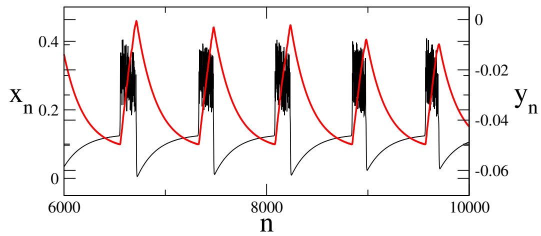

Figure 2 – Example of bursting oscillations produced by the CNV model. This time serie was produced after 6000 transient iterations, here $\sigma = 0.002$.

for each Rulkov neuron i and

$$\begin{aligned} x^{(j)}_{n+1} &= x^{(j)}_n + F(x^{(j)}_n) - y^{(j)}_n - \beta H(x^{(j)}_n - d) \\ &\quad + \varepsilon/N \sum_{k=1}^{N} x^{(k)}_n, &(11)\\ y_{n+1}{}^{(j)} &= y^{(j)}_n - \sigma^{(j)}(x^{(j)}_n - J), &(12)\end{aligned}$$

for each CNV neuron j, N is the network size.

The $\alpha^{(i)} \in [4.1 : 4.3]$ and $\sigma^{(j)} \in [0.0015 : 0.0025]$ parameters will be randomly determined according with an uniform probability distribution. If for each parameter we have an unique associated bursting frequency ω - what is the case inside the mentioned range - then we generate an uniform bursting frequency distribution.

3. PHASE AND FREQUENCY OF BURSTING OSCILLATIONS

For the Rulkov model a burst starts when the y-variable reachs its maximum value, similarly, in the CNV model the burst starts when the y-variable reachs its minimum value. Based on this, for both models we can define a phase φ considering the critical points of the y-variables [9]

$$\varphi_n = 2\pi k + 2\pi \frac{(n - n_k)}{(n_{k+1} - n_k)}, \quad (13)$$

n_k is the initial time of the k-th burst, $n_k = y(\text{k-th critical point})$. Through the equation (13) it is possible to study phase and frequency synchronization among coupled maps. The average bursting frequency ω will be

$$\omega = \lim_{n\to\infty} \frac{\varphi_n - \varphi_0}{n}, \quad (14)$$

considering φ_0 the initial phase.

4. FREQUENCY SPLITTING EFFECT

Despite of the bursting similarities, these model have different dynamical properties. Both models show a similar phase synchronization profile, however, their bursting frequency behavior after phase synchronization is completely different.

As we show in the Figure 3, when the coupling strength ε is decreased the bursting frequency splits, this can be called Frequency Splitting Effect. This effect show us that similar phase synchronization profiles can mask important features of the neuronal activity during synchronization process.

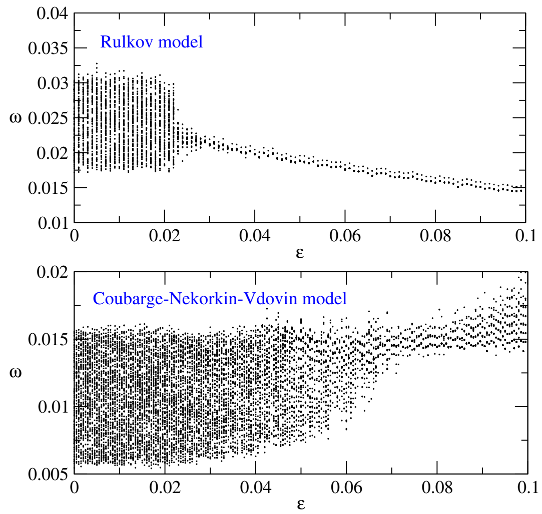

Figure 3 – Comparing the Frequency Splitting Effect between the Rulkov and CNV model. The points here are the bursting frequency ω of each neuron of the network. $N = 100$**.**

5. CONCLUSION

The frequency splitting analyses could be a mechanism for distinguish different types of bursting neurons. The observed effects here are rather different from the frequency splitting observed in coupled Kuramoto oscillators.

ACKNOWLEDGMENTS

This work was made possible through financial support from the Brazilian Research Agencies, CAPES and CNPq. We thanks Ruedi Stoop, co-author of the paper [10], and his research group for their contributions and relevant discussions on this subject.

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