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NONLINEAR DAMPING IN MEMS/NEMS BEAM RESONATORS RESULTING FROM CLAMPING LOSS

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Abstract: We present two new energy loss mechanisms that induce nonlinear damping in suspended beam micro- and nanoresonators. Both mechanisms involve acoustic losses through the clamping region. In one mechanism the axial stress due to the geometric nonlinearity produces the acoustic waves at the clampings. In the second mechanism energy is lost due to the vibrations induced at the clamping region by the higher frequency harmonics present in the beam vibration resulting from the nonlinear dynamics. Analytical expressions for the average energy dissipated, quality factor and nonlinear dissipation coefficients are derived.

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INTRODUCTION

The nonlinear and chaotic dynamics of micro- and nanoelectromechanical (MEMS/NEMS) beam resonators has been investigated due to the need to establish the regime of linear operation, and due to the potential new applications of nonlinear phenomena, for example, to obtain efficient signal filtering and improved sensitivity in sensors, and applications of the chaotic dynamics for signal encryption and random number generation [1, 2]. Two well known sources of nonlinearity in MEMS/NEMS beam resonators are the electrostatic force used to actuate and sense the beam displacement and the nonlinear elastic deformation. More recently, nonlinear damping has also been considered as a relevant source of nonlinear behavior. More importantly, strong nonlinear damping has recently been observed experimentally in both micro [3] and nanoresonators [4]. By the time of these experiments there was no nonlinear damping mechanism that could explain the observations. Later, Croy *et al.* [5], motivated by the observations of Eichler *et al.* [4] for a graphene nanoresonator, investigated how the phonon emissions through the clamping region could induce nonlinear damping on a suspended graphene sheet clamped at both ends (bridge resonator). They modeled the graphene sheet using an elastic continuum model and derived a pair of coupled nonlinear partial differential equations (PDEs) as the equations of motion for the out-of-plane and in-plane vibrations. The dissipation of mechanical energy occurred because the out-of-plane motion was converted into in-plane phonons in the suspended region resulting in phonon emission (acoustic energy loss) through the supporting region. In order to calculate the acoustic emissions it was necessary to discretize the Fourier transformed response of the substrate to the applied stress and obtain a linear system of equations for the in-plane response function that had to be solved numerically. A characteristic of this model is that the parameters characterizing the linear and nonlinear dissipation can not be easily obtained, and no analytical expressions for such parameters as a function of geometrical and material parameters has been derived. Also, no comparison with the experimental data of Eichler *et al.* [4] was made which could establish the relevance of the proposed nonlinear damping (NLD) mechanism.

Fortunately, in the case of more conventional beam resonators, with attachments imbedded directly on the substrate, like in the experiment by Zaitsev et al. [3], the analysis can be considerably simplified. For such resonators we identify two acoustical energy loss mechanisms which are presented in the next sections.

NLD INDUCED BY THE GEOMETRIC NONLINEARITY

In this section we present the NLD mechanism recently proposed by one of the authors [6]. The NLD originates from the axial stress induced along the beam due to the geometric nonlinearity, evidenced in the second term of the partial differential equation (PDE) that models the beam dynamics

$$EIw'''' - (N_0 + N(t))\, w'' + \rho A\ddot{w} + c\dot{w} = \mathcal{F}(x,t)\,. \quad (1)$$

In this PDE we have that $N(t) = EA/(2l) \times \int_0^l w'^2 dx$, $w = w(x,t)$ denotes the transversal displacement of the beam located between $x = 0$ and $x = l$, the overdots and primes represent derivatives with respect to time, t, and space, x, respectively, E denotes the Young modulus, I the geometric moment of inertia, A the cross-sectional area, and ρ the beam density. N_0 is the axial load due to residual or applied stress. Included in the equation is a viscous damping term, which accounts for any existing linear damping mechanisms, and the forcing along the beam $\mathcal{F}(x,t)$.

The time varying axial stress $N(t)$ is the result of the geometric nonlinearity and induces a periodic normal forcing at the clamping region. In its turn, this normal forcing results in acoustic waves that carry away a fraction of the vibrational energy of the beam. Working in the quasi-harmonic approximation for the nonlinear regime we take $w(x,t) = u_0\phi_n(x)\cos(\omega_n t)$, where $\phi_n(x)$ correspond to the mode-shapes of the Euler-Bernoulli beam. The normal force becomes $N(t) = EAI_1^n/(4l^2)u_0^2\cos(2\omega_n t) = F\cos(2\omega_n t)$, with $I_1^n = \int_0^1 [\phi_n'(y)]^2 dy$. The acoustic emissions are going to the depend upon the details of the geometry of the clamping region. Here we consider the case of of a beam clamped to semi-infinite thin plates of thickness h. In this case the average emitted acoustic power due to a harmonic normal force with amplitude F is given by

$$\Pi^p = \frac{2\phi_0(\gamma)}{\pi}\frac{1+\nu}{E(1-\nu)}\,\omega\frac{F^2}{h}\,, \quad (2)$$

where ν denotes the Poisson ratio, $\phi_0(\gamma)$ is a suitable integral, and ω the frequency of the forcing. To obtain the quality factor for the $n-th$ mode, given by $Q_n = \omega_n U/\Pi^{s,p}$, for the loss taking place at both ends of the beam, we have to determine the maximum mechanical energy U. Considering that the nonlinear PDE can be reduced using the Garlekin method to the following nonlinear ordinary differential equation (ODE)

$$m\ddot{u}_n + 2b_{11}\dot{u}_n + k_1 u_n + k_3 u_n^3 = F\cos(\omega_n t)\,, \quad (3)$$

U can be approximated by $U = u_0^2(k_1 + \frac{k_3}{2}u_0^2)/2$. The resulting quality factor for a beam with rectangular cross section (width w and thickness h) is

$$\begin{aligned} Q_n &= \frac{\pi\rho(1-\nu)l^5\omega_n^2}{\phi_0(\gamma)(I_1^n)^2E(1+\nu)w\,u_0^2} + \frac{\pi(1-\nu)l}{4\phi_0(\gamma)(1+\nu)w} \\ &= Q_{n,1} + Q_{n,2}\,. \end{aligned} \quad (4)$$

We can see that Q_n can be separated into an amplitude dependent term $Q_{n,1}$ and a constant term $Q_{n,2}$. The amplitude dependent Q is a characteristic of the presence of nonlinear damping. In fact, such dependence with u_0^{-2} is compatible with the presence of nonlinear damping terms of the form $b_{31}u_n^2\dot{u}_n + b_{32}\dot{u}_n^3$ in the reduced order model. By comparison of $Q_n$with the corresponding expression for the nonlinear Q obtained from the reduced order model $Q = 4k_1/(\omega_n b u_0^2) + 2k_3/(\omega_n b)$ we can obtain an expression for the effective nonlinear damping parameter $b_n = b_{31} + 3b_{32}\omega_n^2$

$$b_n = \frac{4\phi_0(\gamma)(I_1^n)^2(1+\nu)Ehw^2}{\pi(1-\nu)l^4\,\omega_n}\,. \quad (5)$$

In order to illustrate the potential relevance of the proposed nonlinear damping mechanism, in Figure 1 we present the frequency-response curve for a realistic MEMS resonator with and without the contribution of the nonlinear damping given by Eq. (5). We can see a significant effect at the onset of bistability.

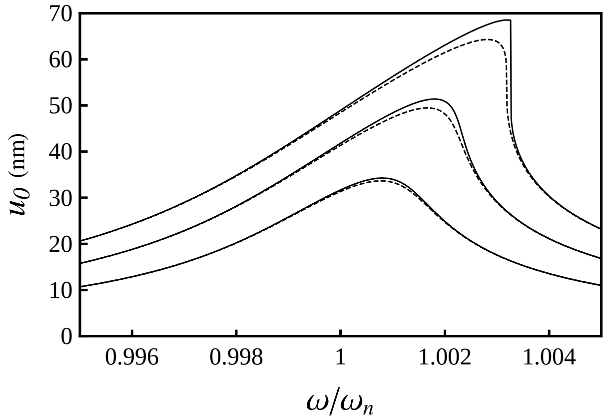

Figure 1 – Frequency-response curves for a microresonator made from polysilicon with dimensions $h = 1\,\mu$m, $l = 30\,\mu$m, and $w = 2\,\mu$m oscillating in its first mode $n = 1$ and with an intrinsic quality factor $Q_0 = 300$. The lower, middle, and higher curves have been obtained for an harmonic forcing with amplitude $F = 6\times10^{-8}, 9\times10^{-8}$, and 12×10^{-8} N, respectively. Continuous lines represent the response in the absence of nonlinear damping ($b = 0$) and the dashed ones with nonlinear damping included.

ENERGY LOSS THROUGH HIGHER HARMONICS

We have been also investigating another energy loss mechanism that leads to nonlinear damping. The nonlinear damping emerges when we incorporate the effect of the presence of the higher harmonics into the analysis of the conventional clamping loss mechanism. In this mechanisms the transversal vibration of the beam induces shear and bending moments at the clamping region, which result into the emission of acoustic waves that carry away a fraction of the vibrational mechanical energy. So far, only the harmonic vibration of the beam has been considered in calculating the emitted acoustic energy [7, 8]. However, both bridge and cantilevered beam resonators may develop nonlinear vibrations due to the nonlinearity of the applied external forces, p. ex. the electrostatic force, or due to the geometric nonlinearity, in the case of bridge resonators.

In the nonlinear regime and in the steady state, the displacement in the reduced order model can be described by a Fourier series

$$u(t) = \sum_{p=0}^{\infty} A_p \cos(p\omega t + \phi_p)\,, \tag{6}$$

where A_p denotes the amplitude of the $p-th$ harmonic. This anharmonic displacement results in an anharmonic forcing at the clamping region. Considering the same case we analyzed in the previous NLD mechanism, of a beam clamped to a semi-infinite plate and performing in-plane vibrations, we have that the main contribution to the acoustic energy loss comes from the shear force, the bending moment having a negligible contribution [7]. The shear force at the clampings $(x = 0, l)$ is given by $S = EIw'''|_{x=0,l}$ = 2 E I $(\kappa_n/l)^3 \chi_n u_n(t)$ where, p. ex., $\kappa_n = 4.7300, 7.8532, \ldots$ and $\chi_n = -0.9825, -1.0008, \ldots$ for the modes $n = 1, 2, \ldots$ of a bridge with $N_0 = 0$. Because the process of acoustic emission is a linear process for the small forces produced by the vibrating beams, we can apply the superposition principle and the total emitted acoustic power is given as the sum of the power emitted at each frequency of the forcing. The average emitted acoustic power through one of the clampings, at a frequency close to that of a resonant mode therefore is,

$$\Pi_n = \frac{\phi(\gamma)}{18\pi}\,\frac{E(1+\nu)}{1-\nu}\,\kappa_n^6 \chi_n^2 \omega_n \frac{hw^6}{l^6} \sum_{p=1}^{\infty} p A_p^2\,, \tag{7}$$

where $\phi(\gamma)$ represents an appropriate integral. If we consider now that external forces, such as the electrostatic force, can result in an additional quadratic restoring force $k_2 u_n^2$ to Eq. (3), and that even in a comparatively strong nonlinear regime $A_1 \gg A_2, A_3, \ldots$, the maximum mechanical energy can be well approximated by $U = A_1^2(k_1/2 + k_2 A_1/3 + k_3 A_1^2/4)$. We thus obtain for a bridge (with $N_0 = 0$) or cantilever resonator with a rectangular cross section the quality factor

$$Q_n = C_n \frac{1-\nu}{E(1+\nu)}\,\frac{l^6}{hw^6}\,\frac{k_1/2 + k_2 A_1/3 + k_3 A_1^2/4}{1 + 2(A_2/A_1)^2 + 3(A_3/A_1)^2}\,. \tag{8}$$

In this expression for Q_n we have that $C_n = 28.30/[\kappa_n^6 \chi_n^2 \phi(\gamma)]$ in the case of a bridge with $N_0 = 0$, and $C_n = 18\pi/[\kappa_n^6 \chi_n^2 \phi(\gamma)]$ in the case of a cantilever.

In the above expression for Q we have limited the number of harmonics to 3 because even in a strong nonlinear regime the condition $A_1 \gg A_2, A_3, \ldots$ is well satisfied. The amplitudes A_1, A_2 and A_3 have not, so far, been derived analytically as a function of all parameters appearing in Eq. (3). Only approximate results, valid in the weak nonlinear regime and for frequencies very close to the resonant frequency have been derived [9]. For this reason, we have developed a new approach, based on the harmonic balance method, that allow us to obtain very general and, in some cases, very simple analytical expressions for the A_p. Our approach is based on the fact that the set of nonlinear algebraic equations that is usually solved numerically to obtain such coefficients can be separated into subsets of one or two equations whose solution give us A_0, A_2, A_3, etc, as a function of the dominant amplitude A_1.

For example, let us consider the case in which $k_2 = 0$. Because the resulting nonlinear ODE, Eq. (3), is odd, we can assume a steady state solution of the form $u(t) = a_1 \cos(\omega t) + b_1 \sin(\omega t) + a_3 \cos(3\omega t) + b_3 \sin(3\omega t)$, with $A_p = \sqrt{a_p^2 + b_p^2}$ and $\phi_p = \tan^{-1}(-b_p/a_p)$, $p = 1, 3$. Substitution of this solution in Eq. (3) results in a set of four nonlinear algebraic equations. Two equations of the set are strong functions of a_1 and b_1, and can be used to give approximate results to these two coefficients if we take $a_3 = b_3 = 0$. This is justified because a_3 and b_3 are expected to be much smaller than a_1 and b_1. The two remaining equations are strong functions of a_3 and b_3, and can be used to give these coefficients as a function of a_1 and b_1. We have been able to obtain analytical results for a_1 and b_1 and, consequently, to A_1. However, the expression is too large to be reproduced here. The resulting expression for A_3 is much simpler

$$A_3 = \frac{k_3 A_1^3}{2[4(9c^2\omega^2 + K^2) + 3k_3 A_1^2(4K + 3k_3 A_1^2)]^{\frac{1}{2}}}\,, \tag{9}$$

where $K = k_1 - 9m\omega^2$. This equation provides the magnitude of the third harmonic as a function of all parameters in Eq. (3), including the excitation frequency, except for the forcing F that enters indirectly trough A_1. This result was compared with A_3 obtained from the numerical solution of the ODE and gives excellent results, even in the strong nonlinear regime. We have also derived analogous equations for A_0 and A_2. As we calculate the quality factor at the resonant frequency, we can substitute in the equation for Q_n simplified expressions for the A_p, namely, $A_2 = k_2/(6k_1)A_1^2$ and $A_3 = k_3/(32k_1)A_1^3$. Therefore, the Q_n can be written as

$$Q_n = C_n \frac{1-\nu}{E(1+\nu)}\,\frac{l^6}{hw^6}\,\frac{k_1/2 + k_2 A_1/3 + k_3 A_1^2/4}{1 + \frac{1}{18}\left(\frac{k_2}{k_1}\right)^2 A_1^2 + 3\left(\frac{k_3}{32k_1}\right)^2 A_1^4}\,. \tag{10}$$

We have thus evidenced the amplitude dependence of the quality factor. Therefore, the energy loss through the higher harmonics results in nonlinear damping. This time, however, Q depends not only on A_1^{-2}, but also on A_1^{-4}. In fact, if we considered higher frequency harmonics, higher powers of A_1

would appear in the denominator of Eq. (10) . Therefore, the NLD due to the acoustic emission through the higher harmonics can be incorporated into the reduced other model, Eq. (3), including terms with powers of $\dot{u}_n$ or the product of powers of this velocity with powers of the displacement u_n multiplied by an appropriate coefficient, which could be determined as we have done for b_n in the previous section. Eq. (10), also indicates that the NLD can be relevant when the ratios k_2/k_1 and k_3/k_1 are largest.

Further simulations of realistic MEMS/NEMS resonators are required to establish the actual relevance of the proposed NLD mechanisms. Also, other configurations of clamping, such as the clamping to semi-spaces considered in Ref. [6] for the first NLD mechanism, could be considered for the second mechanism we have proposed. Also, for this last case, the NLD for beams attached to a plate and vibrating transversaly should be considered.

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