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ON NONLINEAR OSCILLATIONS MODELING IN STRUCTURAL ENGINEERING AND SOLAR CORONA

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Abstract: The goal of this work is to discuss the possibility of applying models for vortex-induced vibration, usually associated with structural engineering problems, in a new area. The models could be useful to model the dynamics of nonlinear oscillation in coronal loops. Models for vortex-induced vibration use two-nonlinear coupled oscillators to model the system: one oscillator describing the oscillation of the structure and other one describing the vortex dynamics in a fluid. The coupling of nonlinear oscillators like Van der Pol, Duffing and Duffing-Van der Pol could be useful in this modeling.

Keywords: Analysis and control of nonlinear dynamical systems with practical applications, bifurcation analysis and applications, fluids dynamics, plasma and turbulence, Van der Pol, vortex.

1. INTRODUCTION

Vortex-induced vibration (VIV) and vortex-induced motion (VIM) are complex problems that occur when there is a turbulent flow and fluid-solid interaction phenomena.

In structural engineering, for example, it is a serious problem trying to minimize the oscillation of platforms caused by Vortex shedding from ocean currents.

On the other hand, on the solar corona, there is the interaction between the plasma of the solar surface with the plasma inside of the magnetic structures called coronal loops.

Votyakov and Kassinos [1] discussed the analogies between the magnetohydrodynamics flow moving through a region where an external local magnetic field (magnetic obstacle) is applied and the hydrodynamic flow around a solid obstacle.

Based on these analogies, we propose the application of the methods used in engineering in the new area of the coronal seismology. The coronal seismology uses the oscillations observed as a diagnostic tool to determine the physical condition of the coronal plasma; for this reason the study of the nonlinear dynamic of oscillation in the coronal loops is relevant.

2. VORTEX-INDUCED VIBRATION IN ENGINEERING

The induced vibration in elastic structures of engineering by vortex shedding of a fluid flowing around this structure is of practical importance due its potentially destructive effects on bridges, stacks, towers, offshore pipelines [2].

The vortex shedding frequency (frequency of formation of vortex) is given by the empirical formula

$$f_s = V.\, S_t \,/\, D, \qquad (1)$$

where the non-dimensional parameter S_t is the Strouhal number, V is the free-stream velocity and D is the cylinder's diameter (or the structure's dimension).

When the frequency of vortex shedding f_s matches with the natural frequency of oscillation of the structure f_0, the resonance or "lock in" phenomenon occurs and the structure can oscillate with large amplitude.

The Reynolds number $Re = V.D / \nu$, where ν is the fluid kinematic viscosity, not only determines the flow patterns, but it also has a direct influence on the Strouhal number, that by its turn modifies the frequency of vortex shedding f_s.

On fig.1 and fig.2, we can see the different regime of flow and the dependence of the Strouhal number on the Reynolds number, respectively.

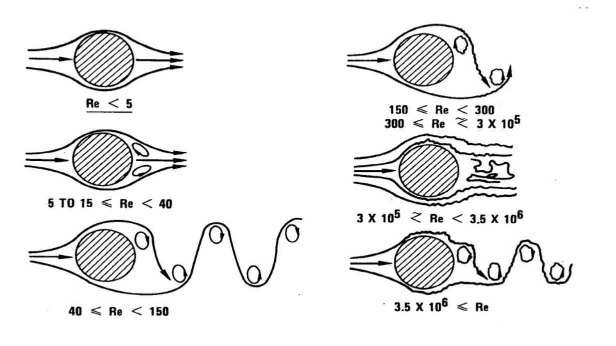

Figure 1: Dependence of the vortex street with the Reynolds number [2]

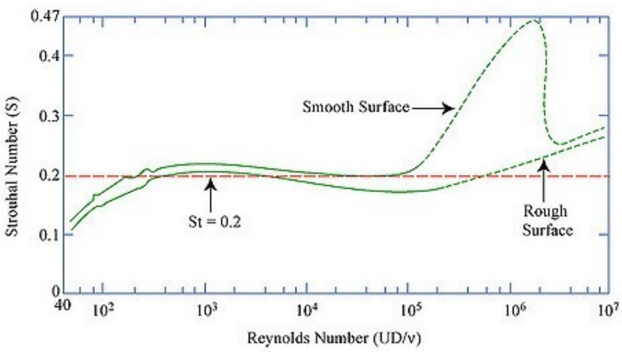

Figure 2: Relation between Strouhal number and Reynolds number for circular cylinders [2].

3. THE MODEL AND EQUATIONS

The nonlinear dynamics of oscillators is used like a first approach to the system resulting of the interaction between an elastic structure and a fluid flowing around this structure. On the following, we present the equations involved on this modeling.

3.1 Nonlinear oscillators

The nature of evolution of physical systems depends upon the nature of the forces acting on them and on their initial state. For the one-dimensional case, we can write this force as

$$F(y,y',t) = F(y,y') + F(t). \tag{2}$$

and Newton's second law becomes

$$m\, d^2y / dt^2 - F(y,y') = F(t). \tag{3}$$

Using the Taylor's series expansion of $F(y,y')$, we get

$$F(y,y') = F(y_0,v_0) + [a_1y + b_1y'] + [a_2y^2+c_{11}yy'+b_2y'^2]+[a_3y^3+c_{21}y^2y'+c_{12}yy'^2+b_3y'^3]+\ldots \tag{4}$$

where we can see the first-order terms in the first bracket, and so on.
According to the terms kept in the Taylor's series, we get a different nonlinear oscillator, for example:

a) Van der Pol oscillator

$$F(y,y') = [a_1y + b_1y'] + c_{21}y^2y' \tag{5}$$

b) Duffing oscillator

$$F(y,y') = [a_1y + b_1y'] + a_3y^3 \tag{6}$$

c) Duffing-Van der Pol oscillator

$$F(y,y') = [a_1y + b_1y'] + a_3y^3 + c_{21}y^2y' \tag{7}$$

all these oscillators have nonlinearities in terms of third order.
We can find on the literature much information about the dynamics of the Van der Pol and Duffing oscillators. However, studies about the Duffing-Van der Pol oscillator are a recent subject of research.

3.2 Coupled nonlinear oscillators

We use two-nonlinear coupled oscillators to model the system: one oscillator describing the oscillation of the structure and other one describing the vortex shedding in a fluid:

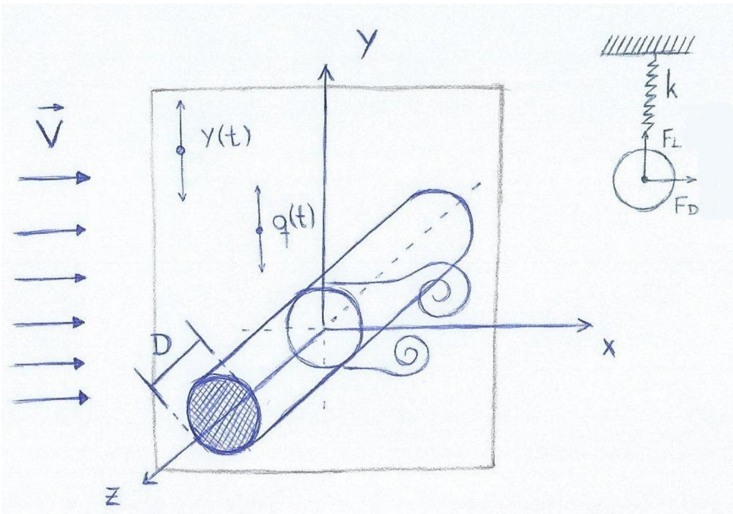

Figure 3: Forces on the structure, where F_L and F_D are the lift force and the drag force, respectively.

a) Oscillator describing the structure

$$(m + m_a)\, d^2y / dt^2 - F_y(y, y') = F_q(q, q', t) \tag{8}$$

b) Oscillator describing the vortex shedding dynamics in a fluid (the wake oscillator model)

$$(m + m_a)\, d^2q / dt^2 - F_q(q, q') = F_y(y, y', t), \tag{9}$$

where m_a is the added mass or hydrodynamic mass. In the case of a cylinder we have $m_a = \rho\pi r^2$.
The wake oscillator model is a heuristic model that uses

an oscillator with a single degree of freedom to represent the dynamic associated with the vortex shedding in a fluid when this fluid flows around a solid structure, a stationary circular cylinder on the present case.

It is usual to keep the nonlinearity only on the oscillator describing the fluid [2]. However, the normal frequency of oscillation of the structure can be modified when effects of nonlinearity are included on the structure too. Studies with the Duffing oscillator show that this actually occurs. If this fact is considered, the resonance condition will be completely modified. One of our future goals is to analyze these modifications on the context of the coronal loops oscillations. The mechanisms of energy exchange between the plasma of solar surface and the plasma on the coronal structures can be modified by these nonlinear effects and a better understanding of the solar eruptions would be possible.

4. OSCILLATIONS IN THE SOLAR CORONA

The plasma can be seen like a charged fluid that involves many of the dynamical behavior of a neutral fluid, meanly the nonlinearities of hydrodynamics equations. The fact that the plasma is a charged fluid makes the problem more complex. The use of the magnetohydrodynamics to describe the coronal plasma is usual and it holds due to the plasma density, intensity of the magnetic field and conductivity on this region.

On the solar corona there are magnetic structures called coronal loops. It is believed that oscillations on these structures can contribute to the determination of the mechanisms of emission of radiation by the sun.

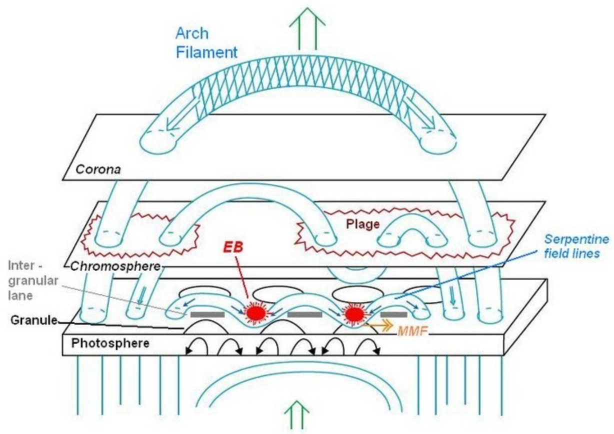

Figure 4: Sketch of the emergence of magnetic flux through the lower layers of the solar atmosphere. Extracted figure from ref. [5].

Few works [3,4] discuss the role of vortex shedding in the excitation of transverse oscillation on the coronal loops. However, the role of the nonlinearity was considered on the models of these works. In general, a simple linear forced oscillator is used [3]. Different than what is found on the engineering literature, where the use of two coupled oscillators have been used to describe the interaction of structure-fluid, but with the nonlinearity included only on the description of the fluid.

5. CONCLUSION

The structure of equations proposed on section three is useful to take into account the role of nonlinearity in a growing degree of complexity. On both oscillators, which describe structure and fluid, it is necessary to include nonlinear terms in order to get a more realistic description of the system and a better understanding of the resonance phenomena.
On the area of engineering, the nonlinearity has been included only on the oscillator describing the fluid, more specifically, using the Van der Pol oscillator. On the area of solar physics nonlinear effects have not been taking into account yet. We propose the use of two coupled nonlinear oscillators to describe the dynamics of the oscillation on coronal loops, with inclusion of nonlinearity on both oscillators. The coupling of nonlinear oscillators like Van der Pol, Duffing and Duffing-Van der Pol could be useful in this modeling. In particular, we can use the Van der Pol oscillator to model the fluid and the Duffing oscillator to model the elasticity of the coronal loops.

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