INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – May 16-20, 2016

IIR EQUALIZATION BASED ON COMPLEXITY MEASURES IN THE CONTEXT OF CHAOTIC INFORMATION SOURCES

Patrick Franco Coutinho[1], Diogo Coutinho Soriano[2], Filipe Ieda Fazanaro[2], Romis Attux[1]

1 DCA/FEEC/UNICAMP, Campinas, Brazil, attux@dca.fee.unicamp.br
2 CECS/UFABC, Santo André, Brazil, {diogo.soriano, filipe.fazanaro}@ufabc.edu.br

Abstract: In this work, it was analyzed the use of complexity measures derived from recurrence quantification analysis to solve the problem of blind IIR equalization of FIR channels for chaotic signals. The results show that measures related to the complexity of the ensuing recurrence plot allow perfect equalization when this is structurally feasible.

keywords: analysis and control of nonlinear dynamical systems with practical applications, channel equalization, modeling, nonlinear dynamics and complex systems, recurrence quantification analysis.

1. INTRODUCTION

The problem of channel equalization can be defined as an information source *s(n)* sent through a communication channel with impulse response *h(n)* and, potentially, additive noise (this will not be the case in this work). The signal is received as a distorted version *x(n)*, and the aim is to invert the effects of the channel in order to recover the desired information. This is done by carefully designing a filter, termed *equalizer*.

When it is possible to have access to samples of *s(n)*, the equalization task is considered to be *supervised*. If this is not the case, and only general *a priori* information is available, the task is referred to as *blind equalization* [1].

The problem of blind equalization is closed related to another inverse problem, that of *Blind Source Separation* (BSS) [2]. BSS is characterized by the need for recovering, in a blind fashion, a set of information sources from mixtures of them. Therefore, blind equalization is a sort of "BSS in the time domain" [2].

Classically, BSS is associated with Independent Component Analysis (ICA) in view of the usual hypothesis that the sources are mutually independent [1]. However, there are practical instances in which this hypothesis is not valid, which demands a distinct prior. The use of sparseness or non-negativity are examples of popular alternatives [3].

Interestingly, in 1998, Pajunen [4] proposed a different perspective to the handling of the BSS problem. He built a separation criterion based on algorithmic complexity, conjecturing that, at least for natural signals, the complexity of a mixture tends to be higher than that of its components. To put in numerical terms the rather ethereal notion of complexity, he quantized the focused signal and used Lempel-Ziv coding.

In 2011, Soriano et al. [5] revisited this framework by employing complexity measures derived from Recurrence Quantification Analysis (RQA) to solve the BSS problem concerning mixtures of deterministic and stochastic signals. Afterwards, the work developed in Ref. [6] have shown that the classical recurrence-based measures could also be used for separating mixtures involving just deterministic signals. In 2015, Soriano et al. [7] adopted a measure more akin to that of Pajunen, the Lempel-Ziv compression rate of the recurrence plot of a given signal, which connected complexity algorithm theory to the RQA framework.

The approaches developed by Soriano and his collaborators were originally applied in the context of blind equalization of an infinite impulse response (IIR) channel in a preliminary work by Coutinho et al. [8]. The obtained results were quite encouraging, but there remained important open questions, such as the possibility of performing IIR equalization of a Finite Impulse Response (FIR) channel using the same approach. In this work, we deal exactly with this problem, aiming at analyzing the soundness of complexity-based channel equalization, a brand new research line within the theory developed under the aegis of this most relevant problem.

In section 2, we discuss the notion of recurrence quantification analysis together with some important measures. In section 3, the proposal is presented and discussed. Section 4 brings the results and section 5 the conclusions.

2. RECURRENCE QUANTIFICATION ANALYSIS AND COMPLEXITY MEASURES

Recurrence Quantification Analysis (RQA) consists of a method for time series analysis that aims to characterize the recurrence structure of an observation. The technique usually employs specific statistical measures taken from the Recurrence Plot (RP) [9], which is a binary matrix (R(i,j)) that contains the result of the comparison between the distance $\mathbf{x}(i)$ to $\mathbf{x}(j)$ and a chosen threshold ε, in which j = 1, 2, …, N. The comparisons between $\|\mathbf{x}(i) - \mathbf{x}(j)\|$ and ε obey:

$$R(i, j) = \begin{cases} 1; \text{ if } \|\mathbf{x}(i) - \mathbf{x}(j)\| < \varepsilon \\ 0; \text{ otherwise} \end{cases} \tag{1}$$

being $\mathbf{x}(k)$ the state space vector obtained after Takens' embedding:

$$\mathbf{x}(k) = [x(k) \;\; x(k-\tau) \;\; \ldots \;\; x(k-(d_e - 1)\tau)] \tag{2}$$

in which d_e is the embedding dimension and τ the lag between samples to engender the other coordinates of the state space.

In the recurrence domain, each specific signal will generate a binary "fingerprint" related to its generative dynamics, with many characteristics being mapped onto specific RP features. In particular, diagonals parallel to the main one are related to the coevolution of states, which provides longer structures for more regular (*e.g.* sinusoidal) signals. Interpretation of typical RP structures can be found in [9].

Once that the recurrence matrix is available, typical RQA measures can be obtained in terms of diagonal length distribution, defining different measures of signal complexity *i.e.* different quantifiers for its organization [9]. Having this is mind, a previous work [7] has introduced the possibility of computing algorithmic complexity of an RP based on ZIP coding strategy that employs a combination between Lempel-Ziv and Huffman coding [10] for describing the sequence of bits. Our approach here relies on generating a single string of bits ***s*** from the concatenation of the diagonals in the superior triangular part of R(i,j) – Figure 1 –, capturing the coevolution of states. The string is then zipped and size of the produced file is then compared to the size of original string, leading to the expression:

$$\text{ZIP}_{\text{index}} = \frac{\text{size}_{\text{ZIP}}}{\text{size}_{\text{original}}} \tag{3}$$

Therefore, rich coevolution profiles (*i.e.* large variety of diagonals lengths) tend to produce maps that are more easily compressed, leading to a ZIP_{index} close to zero, while strings poor in diagonals diversity tends to present a higher ZIP_{index}.

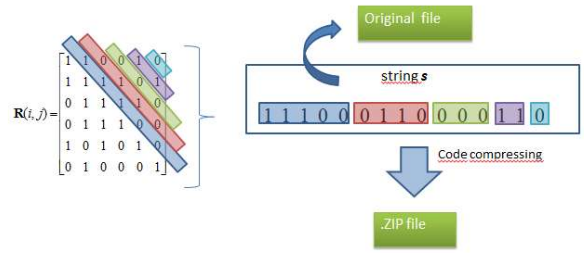

Figure 1 - Concatenation of diagonals in RP in order to form a single string to be ZIP compressed and compared to the size of original string.

3. BLIND IIR EQUALIZATION OF FIR CHANNELS BASED ON COMPLEXITY MEASURES

Following the assumption presented by Pajunen [4], it is assumed that the intersymbol interference introduced by the channel will tend to generate a received signal more complex than the original information source. Therefore, the main idea to achieve equalization in a blind fashion by minimizing a complexity measure – here, (3) was employed.

Differently from the *IIR channel* [8], the *FIR channel* does not impose an extremely long impulse response to the signal being transmitted. The implication thereof is that the complexity of the signal at the receiver is just slightly larger than that at the transmitter. Hence, to perform an efficient recovery of the source signal, the choice of the parameters of the RPs must be made carefully. For some combinations of τ and ε, the complexity of the transmitted signal may be lesser than that of the source. This situation completely misleads the process of equalization, once the main hypothesis is not satisfied.

4. RESULTS

To evaluate the proposed idea, the following numerical experiment was performed. It was considered s(n) to be a normalized version (zero mean and unit variance) of *500* samples derived from the *logistic map*, which is described by the following equation:

$$x_{n+1} = \mu x_n (1 - x_n) \tag{4}$$

where $\mu = 4$. The channel is a FIR filter with unit norm coefficient vector and the following transfer function:

$$H(z) = \cos(\varphi) - \sin(\varphi) z^{-1} \tag{5}$$

in which $\varphi = \pi/6$. No additive noise was taken into account. The IIR filter that models the equalizer is analogously structured in terms of a single variable θ:

$$W(z) = \frac{1}{\cos(\theta) - \sin(\theta) z^{-1}} \tag{6}$$

To perform blind equalization, an exhaustive search over θ is carried out. Due to the IIR filter, the range of the θ parameter is from 0 to $\pi/4$, which ensures system stability. 100 values of θ in the specified range were used. For each θ, the RP of the recovered signal is generated and its complexity is evaluated. The relation between the compressed and the uncompressed file that represents the RP is taken as an estimate of the subjacent complexity - the ZIP_{index} in (3). It should be noted that, for $\theta = \pi/6$, the equalizer perfectly inverts the channel *i.e.* $H(z)W(z) = 1$.

In respect to the RP parameters, the choice of the *threshold* relies on the simple "rule of thumb" of considering values varying from 20% to 40% of the signal's standard deviation [11]. On the other hand, the selection of the *delay time* value was made based on preliminary tests, in which results of the blind equalization for each value of τ were examined.

The variation of the complexity measures for values of ε ranging from 0.2 to 0.4, $\tau = 2$, $d_e = 2$, and θ changing as aforementioned is shown below (Figure 2). The figure depicts how the complexity measure changes depending on the values of ε and θ. As expected, the minima in all cases are close to the value of $\pi/6$ for θ. Figure 3 compares the three involved signals, $s(n)$, $x(n)$ and the source estimate $y_{rec}(n)$ in a globally optimal situation. It is visually clear that equalization is duly performed.

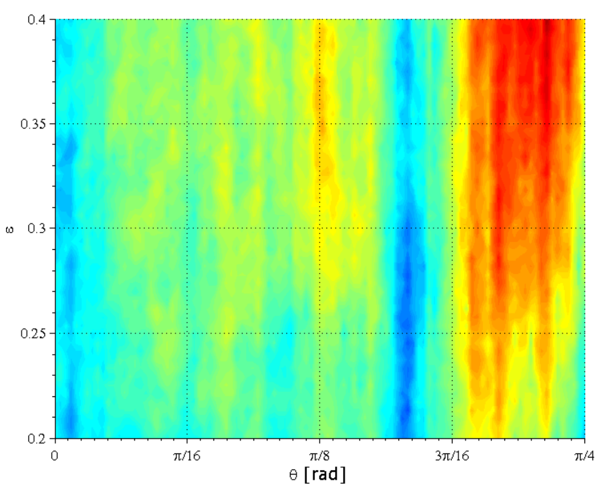

Figure 2 - Variation of the complexity measure - estimated via ZIP_{index} - when the blind equalizer parameters are modified. All cases show minima around the value $\theta = \pi/6$. At these points, channel inversion is attained.

5. CONCLUSIONS

In this work, it is shown that it is possible to perform blind equalization of a chaotic signal transmitted over a FIR channel using an IIR equalizer and a properly tuned complexity measure. This complements the work [8] in that the case of a channel with a small impulse response is dealt with. The results were satisfactory, but it should be remarked that there is a noticeable sensitivity with respect to the parameters of the recurrence plot. This indicates the need for further investigation in this scenario and also in the more complex case of FIR equalization of a FIR channel.

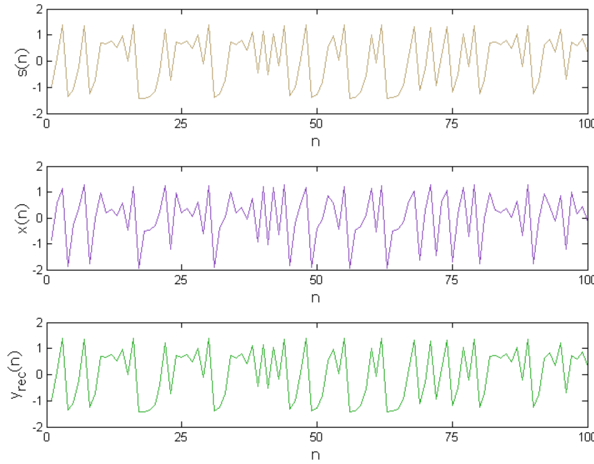

Figure 3 – 100 samples of each signal: $s(n)$, $x(n)$ and $y_{rec}(n)$. $s(n)$ and $y_{rec}(n)$ are both normalized. At the point of minimum complexity, for each value of ε, the recovered signal is an excellent estimative of the source signal.

ACKNOWLEDGMENTS

The authors thank CNPq (449467/2014-7, 449699/2014-5, 305621/2015-7) for the financial support.

REFERENCES

[1] J. M. T. Romano, R. Attux, C. C. Cavalcante, R. Suyama, *Unsupervised Signal Processing: Channel Equalization and Source Separation*, CRC Press, 2010.

[2] A. Hyvärinen, J. Karhunen, E. Oja, *Independent Component Analysis*, Wiley, 2001.

[3] P. Comon, C. Jutten, *Handbook of Blind Source Separation: Independent Component Analysis and Applications*, Academic Press, 2010.

[4] P. Pajunen, "Blind Source Separation Using Algorithmic Information Theory", Vol. 22, Nos. 1 – 3, pp. 35 – 48, 1998.

[5] D. C. Soriano, R. Suyama, R. Attux, "Blind Extraction of Chaotic Sources from Mixtures with Stochastic Signals Based on Recurrence Quantification Analysis", Digital Signal Processing, Vol. 21, pp. 417-426, 2011.

[6] D. C. Soriano, R. Suyama, R. A. Ando, R. Attux and L.T. Duarte, "Blind Source Separation in the Context of Deterministic Signals", In: Chaotic Signals in Digital Communication, Ed: M. Eisencraft, R. Attux and R. Suyama, CRC Press, pp. 325-348, 2013.

[7] D. C. Soriano, F. I. Fazanaro, R. Attux, R. Suyama, "Estimating Algorithmic Complexity of Dynamical Systems and Time Series Using Recurrence Plots", Sixth International Symposium on Recurrence Plots Grenoble – France, 17-19/06/2015.

[8] P. F. Coutinho, F. I. Fazanaro, D. C. Soriano, R. Suyama, R. Attux, "Preliminary Results Concerning the Use of Complexity Measures in Blind Equalization", Proceedings of the VII Simpósio de Instrumentação e Imagens Médicas (SIIM) / VI Simpósio de Processamento de Sinais da UNICAMP (SPS), Vol. 2, p. 84, 2015.

[9] N. Marwan, M. C. Romano, M. Thiel and J. Kurths, "Recurrence Plots for the Analysis of Complex Systems", Physics Reports, Vol. 438, pp. 237-329, 2007.
[10] T. M. Cover, J. A. Thomas, *Elements of Information Theory*, Wiley, 2006.
[11] S. Schinkel, O. Dimigen, and N. Marwan, "Selection of Recurrence Threshold for Signal Detection", The European Physical Journal Special Topics, Vol. 164, No. 1, pp. 45-53, 2008.