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Nonlinear Dynamics of an Origami Structure Coupled to Smart Materials

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Abstract: Self-folding structures obtained from origami concepts require small actuators to transition between shapes assumed by the structure. This paper deals with the dynamic analysis of an origami-wheel actuated by shape memory alloys. Numerical simulations are carried out analyzing mechanical and thermal external excitations. The combination of strong geometrical and constitutive nonlinearities is responsible for rich dynamics that should be properly investigated in the system design. Folding and operational excitations can affect the system response being critical to several applications.

keywords: Nonlinear Dynamics, Analysis of Nonlinear Dynamical Systems, Practical Applications of Nonlinear Systems, Origami structure, Smart Materials

1. INTRODUCTION

Origami (*oru* – fold and *kami* – paper) is an ancient art from Japan that creates a sculpture representation through pattern folds of a flat paper sheet, producing a desired shape. Origami art starts to inspire researchers in order to produce systems and devices for various fields of science and technology [1–2]. Some characteristics inherent to these structures, such as self-adaptability and compacting capacities, are motivating applications in several engineering fields.

Smart materials are essential for actuation of origami system. The use of shape memory alloys (SMAs) and magnetic materials allows the change between two different configurations of the origami structure by applying a thermal field, an electric current or a magnetic field.

[3] studied the kinematic behavior of the origami's geometry, aiming its application in adaptive frontage with solar skin. [4] fabricated a sheet that can be thermally activated to self-fold into a miniature origami robot that is controlled by an alternating external magnetic field. Origami design also inspired the development of a new type of stent graft made from foldable NiTi foil [5].

Dynamics of origami systems is important especially due to its thin structure. The combination of strong geometrical and constitutive nonlinearities is responsible for a rich dynamic that should be properly investigated in the system design. Folding and operational excitations can affect the system response being critical to several applications.

This paper deals with the dynamic analysis of a deformable wheel robot that is constructed employing the magic-ball pattern, a well-known pattern that changes its shape from a long cylindrical tube to a flat circular tube.

2. ORIGAMI WHEEL

The origami wheel uses the well-known waterbomb pattern being composed of basic element represented by Figure 1. The magic-ball pattern is built by repeatedly adding up the waterbomb pattern, with end of the wheel structure pattern redesigned [6]. Therefore, it is possible to change between the two main configurations (cylindrical and flat circular tube).

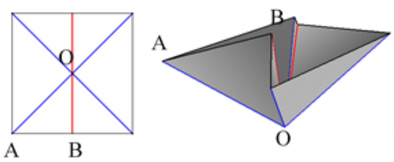

Figure 1 – Waterbomb pattern (left – [7]). Blue line means valley fold and red line means mountain fold. Shape of the unitary component (right).

2.1. Geometrical modeling of the structure

The origami-wheel is not symmetric due to the non-symmetrical distribution of the applied forces. Nevertheless, one can assume that this is a minor effect that can be neglected. Furthermore, it is reasonable to assume that the fold motion occurs only in the crease and the facets remain straight. These assumptions are assumed in order to establish relations between angles and distances of the origami.

Figure 2 shows the basic component of the origami wheel. The unitary cell is assumed to be a square with size $2a$. Figure 3 shows some views of the origami where L_1 is the length of the SMA actuator, L_2 is the half-length of the elastic actuator, R_1 and R_2 are respectively internal and external radius, and b is the radius of the circle circumscribed to the octagonal acrylic plate.

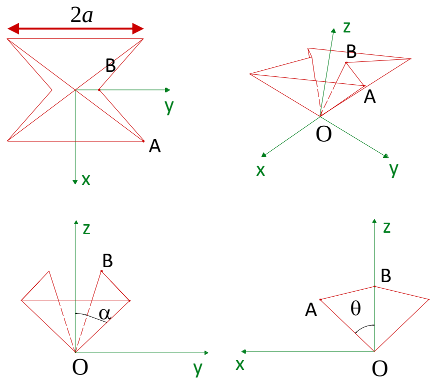

Figure 2: Analysis of the unitary cell of the origami wheel.

From Figure 2, it is possible to establish relation between the angles α_0 and θ_0. Note that $|AB|=a$, $A = (a\sin\theta, a, a\cos\theta)$ and $B = (0, a\sin\alpha, a\cos\alpha)$.

$$\cos\theta\cos\alpha + \sin\alpha = 1 \qquad (1)$$

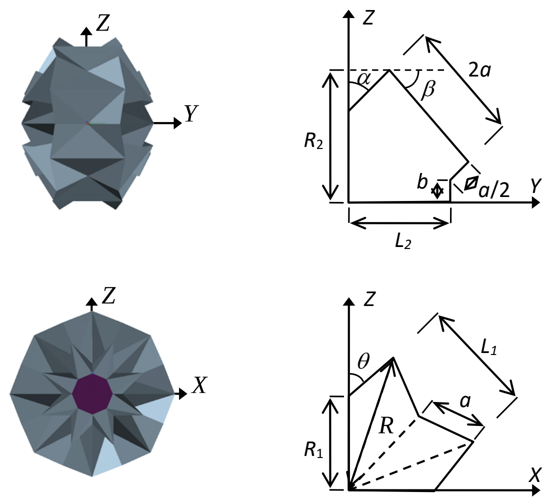

Figure 3: Kinematic analysis of the origami wheel in *Y-Z* (axial) and *X-Z* (circumferential) planes.

From *Y-Z* plane, it is possible to obtain relations for L_2 and R_2, as functions of the angles and sizes a and b.

$$L_2 = a\sin\alpha + 2a\cos\beta - 0.5a\sin\beta \qquad (2)$$

$$R_2 = b + 2a\sin\beta + 0.5a\cos\beta \qquad (3)$$

From *X-Z* plane, it is possible to obtain expressions for R_1, L_1 and a relation between R_1 and R_2.

$$R_1 = a\frac{\sin(\theta - \frac{\pi}{8})}{\sin\frac{\pi}{8}} \qquad (4)$$

$$R_2 - R_1 = a\cos\alpha \qquad (5)$$

$$L_1 = 2a\sin\theta \qquad (6)$$

Using equations (1) to (6), an explicit relation between L_2 and L_1 is obtained. A mounting configuration has been assumed in such a way that the SMA spring has a residual deformation at a low temperature (T = 288 K, where martensitic phase is stable) in a stress-free state. Figure 4 shows L_2 as a function of L_1 where the point P_0 represents the mounting configuration, b = 0.04m and a = 0.065m; P_2 refers to a condition of maximum opening of the wheel and P_1 refers to a condition of minimum open condition.

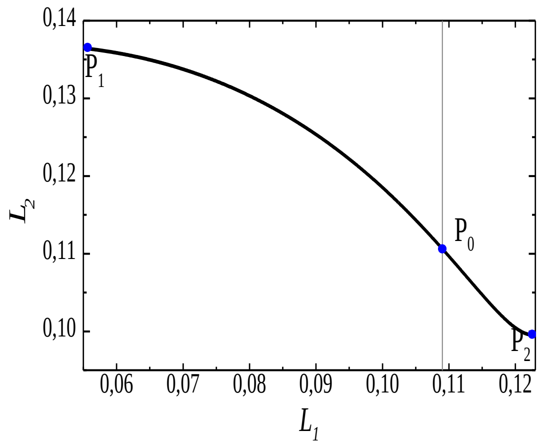

Figure 4: Half-length of the passive elastic spring as a function of the length of the SMA spring.

The transition $P_0 \rightarrow P_1$ occurs by heating the SMA actuator, inducing a phase transformation from twinned martensite (M) to austenite (A), recovering the residual deformation. In other way, if the SMA is cooled, reverse transformation is induced and the wheel comes back to the initial configuration (P_0).

2.2. Dynamic model

Figure 5 shows the origami wheel with the SMA actuators and the passive elastic spring. By assuming a hypothesis that the origami has a symmetric force distribution and folds occur only on the creases, spring displacements, L_1 and L_2, are related to efforts transmission.

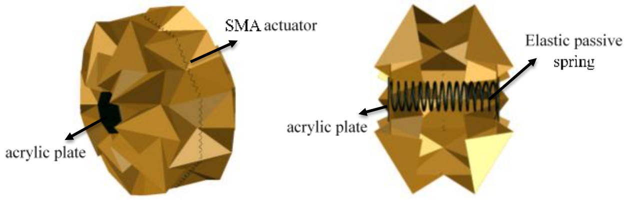

Figure 5: Origami wheel for dynamic analysis, with the SMA actuators and passive elastic spring.

Figure 6 presents a model representation of the forces where E_2 is a force applied in the acrylic plate and E_1 is the corresponding force acting in the radial direction. The relation between E_1 and E_2 is provided by the following equation, obtained by considering homogeneous distribution of efforts in all structure.

$$E_1 = E_2 \frac{\cos\alpha + \sqrt{2}\cos\theta}{4\cos\beta} \tag{7}$$

Moreover, it is defined F_E as the force of the elastic spring (L_2) and F_{SMA} as the force of the SMA spring (L_1).

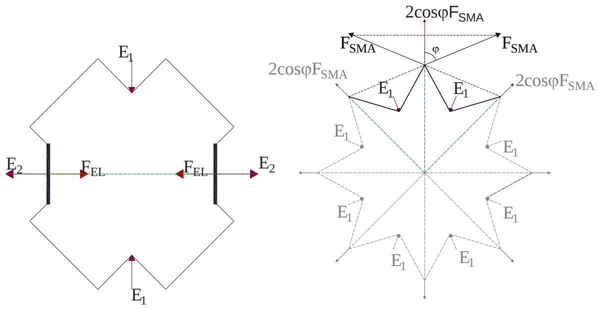

Figure 6: Free-body diagram in the longitudinal and radial views.

The displacement of the passive elastic spring and the SMA spring are given by $u_E = L_2 - L_2^0$ and $u = L_1 - L_1^0$ respectively, where L_2^0 is the half-length of the elastic spring at P_0 point and L_1^0 is the length of the SMA spring also at P_0 point. Each acrylic plate has a mass M and the structure mass is assumed to be discretely distributed around the radial view, and each one is represented by m.

SMA spring nonlinear behavior is described by the polynomial constitutive model [8]. By assuming a homogeneous distribution of the phase transformation along the wire cross-section, force-displacement relation is similar to stress-strain constitutive equation [9-10].

Under these assumptions, and with the help of Figures 3 and 6, the dimensionless equation of motion can be written as a first order differential equations system, where $\ddot{u}_E = f_1\ddot{u} + f_2\dot{u}^2$ and $h = \frac{(\cos\alpha + \sqrt{2}\cos\theta)}{4\cos\beta}\cos\frac{\pi}{8}$.

Dimensionless displacement U and time τ are assumed such that $U=u/L$ and $\tau = t\,\omega_{REF}$.

$$\begin{cases} \dot{U} = V \\ \dot{V} = \delta(t) + \dfrac{(\gamma_1(T-T_M)U - \gamma_2 U^3 + \gamma_3 U^5) - h\eta f - \xi V - {}^{M}\!/_{m}\, h f_2 V^2}{{}^{M}\!/_{m}\, h f_1 - \sqrt{{}^{2+\sqrt{2}}\!/_{2}}} \end{cases} \tag{8}$$

3. NUMERICAL SIMULATIONS

The origami-wheel is extremely nonlinear, and it has a rich dynamic behavior. External mechanical forcing is represented by a harmonic series and, for the sake of simplicity, only two terms are employed. Parameters presented in Table 1 are employed and a linear viscous damper with coefficient ξ=0.1.

Table 1: Parameters adopted in numerical simulations

m (kg)	M (kg)	T_M (K)	T_A (K)	c_1 (MPa/K)
0.08	0.12	291.4	326.4	50
c_2 (MPa)	c_3 (MPa)	N_E	G_E (GPa)	d_E (m)
7×10^5	7×10^7	30	80	3×10^{-3}
D_E (m)	N_{SMA}	d_{SMA} (m)	D_{SMA} (m)	L (m)
20×10^{-3}	10	2×10^{-3}	3.5×10^{-3}	0.089

The transition between origami shapes can be seen in Figure 7, where the SMA is heated to 373 K. Residual strain is totally recovered and the structure assumes a new stable configuration (P_1). After that, the SMA is cooled again, returning to the initial configuration (P_0). Note oscillations due to inertia effects.

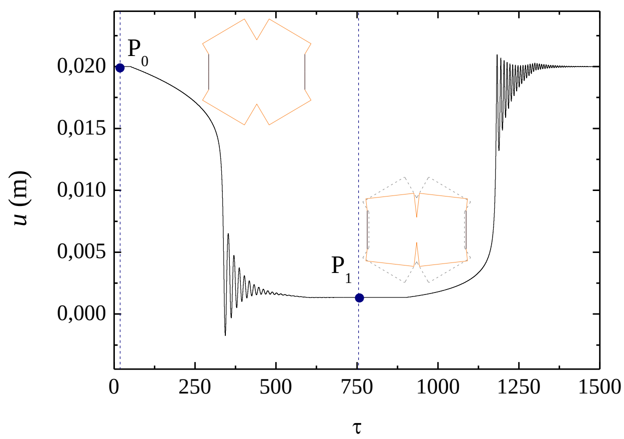

Figure 7: Origami behavior due to temperature change.

Bifurcation diagram presented in Figure 8 shows a global view of the nonlinear dynamics of the origami system. Basically, it varies δ_1 with ω_1=0.8, ω_2=3, $\delta_2=7\times10^{-4}$, T = 288K.

For low values of δ_1 ($\delta_1 < 7.5\times10^{-3}$) the system has a periodic response. For higher values, the sytem presents bifurcations (between $\delta_1=10\times10^{-3}$ and $\delta_1=15\times10^{-3}$) reaching a chaotic-like behavior.

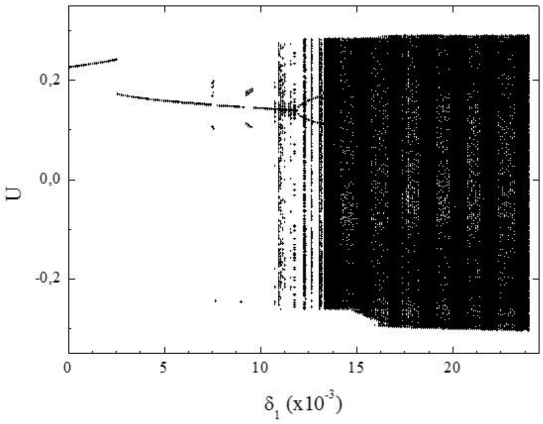

Figure 8: Bifurcation diagram for T=288K and ξ=0.1

Figure 9 shows the Poincaré section for δ_1=0.024. Note a chaotic, strange attractor with lamellar structure.

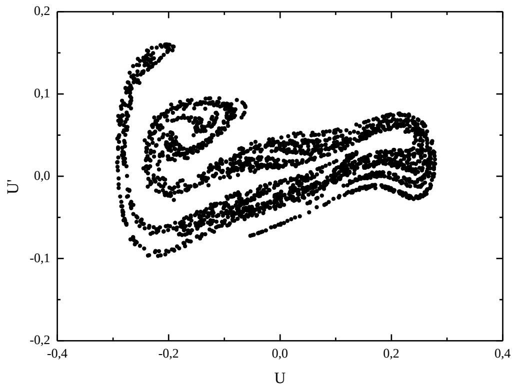

Figure 9: Strange attractor for δ1=0.024

Dynamical jumps are also present on system response. Figure 10 shows a bifurcation diagram built considering the maximum amplitude under a slow quasi-static variation of frequency ω_1 and a constant temperature *T*=288K. Up-sweep and down-sweep tests allow one to observe one jump between 0.7 and 0.8.

3. CONCLUSIONS

This paper deals with the dynamical behavior of an origami wheel. A one degree of freedom model is proposed allowing a proper understanding of the nonlinear dynamics of the origami system. Numerical simulations are carried out showing a rich dynamic behavior by considering different operational situations.

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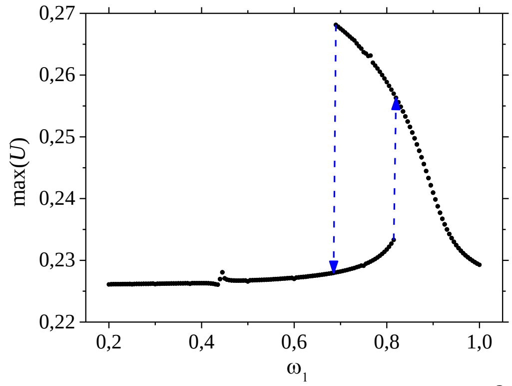

Figure 10: Dynamical jumps for ξ=0.01, δ_1=10^{-3} and δ_2=7×10^{-4}.

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