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EIGENVALUE ANALYSIS OF A SIMPLE FLEXIBLE ROTOR

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Abstract: There are several methods available to solve the eigenvalue problem for a rotor system. In this work the finite element formulation of a simple rotor system is performed. After that, the subspace iteration method is applied to calculate the Natural Frequencies, which are obtained for different numbers of elements. Finally the results are compared between the various discretization performed.

keywords: Dynamical Systems, Modeling, Numerical Simulation.

1. INTRODUCTION

The eigenvalue problem is widely discussed in several sorts of disciplines. Regarding dynamical problems, it is necessary to solve the eigenvalue problem in order to obtain natural frequencies and vibration modes of structures, which can be critical if not treated properly.

For a rotor system it is crucial to determine critical speeds. For this purpose the damping matrix must be considered and the simple eigenvalue problem becomes a quadratic eigenproblem.

Consequently a mathematical model is necessary, in this case it will be used finite element formulation for rotor system which is very well explained in [1]. It takes into account the inertial and gyroscopic effects allowing the determination of mass and stiffness matrices, from whom natural frequencies and vibration modes are obtained, only lateral vibration are addressed in this work.

Additionally, it is important to state that the rotor, as a continuous body, has infinite degrees of freedom and consequently infinite natural frequencies. However the interest are the most relevant vibration modes, which are the lower ones.

In [2] it is widely discussed the eigenvalue problem and its application in dynamic problems. And [3] contains a straightforward analysis of vibration for rotor systems. The aim of this work is to investigate how the finite element modeling can affect the results of the eigenvalue problem, which was done by several simulations and comparison.

2. METHODS

2.1. Rotor

The rotor system which is modeled by finite element is shown in Figure 1. The rotor dimensions are show in Table 1 and the material properties of both the shaft and the disk are shown in Table 2. Both K_{xx} and K_{zz} worth 7.73 10^5 N/m.

The analysis consist in investigating whether each modeling is good or not to achieve natural frequencies values, several modelings were performed, considering different number of elements. Table 3 lists the different discretizations performed where Ne is the number of elements, the simplest was a two element model and the the largest had a hundred and twenty elements, for the non rotational case, when considering rotation the largest model had sixty elements,Nn is the number of nodes and elength is the elementar length. Bearing damping and gravitational force were neglected in the modeling.

The equation of motion for the rotor system is given by:

$$[M]\ddot{u} + [\dot{\phi} C_g]\dot{u} + [K]u = F_{ext}, \tag{1}$$

where $[M]$ is the mass matrix, $[C_g]$ is the gyroscopic matrix, $[K]$ is the stiffness matrix, u is the generalized displacement

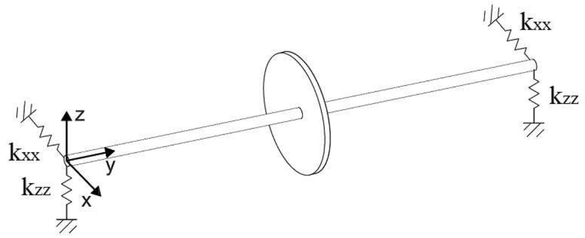

Figure 1 – Finite Element discretization.

Table 1 – Rotor Characteristics

Shaft		Disk	
$Mass(kg)$	0.339	$Mass(kg)$	1.990
$Length(m)$	0.550	$Thickness(m)$	0.010
$Diameter(m)$	0.010	$Diameter(m)$	0.180

Table 2 – Rotor Material Properties

Young Modulus (N/m^2)	2.05 10^{11}
Poisson Coefficient	0.30
Density (kg/m^3)	7850

Table 3 – Finite Element Models

Model	Ne	Nn	elength (mm)
1	2	3	275 .00
2	4	5	137.50
3	6	7	91.67
⋮	⋮	⋮	⋮
60	120	121	4.58

vector, F_{ext} is the resultant external force and $\dot{\phi}$ is the angular speed. Given that the purpose of this work is to obtain the critical speeds, the damping from the bearing was neglected. Gravitational forces are also not considered.

For the rotor system the generalized displacement vector is chosen as:

$$u = [x\, z\, \theta_x\, \theta_z]^T. \tag{2}$$

In order to achieve the natural frequency for the non-rotational case, the equation to be handled is:

$$[M]\ddot{u} + [K]u = 0. \tag{3}$$

There are numerous methods to solve this simple eigenvalue problem, such as Lanczos and QR algorithms, in this work, the subspace iteration method was chosen since it had shown fast convergence for the lower modes.

When the rotor rotates the achievement of the natural frequency requires a different procedure.This occurs due to the contribution of the gyroscopic effect in the equation of motion, which is treated in [4]. The natural frequencies can be found by solving the following equation:

$$[M]\ddot{u} + [\dot{\phi}C_g]\dot{u} + [K]u = 0. \tag{4}$$

In order to have different solution then the trivial one, it is necessary that the frequency equation is satisfied:

$$det([K] + p[C_g] + p^2[M])u = 0, \tag{5}$$

where p is equal to the natural frequency.

This quadratic eigenproblem need first to be transformed into a generalized eigenproblem, this is called the direct method. Where we reach:

$$AX = \lambda BX \tag{6}$$

where

$$A = \begin{bmatrix} 0 & M \\ M & C \end{bmatrix}, \tag{7}$$

$$B = \begin{bmatrix} M & 0 \\ 0 & -K \end{bmatrix}, \tag{8}$$

$$\lambda = \frac{1}{p} \tag{9}$$

and

$$X = \begin{bmatrix} pu \\ u \end{bmatrix}. \tag{10}$$

Finally, the subspace iteration method can now be used to find the natural frequencies. The results are expected to be imaginary and for this case, the natural frequency is the imaginary part of the eigenvalues.

3. RESULTS

3.1. Non rotational case

The natural frequencies obtained by several models are summarized in Table 1.

It can be observed that all models are able to give accurate values for the first vibrational frequency, however the simpler models failed to obtain the higher modes. Other conclusion from this simulation is that after the model with Ne=20 only minor variations occurred. And since the increase in the processing time is significant, this model is a good one to work with this kind of system.

The shape of the first seven vibrational modes are shown in Figure 2, they were obtained from the eigenvectors obtained from (3).

3.2. Rotation case

The natural frequencies were achieved for the rotor with 5000 rpm, the results are shown in Table 2. The pattern is very similar to the stationary case. For this case the maximum number of elements simulated had Ne=60, that is because, as explained in prior section, the size of the matrix is doubled to solve the quadratic problem.

Table 4 – Simulated Natural Frequencies (rad/s)

Mode	Ne=2	Ne=4	Ne=6	Ne=8	Ne=20	Ne=40	Ne=60	Ne=80	Ne=100	Ne=120
1	114.91	114.9	114.9	114.9	114.9	114.9	114.9	114.9	114.9	114.9
2	2903.12	2428.45	451.92	451.92	451.91	451.91	451.91	451.91	451.91	451.91
3	9031.42	6205.54	2412.3	2409.42	2408.6	2407.97	2407.96	2407.96	2407.96	2407.96
4	-	13654.28	2420.28	2417.39	2416.57	2415.93	2415.92	2415.92	2415.92	2415.92
5	-	37109.3	6053.91	6012.72	6000.28	5990.09	5989.98	5989.95	5989.93	5989.93
6	-	-	6073.9	6032.73	6020.29	6010.08	6009.97	6009.93	6009.92	6009.91
7	-	-	11487.57	11455.66	11396.95	11338.65	11337.99	11337.79	11337.71	11337.66

Table 5 – Simulated Natural Frequencies (rad/s) **with 5000 rpm rotation**

Mode	ne=2	ne=4	ne=6	ne=8	ne=10	ne=20	ne=30	ne=40	ne=50	ne=60
1	4.78	4.78	4.78	4.78	4.78	4.78	4.78	4.78	4.78	4.78
2	114.81	114.59	114.23	113.77	113.23	110.37	108.74	108.5	108.99	109.71
3	115	115.22	115.58	116.05	116.6	119.61	121.39	121.66	121.12	120.33
4	2901.43	2424.32	2403.78	2395.14	2382.05	2350.91	2330.1	2326.86	2332.88	2341.92
5	2904.79	2380.83	2372.46	2375.71	2387.57	2305.76	2284.16	2280.8	2287.06	2296.47
6	2862.71	2383.69	2360.36	2351.56	2343.76	2420.34	2443.2	2446.76	2440	2429.98
7	2859.33	2432.59	2420.87	2423.83	2429.9	2466.8	2488.87	2492.32	2485.82	2476.14

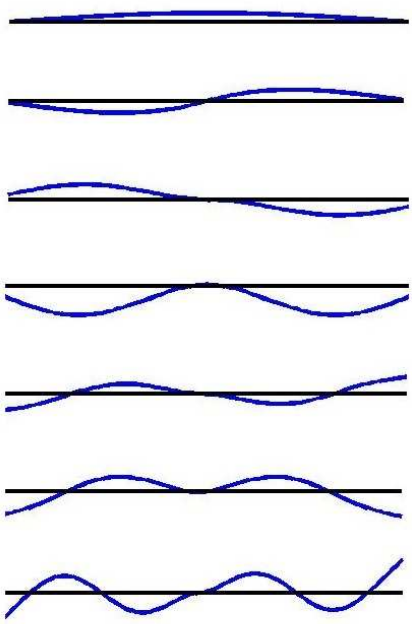

Figure 2 – Vibration Modes

4. CONCLUSION

From all the analysis performed it is can be concluded that the number of elements in a modeling procedure need not to be so high if the intention is to obtain few lower frequencies, but if the need is for higher modes, then it is important to use more elements in the model, this conclusion is valid both for stationary and rotational state of the rotor.

It can be observed also that increasing excessively the number of elements has no practical effects.

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