INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – May 16-20, 2016

PERIODIC CONTROL APPLIED TO ATTITUDE CONTROL OF THE SERPENS II MISSION

Felipe Coelho[1], *André Luís da Silva*[2], *José Renes Pinheiro*[3]

[1] Federal University of Santa Maria, Santa Maria, Brazil, felipesn.coelho@gmail.com
[2] Federal University of Santa Maria, Santa Maria, Brazil, taurarm@gmail.com
[3] Federal University of Santa Maria, Santa Maria, Brazil, jrenespinheiro@gmail.com

Abstract: This paper presents a periodic controller design for attitude control of a small spacecraft equipped with electromagnetic actuators only. This approach considers a discrete linear time-varying model of the spacecraft and a periodic approximation of the geomagnetic field. The method is based on the solution of the periodic Riccati equation. Numerical simulations were conducted in MATLAB/Simulink for validation. Results have shown good performance. The designed periodic controller coped with significant initial conditions, stabilizing and converging to the nadir-attitude within less than 5 orbits.

keywords: nonlinear dynamical systems, modelling, numerical simulation, periodic control.

1. INTRODUCTION

The number of small satellite missions has considerably increased in the past years, even more owing to the CubeSat standard satellites which have drawn the attention of several universities and industry around the world.

Usually, small satellites are designed for Low-Earth Orbit (LEO) missions, but these satellites have limited instrumentation due to dimensional and power limitations, low-cost budget and available technologies. Considering this scenario, magnetic actuation becomes appreciable for attitude control. In such cases, electromagnetic devices are used to perform stabilization and corrective maneuvers. However, critical issues arise as electromagnetic actuators are limited to produce control torque perpendicular to the geomagnetic field only. Because of that magnetic actuated spacecraft are said to be underactuated.

Several papers investigating control strategies for LEO spacecraft applications have been carried out. A quaternion based modelling and LQR control analysis is discussed in [1]. Also, important analysis regarding the controllability of spacecraft using electromagnetic actuators is investigated in [2]. In [3] is discussed a controller design based on LQ-tracker and $\boldsymbol{H}_\infty$ techniques for a magnetic actuated 1U Cubesat. Taking into account the periodicity of the geomagnetic field, periodic controller design is proposed in [4] solving the periodic Riccati equation in continuous time domain. Motivated by the latter, [5] proposed an efficient algorithm for solving the periodic Riccati equation in the discrete time domain.

The main contribution of this work lies on the design of a periodic controller combining the methods presented in [4] and [5] applied in the context of the SERPENS II Mission [6].

These are the outlines of this paper: Section 2 establishes the reference frames defined for this work. Section 3 covers the spacecraft modelling and model linearization. Periodicity of the geomagnetic field is investigated in Section 4. Moving to Section 5, the discrete solution of the periodic Riccati equation is presented. Design details and simulation results are shown in Section 6. Section 7 concludes this paper.

2. REFERENCE FRAMES

Let the reference frames considered in this work be defined as follows.

2.1 Inertial frame

The inertial frame refers to the coordinate system placed at the centre of the Earth and fixed in space. Consider subscript ‘i’ to the Inertial frame. The z_i^+-axis points along the Earth’s rotation axis; the x_i^+-axis lies on the Equator plane pointing towards the vernal equinox; and the y_i^+-axis completes the Cartesian coordinate system.

2.2 *Local-Vertical and Local-Horizontal (LVLH) frame*

The LVLH frame, also known as the local orbit frame, is placed at the spacecraft centre of mass and fixed along the orbit path. Consider subscript 'o' to the LVLH frame. The z_o^+-axis points towards the centre of the Earth; the x_o^+-axis points forwards, tangent to the orbit path; and the y_o^+-axis completes de Cartesian coordinate system.

2.3 *Body frame*

The Body frame is placed at the centre of mass of the spacecraft and remains attached to the body. Consider subscript 'b', (x_b, y_b, z_b) matches with the spacecraft's axes of inertia. For an inertial-pointing spacecraft, the attitude is defined as the rotation from the inertial to the body frame, whereas for a nadir-pointing spacecraft the attitude is defined as the rotation from the LVLH frame to the body frame. This definition is important in the sequel.

3. SYSTEM MODELLING

We will consider a nadir-pointing 3U Cubesat shaped spacecraft. Assuming the spacecraft as a rigid body, let $\mathbf{J}$ be the approximated inertia matrix defined as

$$\mathbf{J} = \begin{bmatrix} J_{11} & 0 & 0 \\ 0 & J_{22} & 0 \\ 0 & 0 & J_{33} \end{bmatrix} \tag{1}$$

Let $\bar{\mathrm{q}} = [q_0\ q_1\ q_2\ q_3]^{\mathrm{T}} = [q_0\ \mathrm{q}^{\mathrm{T}}]^{\mathrm{T}}$ be the quaternion representing the spacecraft attitude and $\omega_{\mathrm{ob}}^{\mathrm{b}} = [\omega_1\ \omega_2\ \omega_3]^{\mathrm{T}}$ be the body rates with respect to the LVLH frame, expressed in the body frame. Knowing the normalized quaternion property [1][3], i.e. $q_0 = \sqrt{1 - q_1^2 - q_2^2 - q_3^2}$, the kinematic equations can be described in the reduced form as follows:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{bmatrix} \omega_{\mathrm{ob}}^{\mathrm{b}} \tag{2}$$

Equation (3) describes the dynamic equations with respect to the LVLH frame:

$$\begin{aligned} \mathbf{J}\dot{\omega}_{\mathrm{ob}}^{\mathrm{b}} = {} & \mathbf{S}(\mathbf{J}\omega_{\mathrm{ob}}^{\mathrm{b}})\omega_{\mathrm{ob}}^{\mathrm{b}} + \mathbf{S}(\mathbf{J}\omega_{\mathrm{o}}^{\mathrm{b}})\omega_{\mathrm{ob}}^{\mathrm{b}} - \mathbf{S}(\omega_{\mathrm{o}}^{\mathrm{b}})\mathbf{J}\omega_{\mathrm{ob}}^{\mathrm{b}} \\ & - \mathbf{J}\mathbf{S}(\omega_{\mathrm{o}}^{\mathrm{b}})\omega_{\mathrm{ob}}^{\mathrm{b}} + \mathbf{S}(\mathbf{J}\omega_{\mathrm{o}}^{\mathrm{b}})\omega_{\mathrm{o}}^{\mathrm{b}} + \mathrm{T} \end{aligned} \tag{3}$$

where $\mathbf{S}(*)$ denotes the cross-product operator; T is the total torque applied on the spacecraft; and $\omega_{\mathrm{o}}^{\mathrm{b}} = \mathbf{R}_{\mathrm{o}}^{\mathrm{b}}\omega_{\mathrm{o}}$ is the orbital rate ω_{o} with respect to the inertial frame, expressed in the body frame via the attitude matrix

$$\mathbf{R}_{\mathrm{o}}^{\mathrm{b}} = \begin{bmatrix} 2q_0^2 - 1 + 2q_1^2 & 2q_1q_2 + 2q_0q_3 & 2q_1q_3 - 2q_0q_2 \\ 2q_1q_2 - 2q_0q_3 & 2q_0^2 - 1 + 2q_2^2 & 2q_2q_3 + 2q_0q_1 \\ 2q_1q_3 + 2q_0q_2 & 2q_2q_3 - 2q_0q_1 & 2q_0^2 - 1 + 2q_3^2 \end{bmatrix} \tag{4}$$

The total torque can be decomposed as $\mathrm{T} = \mathrm{T_d} + \mathrm{T_m}$, where $\mathrm{T_d} \in \mathbb{R}^3$ is resultant from environment disturbance torques which are not discussed in this paper; and

$$\mathrm{T_m} = \mathrm{m} \times \mathrm{b}(t) \tag{5}$$

is the mechanical torque produced from the magnetic dipole moment $\mathrm{m} = [m_1\ m_2\ m_3]^{\mathrm{T}}$, generated by electromagnetic coils, interacting with the time-varying local geomagnetic field $\mathrm{b}(t) = [b_1(t)\ b_2(t)\ b_3(t)]^{\mathrm{T}}$, expressed in the body frame.

The model described in (2)-(3) is naturally non-linear and must be linearized in order to be considered in the controller design equations. Assuming the equilibria points $\bar{\mathrm{q}} = [1, 0, 0, 0]^{\mathrm{T}}$ and $\omega_{\mathrm{ob}}^{\mathrm{b}} = [0, 0, 0]^{\mathrm{T}}$, the linearized model is given by [1][3][4]

$$\dot{\mathrm{x}}(t) = \mathbf{A}\mathrm{x}(t) + \mathbf{B}(t)\mathrm{m_c}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \\ f_{41} & 0 & 0 & 0 & 0 & f_{46} \\ 0 & f_{52} & 0 & 0 & 0 & 0 \\ 0 & 0 & f_{63} & f_{64} & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times3} \\ \boldsymbol{\mathcal{B}}(t) \end{bmatrix} \begin{bmatrix} m_{c1} \\ m_{c2} \\ m_{c3} \end{bmatrix} \tag{6}$$

$$f_{41} = 8\omega_o^2(J_{33} - J_{22})/J_{11} \tag{7}$$
$$f_{52} = 6\omega_o^2(J_{33} - J_{11})/J_{22} \tag{8}$$
$$f_{63} = 2\omega_o^2(J_{11} - J_{22})/J_{33} \tag{9}$$
$$f_{46} = \omega_o(J_{11} - J_{22} + J_{33})/J_{11} \tag{10}$$
$$f_{64} = -f_{46}J_{11}/J_{33} \tag{11}$$

where (7)-(11) includes the linearized model of the gravitational gradient perturbation [3]; $\mathrm{m_c} = [m_{c1}\ m_{c2}\ m_{c3}]^{\mathrm{T}}$ is the magnetic dipole command from controller mapped as [4]

$$\mathrm{m_c} \to \mathrm{m} : \mathrm{m} = \frac{\mathrm{m_c} \times \mathrm{b}(t)}{\|\mathrm{b}(t)\|} \tag{12}$$

and the time-varying matrix $\boldsymbol{\mathcal{B}}(t)$ is derived from (5) and (12), yielding

$$\boldsymbol{\mathcal{B}}(t) = \frac{\mathbf{J}^{-1}}{\|\mathrm{b}(t)\|} \begin{bmatrix} (-b_2^2 - b_3^2) & b_1b_2 & b_1b_3 \\ b_1b_2 & (-b_1^2 - b_3^2) & b_2b_3 \\ b_1b_3 & b_2b_3 & (-b_1^2 - b_2^2) \end{bmatrix} \tag{13}$$

The mapping formula (12) guarantees that magnetic moment m lies on the 2-dimensional manifold perpendicular to the local geomagnetic field $\mathrm{b}(t)$ for any direction of the command vector $\mathrm{m_c}$ [4]. Equation (13) shows the influence of time-varying parameters which will introduce uncertainties to the system and consequently hamper the controller performance [3]. Nevertheless, the geomagnetic field can be considered periodic at some point. Therefore, the section in the sequel will analyse the periodicity of the geomagnetic field in order to contribute with the periodic controller design.

4. GEOMAGNETIC FIELD ANALYSIS

The geomagnetic field considered in this study is generated from the IGRF model. Taking into account the Earth rotation period of 24 hours and assuming a circular orbit with period $P = 2\pi/\omega_o$, one can estimate the periodicity of the geomagnetic field. Considering a spacecraft at 400 km of altitude in a circular orbit, it can be observed that the geomagnetic field starts to repeat after approximately $N = 15$ orbits as shown in Figure 1.

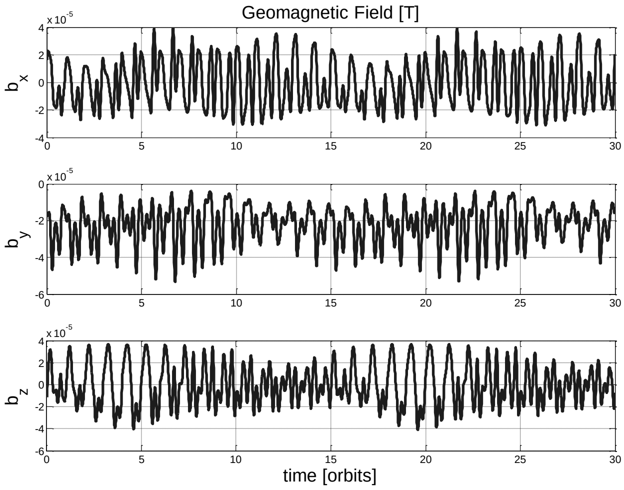

Figure 1 - Geomagnetic field seen from LVLH frame, propagated for 30 orbits (circular and inclination of 50°).

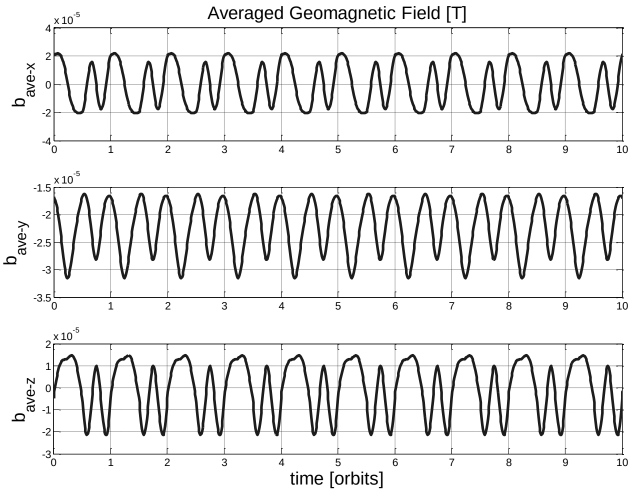

Figure 2 - Averaged geomagnetic field seen from LVLH frame, propagated for 10 orbits.

A periodic approximation of the geomagnetic field is of interest. In [4] the geomagnetic field is parameterized in terms of the mean anomaly $M(t)$ since it has the same period P. Therefore, the linear model from (6) can be referred as a periodic system in (14)

$$\dot{\mathrm{x}} = \mathbf{A}\mathrm{x} + \widetilde{\mathbf{B}}\big(M(t)\big)\mathrm{m_c} \tag{14}$$

$$\mathrm{b}_{ave}(M) = \frac{1}{N}\sum_{i=1}^{N} \mathrm{b}(M + i * P) \tag{15}$$

where $\widetilde{\mathbf{B}}\big(M(t)\big)$ is given by (6) and (13) substituting components of $\mathrm{b}(t)$ by the averaged field components from (15). Figure 2 depicts the averaged geomagnetic field with respect to the LVLH frame with period P. This periodic approximation will play important role in the periodic controller design.

5. DISCRETE SOLUTION OF THE PERIODIC RICCATI EQUATION

Recalling the periodic linear model defined in (14), let k be the discrete time for each sample time t_s and p the number of samples for one period P. Applying Euler discretization we have:

$$\mathrm{x}_{k+1} = \mathbf{A}_k\mathrm{x}_k + \mathbf{B}_k\mathrm{m}_{ck} \tag{16}$$

$$\mathbf{A}_k = \mathbf{A}_{k+1} = \cdots = \mathbf{A}_{k+p} = (\mathbf{I} + \mathbf{A}t_s) \tag{17}$$

$$\mathbf{B}_k = \mathbf{B}_{k+p} = \widetilde{\mathbf{B}}(kt_s)t_s \tag{18}$$

For the LQR state feedback control, the optimal m_{ck} is found minimizing the quadratic cost function

$$\lim_{N\to\infty}\left\{\min\left(\frac{1}{2}\mathrm{x}_N^{\mathrm{T}}\mathbf{Q}_N\mathrm{x}_N + \frac{1}{2}\sum_{k=0}^{N-1}\mathrm{x}_k^{\mathrm{T}}\mathbf{Q}_k\mathrm{x}_k + \mathrm{m}_{ck}^{\mathrm{T}}\mathbf{R}_k\mathrm{m}_{ck}\right)\right\} \tag{19}$$

where

$$\mathbf{Q}_k = \mathbf{Q}_{k+1} = \cdots = \mathbf{Q}_{k+p} \geq \mathbf{0} \tag{20}$$

$$\mathbf{R}_k = \mathbf{R}_{k+p} > \mathbf{0} \tag{21}$$

and optimal feedback m_{ck} is given by [5]

$$\mathrm{m}_{ck} = -\big(\mathbf{R}_k + \mathbf{B}_k^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{B}_k\big)^{-1}\mathbf{B}_k^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{A}_k\mathrm{x}_k \tag{22}$$

where initial condition x_0 is given; and $\mathbf{P}_k$ is the positive semi-definite solution of the discrete Riccati equation

$$\begin{aligned}\mathbf{P}_k = -\mathbf{A}_k^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{B}_k\big(\mathbf{R}_k + \mathbf{B}_k^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{B}_k\big)^{-\mathbf{1}}\mathbf{B}_k^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{A}_k \\ + \mathbf{A}_k^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{A}_k + \mathbf{Q}_k\end{aligned} \tag{23}$$

with boundary condition $\mathbf{P}_N = \mathbf{Q}_N$.

In [5] is demonstrated that the Riccati equation (23) can be solved using a symplectic system associated with (16) and (19) defined as

$$\mathbf{E}_k\mathrm{z}_{k+1} = \mathbf{E}_k\begin{bmatrix}\mathrm{x}_{k+1}\\ \mathrm{y}_{k+1}\end{bmatrix} = \mathbf{F}_k\begin{bmatrix}\mathrm{x}_k\\ \mathrm{y}_k\end{bmatrix} = \mathbf{F}_k\mathrm{z}_k \tag{24}$$

$$\mathbf{E}_k = \begin{bmatrix}\mathbf{I} & \mathbf{B}_k\mathbf{R}_k^{-1}\mathbf{B}_k^{\mathrm{T}}\\ \mathbf{0} & \mathbf{A}_k^{\mathrm{T}}\end{bmatrix} \tag{25}$$

$$\mathbf{F}_k = \begin{bmatrix}\mathbf{A}_k & \mathbf{0}\\ -\mathbf{Q}_k & \mathbf{I}\end{bmatrix} \tag{26}$$

where x_k is the state and y_k is the co-state. Considering the periodicity of the system and that $\mathbf{F}_k = \mathbf{F}_{k+1} = \cdots = \mathbf{F}$ because of (17) and (20), we have that

$$\begin{gathered}\mathbf{E}_k\mathrm{z}_{k+1} = \mathbf{F}_k\mathrm{z}_k\\ \mathbf{E}_{k+1}\mathrm{z}_{k+2} = \mathbf{F}\mathrm{z}_{k+1}\\ \vdots\\ \mathbf{E}_{k+p-1}\mathrm{z}_{k+p} = \mathbf{F}\mathrm{z}_{k+p-1}\end{gathered} \tag{27}$$

yielding

$$\begin{bmatrix}\mathrm{x}_k\\ \mathrm{y}_k\end{bmatrix} = \mathrm{z}_k = \mathbf{\Gamma}_k\mathrm{z}_{k+p} = \mathbf{\Gamma}_k\begin{bmatrix}\mathrm{x}_{k+p}\\ \mathrm{y}_{k+p}\end{bmatrix} \tag{28}$$

with the initial state x_0, the boundary condition $\mathrm{y}_N = \mathbf{Q}_N\mathrm{x}_N$, and

$$\mathbf{\Gamma}_k = \mathbf{F}^{-1}\mathbf{E}_k\mathbf{F}^{-1}\mathbf{E}_{k+1} \ldots \mathbf{F}^{-1}\mathbf{E}_{k+p-2}\mathbf{F}^{-1}\mathbf{E}_{k+p-1} \tag{29}$$

Since the eigen-decomposition is not numerically stable, [5] suggests the Schur decomposition of $\mathbf{\Gamma}_k$ given by

$$\begin{bmatrix}\mathbf{W}_{11k} & \mathbf{W}_{12k}\\ \mathbf{W}_{21k} & \mathbf{W}_{22k}\end{bmatrix}^{\mathrm{T}}\mathbf{\Gamma}_k\begin{bmatrix}\mathbf{W}_{11k} & \mathbf{W}_{12k}\\ \mathbf{W}_{21k} & \mathbf{W}_{22k}\end{bmatrix} = \begin{bmatrix}\mathbf{S}_{11k} & \mathbf{S}_{12k}\\ \mathbf{0} & \mathbf{S}_{22k}\end{bmatrix} \tag{30}$$

where $\mathbf{S}_{11k}$ is upper-triangular and has all of its eigenvalues outside the unique circle [5]. Hence, solution of (23) is given as

$$\mathbf{P}_k = \mathbf{W}_{21k}\mathbf{W}_{11k}^{-1} \tag{31}$$

Functions *schur* and *ordschur* from MATLAB were used to solve (30) and to ensure the eigenvalues outside unique circle criteria for $\mathbf{S}_{11k}$, respectively. Proofs of this method are demonstrated in [5].

6. SIMULATION RESULTS

The method presented in the former sections was used to design the periodic attitude controller for a 3U Cubesat. We define the spacecraft according to the SERPENS II model [6] with approximated inertia matrix $\mathbf{J}$ = diag(0.037, 0.034, 0.0032) kg.m^2 orbiting on a circular Low-Earth Orbit at 400 km of altitude and orbit inclination $i = 50°$. Saturation of each electromagnetic coil is set as $0.1\ Am^2$. The number of samples per orbit-period P is established as $NS = 100$ so that sample time is given as $t_s = P/NS$. Recall the periodic geomagnetic field in Figure 2, the proposed method calculates the Riccati equation solution $\mathbf{P}_k$ for each sample time within one orbit-period and stored in memory. Controller parameters are set as $\mathbf{Q}$ = diag(10^{-5}, 10^{-5}, 10^{-5}, 10^{-3}, 10^{-3}, 10^{-3}) and $\mathbf{R}$ = diag(1, 1, 1).

Figure 3 depicts the time history of (1,1)-component of the Riccati equation solution $\mathbf{P}_k$ within 5 orbits. Simulations results consider the non-linear spacecraft model and the geomagnetic field from the IGRF model. Initial values for the attitude angles and body rates are respectively given as: $(\phi, \theta, \psi) = (40°, 50°, 60°)$ and $(\omega_1, \omega_2, \omega_3) = (0.005, 0.005, 0.005)\ rad/s$.

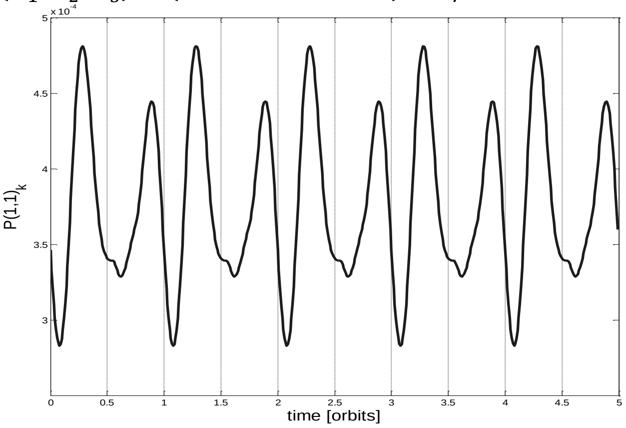

Figure 3 - Time history of $\mathbf{P}_k(1,1)$.

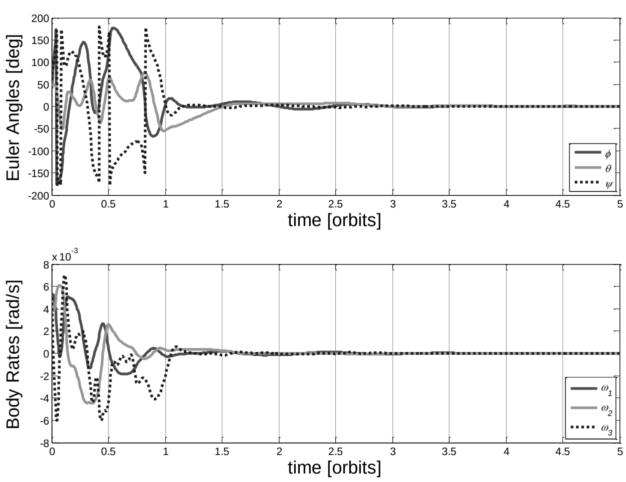

Figure 4 - Performance of the periodic controller.

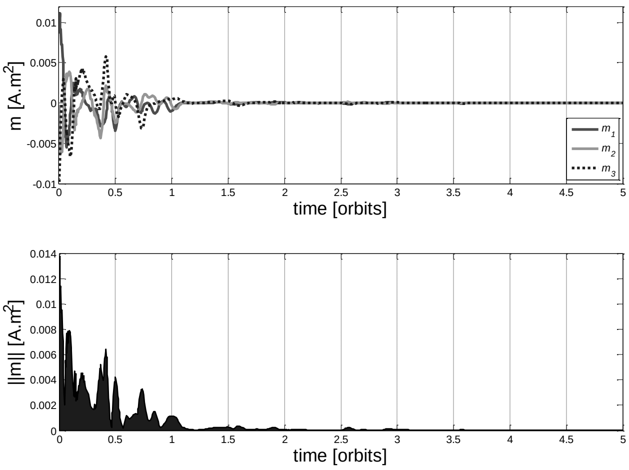

Figure 5 - Magnetic dipole moment produced by the electromagnetic actuators.

Results have shown satisfactory response. In Figure 4 can be seen the performance of the periodic controller. The system converges to the reference within less than 5-orbit time. Figure 5 depicts the magnetic dipole moment generated by the electromagnetic coils during spacecraft stabilization and regulation. No saturation is seen which is appreciable.

7. CONCLUSION

This work has presented a discrete periodic controller design for attitude control using electromagnetic actuators. The chosen scenario meets with the SERPENS II 3U-Cubesat design. A discrete linear time-varying model is derived as well as the discrete solution of the periodic Riccati equation. The presented method considers a periodic approximation of the geomagnetic field. Numerical simulations considered the non-linear spacecraft model as the geomagnetic field is generated from an IGRF model. Results have shown satisfactory performance of the periodic controller for significant initial conditions. The spacecraft is stabilized and its attitude is aligned with the reference within less than 5-orbit time that is very appreciated for a magnetic actuated spacecraft.

References

[1] Yang, Y., "Spacecraft attitude determination and control: quaternion based method," Annual Reviews in Control, Vol. 36, No. 2, 198-219, 2012.

[2] Yang, Y., "Controllability of spacecraft using only magnetic torques" available in arXiv:1507.0693, 2015.

[3] Coelho, F., da Silva, A. L., Pinheiro, J. R., "Optimized Controller Design for Cubesat Attitude Tracking Control Using Magnetic Actuators," to appear in Proceedings Series of the 2nd IAA Latin American Cubesat Workshop, 2016.

[4] Wisniewski, R., "Linear Time Varying Approach To Satellite Attitude Control Using Only Electromagnetic Actuation," Journal of Guidance, Control, and Dynamics, Vol. 23, No. 4, 640-647, July 2000.

[5] Yang, Y., "An Efficient Algorithm for Periodic Riccati Equation for Spacecraft Attitude Control using Magnetic Torque," available in arXiv:1601.01990v1, Jan 2016.

[6] Brum, A. G. V., Barioni, L., da Silva, A. L., de Melo, C. F., Ferreira, E. S., Zanardi, M. C., Celestion, C. C., Pereira, M. C., Spengler, A. W., "Sistema de controle de attitude proposto para a missão espacial SERPENS II," (in Portuguese), Proceedings Series of the Brazilian Society of Computational and Applied Mathematic – SBMAC, São Paulo, 2015.