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CLUSTER FORMATION DYNAMICS OF HETEROGENEOUS AGENTS

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Abstract: This work studies the dynamic of particles under the influence of attraction and repulsion. The main objective is to analyze how the heterogeneity can affect the formation or dissolution of clusters. The dynamic of many particles is impossible to solve by analytical way, so computer simulation is essential. The particles are discs in a plane and similarity or antagonism among them is defined by their colors based on the RGB code. Particles are attracted if their colors are similar and repelled otherwise. Clusters are formed by non-elastic collisions.

Adopting a system with 3 different gray colors and a fixed number of particles, we show that different concentrations can affect the cluster size and the time it is formed.

keywords: Complex Networks, Numerical Simulation, Stochastic Model.

1. INTRODUCTION

We live in a world widely connected and all this connectivity allow us to form many distintc groups. We have close friends on Whatsapp, aquaintances in Facebook and Google+, work relations in Linkedln and ResearchGate and so on. Even out of virtual space we tend to gather with people we share something in common like families, work friends whom we have coffee breakes, coleagues, research groups, sport teams etc. The main objective of this work is to study what are the main causes of the formation of the groups. Why some groups are stronger or more resilient than others? Which are the advantages or disadvantages of a group heterogeneity? How fast can a group be formed or dissoluted?

It may ask that such problem has a strong component of social and human behaviour and it is very difficult to model in a computer. The present work avoid this setback starting from the assumption that a) groups are formed by attraction and repulsion; b) the causes of those attraction and repulsion among agents are not relevant in present study. That way the model of this work was designed to be easily adapted for different conditions and situations, being a tool for testing many scenarios.

One of the most seminal work regarding group formation is based on game theory [1]. In that work, interaction among agents are taken into account as strategies, in a context like the Prisoner's Dilemma game. The main conclusion of this work is to show that an asocial world can evolve under cooperation based on reciprocity. On the other hand, many important aspects of social networks were revealing by the theory of complex networks [2–4]. The concepts as hubs, centrality, beetweeness, small world, preferential attachment and others has became almost mandatory when characterizing the topology of such structures.

In spite of the fact of the two above cited approaches are very grounded and accepted in scientific community, the present work tries to describe the phenomena using a classical tool: the Newtonian mechanic.

2. METHODS: THE MODEL

The model introduced here is based on Newtonian attraction and repulsion among particles in a plane with 2000×2000 pixels. Each particle is represented by a small disc of 18 pixels of diameter. There is no friction among particles and the plane. The particles are under forces defined accordinly with its color. Colors are defined using RGB (red, green, blue) code. In this code all colors can be formed using aditions of 256 $(0 - 255)$ intensities of red, green and blue, as example black=$(0, 0, 0)$, white=$(255, 255, 255)$, light green=$(0, 1, 0)$ and dark green=$(0, 255, 0)$. That way we have $256^3 = 16777216$ different colors. In order to simplify the model, we use a grayscale palette, where all intensities of RGB are equal, reducing the number of colors to 256. The general rule for forces is:

$$F_{i,j} = 1 - 2\frac{|color_i - color_j|}{color_{max}}; \quad (1)$$

where $F_{i,j}$ is the force beetween particle i and j, $color_i$ is the color of particle i, and $color_{max}$ is the maximum number of colors allowed (256 for grayscale). The forces are normalized so if one pair has the same color, $|color_i - color_j| = 0$, $F_{i,j} = 1$ giving maximum attraction. The opposite case $|color_i - color_j| = color_{max}$ and $F_{i,j} = -1$, or maximum repulsion. To force the particles be confined in the 2000×2000 pixels grid, we define a central attraction potential that take place only if the particle are too far from the center, like a horizontal plate with curved borders.

The particles are released on the plane, at random initial positions. The forces (1) among all pairs are computed and the particles acelerate. The colisions among particles are done with a restitution coefficient of $c_r = 0.5$, in order to dissipate energy and promote the formation of clusters. For the model proposed we run simulations with only three colors, in RGB code: $P_1 = (42)$, $P_2 = (126)$ and $P_3 = (210)$, and we choose $color_{max} = 252$. This is done in order to introduce a symmetry with all forces among different particles have the same magnitude. The number of particles is fixed $N = 150$ and we change the concentrations of each color. For a same initial condition we run 1000 ensambles for a better statistic.

3. RESULTS

In order to have some measure of how long the system takes to achieve a stable configuration, we compute the number of colisions, or contacts, among particles during time for 4 distintc concentrations of P_1, P_2 and P_3 (Figure 1). After a transient time all curves tend to a plateau where there is no new contact and the final configuration is achieved. In that plot can be noticed that the more homogenous configuration (triangle up, with $50/50/50$) takes more time to reach the final configuration, and its is done with fewer contacts. The configuration with $25/100/25$ on the other hand, reach the final configuration in a faster way, leading to a configuration with higher contacts, meaning the formation of a larger cluster. The other configurations exhibit intermediate behaviour.

Figure 2 shows the resulting cluster pattern of the cases listed on Figure 1. We point again for the homogenous concentration $50/50/50$ (top right on Figure 2). As showed in Figure 1, the fewer contacts represents the formation of 3 homogenous groups, where all 3 colors are completely separated one from another. The inhomogeneous symmetric case of $25/100/25$ (bottom left on Figure 2) however, leads to a formation of a giant cluster. This giant cluster is evidenced by the highest plateau of this configuration on Figure 1. The other cases results in groups that exclude one of the colors generating a medium cluster.

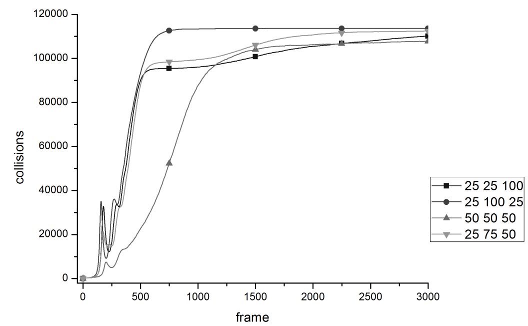

Figure 1 – Contact among particles as a function of time for different concentrations of $P_1/P_2/P_3$ (see box).

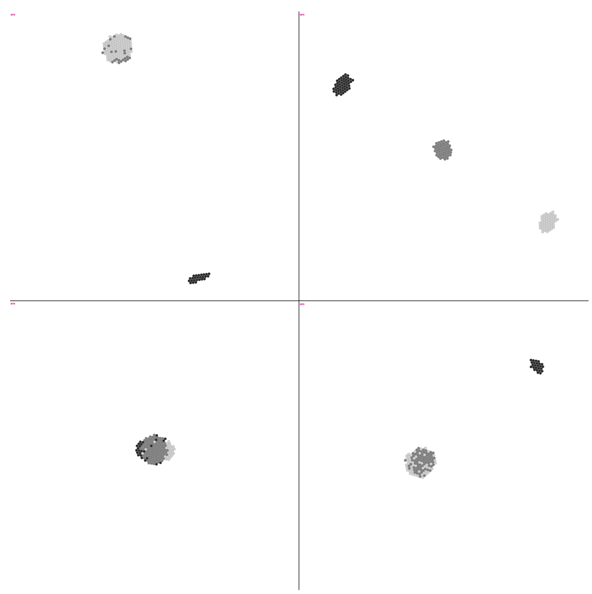

Figure 2 – Cluster pattern formed for the cases showed in Figure 1. The concentrations are: $25/25/100$ top left, $25/100/25$ bottom left, $50/50/50$ top right and $25/75/50$ bottom right.

Finally, in order to show a more complex example, we run a case with 150 particles with color randomly chosen in the range $[0, 252]$. The cluster pattern is showed in Figure 3. It can be noted the formation of a giant cluster with a noteable color gradient inside it. It is easy to see how the light and dark particles take opposite positions in the cluster, mediate by particles with average colors that build a complex frontier among them. A weaker version of such gradient can be viewed also in Figure 2, bottom left.

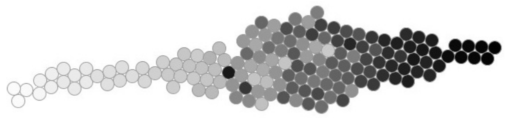

Figure 3 – Cluster pattern formed with 150 particles with random color in the range $[0, 252]$**.**

4. CONCLUSION

With the present model we show that different patterns can be formed with different concentrations of agents. This concentrations, even with only 3 types of particles, also affects how the system reach its equilibrium state. The most homogenous concentration tends to result in separated clusters while inhomogeneous symmetric concentrations leads to giant cluster. This giant cluster is only possible by the presence of a high quantity of "moderate" particles (particles with average colors) that takes a role of a bond beetween antagonic "radical" particles (light and dark particles). The "moderate" particles build a complex boundary among "radical" ones that worth a further investigation.

This work uses Newtonian mechanics as a method for clustering of particles. As mentioned in Introduction, the real nature of attraction and repulsion is not taken into account. The color is just a tool used to indroduce how the particles can colaborate or not. In that way, the model presented here is quite naive and linear, for a more realistic approach maybe the attraction/repulsion based on color scheme should be replace by characteristics related with groups we are interested, as political alignment, sports interests, fields of research, TV shows in common, socio-economic data and so on.

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