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PARAMETRIC EXCITATION OF AN OFFSHORE RISER USING REDUCED-ORDER MODELS BASED ON BESSEL-TYPE MODES: ASSESSMENT ON HYDRODYNAMIC COEFFICIENTS EFFECTS

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Abstract: The effects of the added mass and mean drag coefficients on the post-critical transversal vibrations of an offshore-riser mathematical model are discussed when it is subjected to parametric excitation induced by harmonic axial displacements at the top. A single-degree-of-freedom non-linear reduced-order model is obtained from the continuum-beam equation of motion, using Bessel-type modes. After numerical integration, using a Runge-Kutta scheme, maps of post-critical amplitude are presented as functions of the amplitude and frequency of the parametric excitation. Results are confronted with the standard analysis that usually assumes both hydrodynamic coefficients to be constant.

keywords: reduced-order model, parametric excitation, Bessel-like modes.

1. INTRODUCTION

Parametric excitation happens to be a very common problem in engineering applications. Specifically speaking of ocean engineering, slender structures such as risers and tethers can be subjected to this phenomenon. The lateral motion of these structures are dominated by the normal force (tension). The tension is explicitly dependent of time since the motion of the floating unit imposes a vertical excitation at the top end. Further aspects regarding parametric excitation of risers and tethers can be found in [1], [2], [3], [4] and [5].

Reduced-order models (ROMs) are used as an interesting alternative to the higher-order models (for example, those based on the Finite Element Method) in the study of parametric excitation of offshore structures. In [6] it was investigated a slender, vertical and submerged rod considering a non-linear ROM based on the application of a Galerkin's scheme to the continuum equations of motion. For the sake of simplicity, the authors considered a three-mode ROM based on sinusoidal functions in the Garlerkin's projection.

The same problem was investigated in [7] considering a single mode ROM based on a non-linear Bessel-like mode previously presented in [8]. The authors pointed out that this single mode ROM is capable of capturing some effects that ROMs based on sinusoidal functions only capture with a multi-mode analysis.

All the aforementioned analyses consider that the hydrodynamic coefficients, namely the added mass[1] C_a and the drag coefficients C_d are the same, irrespective the mode considered. However, it is well known in the context of flow around a circular cylinder that these coefficients depend on both oscillation amplitude and frequency (see, for example, [9]). Obviously, these dependence add another source of non-linearity. Most of the riser's dynamic analyses are carried out assuming standard values $C_a = 1$ and $C_d = 1.2$. In [10], the influence of C_a and C_d was systematically studied

[1]The added mass coefficient is the ratio between the added mass and the mass of fluid displaced by the cylinder, both *per* unit length.

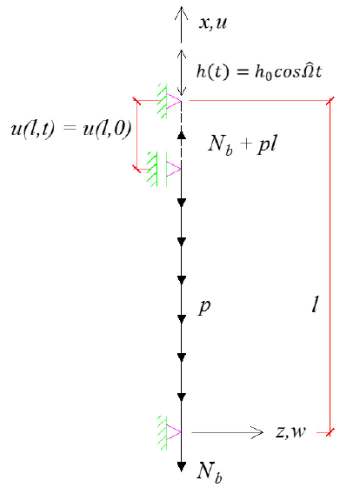


Figure 1 – Sketch of the problem. Extracted from [7].

by evaluating the post-critical oscillation amplitude resulting from the numerical integration of a single-mode ROM based on a sinusoidal function. The focus of this paper is to present a similar analysis, however using a single-mode ROM based on a Bessel-like function. Such a projection function emerges from an appropriate account of extensibility, bending stiffness and self weight effects on the transversal rigidity of the slender structure (see [8]). This is certainly a substantial improvement to the simpler approach used in reference [10].

2. MATHEMATICAL MODEL

Consider the pre-stressed vertical riser immersed in water and subjected to wave induced vertical loading, with external diameter D, length l, linear mass m and linear weight p sketched in Figure 1. For simplicity, direct wave loads effects, as well as ocean current effects, are not considered. The non-linear transverse motion equation of this structure was presented in [8] as:

$$
\begin{aligned}
& m\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} - \\
& \frac{\partial}{\partial x}\left[\left(N_t(0) - px + \frac{EAh(t)}{l}\right)\frac{\partial w}{\partial x}\right] + \\
& \frac{\partial}{\partial x}\left[\frac{EA}{2l}\left(\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 dx\right)\frac{\partial w}{\partial x}\right] \\
&= -C_a\rho_w\frac{\pi D^2}{4}\frac{\partial^2 w}{\partial t^2} - \frac{1}{2}C_a\rho_w D\left|\frac{\partial w}{\partial t}\right|\frac{\partial w}{\partial t},
\end{aligned}
\tag{1}
$$

where $N_t(0)$ is is the static pre-tension applied at the top, ρ_w is the fluid density, EI is the bending stiffness, EA the axial stiffness, $h(t) = h_0\cos\hat{\Omega}t$ is the vertical exciting displacement applied to the top.

A series of parameters is defined in [7] and herein reproduced:

$$
\begin{aligned}
&\alpha = \frac{EI}{m}, \beta = \frac{\bar{N}}{m}, \beta_h(t) = \frac{EAh(t)}{ml}, \gamma = \frac{p}{m}, \mu = \frac{EA}{ml} \\
&v = \frac{w}{D}, \xi = \frac{x}{l}, \tau = \hat{\omega}_1 t, \hat{\omega}_1 = \frac{\omega_1}{\sqrt{1+Ca^*}}, C_a = \frac{m_a}{m_d} \\
&m^* = \frac{m}{m_d}, \omega_1 = \frac{\pi}{2l\sqrt{m}}(\sqrt{N_{b,n}} + \sqrt{N_{t,n}}, C_a^* = \frac{C_a}{m^*} \\
&N_{b,n} = N_{b,o} + \left(\frac{n\pi}{l}\right)^2 EI\left(1 + \frac{3}{16}\eta_n^2\right)^2, N_{b,o} = \bar{N} - \frac{pl}{2} \\
&N_{t,n} = N_{t,o} + \left(\frac{n\pi}{l}\right)^2 EI\left(1 + \frac{3}{16}\eta_n^2\right)^2 N_{t,o} = \bar{N} + \frac{pl}{2} \\
&\eta_n = \frac{w_{o,n}}{r}, \alpha_1 = \frac{C_d\rho_w D}{2rm(1+C_a^*)}, \alpha_2 = \frac{\alpha}{\hat{\omega}_1^2 l^4(1+C_a^*)} \\
&\alpha_3 = \frac{\beta}{\hat{\omega}_1 l^2 l^4(1+C_a^*)}, \alpha_h = \frac{EAh_0}{m\hat{\omega}_1^2 l^3(1+C_a^*)} \\
&\alpha_4 = \alpha_5 = \frac{\gamma}{\hat{\omega}_1^2 l(1+C_a^*)}, \alpha_6 = \frac{\mu r^2}{\hat{\omega}_1^2 l^3(1+C_a^*)}.
\end{aligned}
$$

In the above definitions, $m_d = \rho\pi D^2/4$ is the mass of fluid displaced by the cylinder *per* unit length and the time was normalized with respect to the first natural frequency (in still water) of the beam ($\hat{\omega}_1$); $w_{o,n}$ is the dimensional modal amplitude associated to the n^{th} sinusoidal mode (i.e, $w_{o,n}\sin(n\pi\xi)$); $r = \sqrt{I/A}$ is the gyration radius of the cross-section.

Eq. 1 is re-written in non-dimensional form:

$$
\begin{aligned}
&\frac{\partial^2 v}{\partial \tau^2} + \alpha_1\left|\frac{\partial v}{\partial \tau}\right|\frac{\partial v}{\partial \tau} - \left[\alpha_3 + \alpha_h\cos\left(\frac{\hat{\Omega}}{\hat{\omega}}\tau\right)\right]\frac{\partial^2 v}{\partial \xi^2} \\
&+ \alpha_2\frac{\partial^4 v}{\partial \xi^4} - \alpha_4\left(\frac{1}{2} - \xi\right)\frac{\partial^2 v}{\partial \xi^2} + \alpha_5\frac{\partial v}{\partial \xi} - \\
&\alpha_6\frac{\partial^2 v}{\partial \xi^2}\int_0^1\left(\frac{\partial v}{\partial \xi}\right)^2 d\xi = 0.
\end{aligned}
\tag{2}
$$

In [8], the n^{th} Bessel-type non-linear mode associated to Eq. 2 is given by

$$
\psi_n(\xi, \eta_n) = (1 + al\xi)^{-1/4}\sin(b(\sqrt{(1+al\xi)} - 1)), \tag{3}
$$

where

$$
a = \frac{p}{N_{bn}}, b = \frac{n\pi}{\sqrt{1+al} - 1}. \tag{4}
$$

It is interesting to note that Eq. 3 has the same general form of a WKB solution, valid for natural modes of suspended cables, presented in [11].

Eq. 3 is re-normalized through:

$$
\phi_n(\xi, \eta_n) = \frac{\psi_n(\xi, \eta)}{\psi_n(\bar{\xi}, \eta)}, \tag{5}
$$

where $\bar{\xi}$ is the point at which the n^{th} mode attains its maximum value. For comparison sake, a gauge cross section, based on a simple sinusoidal mode, is chosen as $\bar{\xi} = 1/2$.

Projecting the dynamics modeled by Eq. 2 onto a Bessel-type mode given by Eq. 5, according to Galerkin's method, the following non-linear ordinary differential equation is obtained

$$\begin{aligned} &\ddot{\eta_n} I_0 + \dot{\eta_n}|\dot{\eta_n}|\alpha_1 I_1 - \eta_n^3 \alpha_6 I_6 \\ &+ \eta_n \{\alpha_2 I_2 - \left(\alpha_3 + \alpha_h \cos\left(\frac{\hat{\Omega}}{\hat{\omega}_1}\tau\right)\right) I_3 \\ &- \alpha_4 I_4 + \alpha_5 I_5\} = 0. \end{aligned} \quad (6)$$

where $\eta_n = w_{o,n}/r$ is the normalized modal amplitude with respect to the gyration radius, $r = \sqrt{I/A}$, and $I_k, k = 0, \ldots, 6$ are modal integrals, given by:

$$\begin{aligned} I_0 &= \int_0^1 \phi_n^2 d\xi, I_1 = \int_0^1 \phi_n|\phi_n| d\xi, I_2 = \int_0^1 \phi_n \phi_n^{IV} d\xi \\ I_3 &= \int_0^1 \phi_n \phi_n^{II} d\xi, I_4 = \int_0^1 \left(\frac{1}{2} - \xi\right) \phi_n \phi_n^{II} d\xi \\ I_5 &= \int_0^1 \phi_n \phi_n^{I} d\xi, I_6 = \int_0^1 \phi_n \phi_n^{II} \left(\int_0^1 (\phi_n^I)^2 d\xi\right) d\xi. \end{aligned}$$

Notice that as a single mode response is assumed, discussion regarding modal orthogonality has not been made. A multimodal analysis of the problem herein investigated certainly would fall in a coupled system of ordinary differential equations of motion. This aspect would certainly turn the mathematical model much more complex than the one investigated in this paper.

3. RESULTS

The first result presented in this paper is the post-critical amplitude response, emerged from the steady-state solution of Eq. 7, at the principal Mathieu's instability region, as functions of C_d and C_a^*. Fig. 2 was obtained at $\hat{\Omega} : \hat{\omega}_1 = 2 : 1$ and clearly reveals two aspects. The first one is related to a very limited region in the plane of parameters C_d *vs* C_a^* in which the steady-state oscillation amplitude varies. The range of C_d simulated is the same observed in experimental analysis of oscillating cylinders ([12]).

The second aspect is a well-defined value $C_a^* \approx 0.4$ beyond which a non-oscillatory region appears. This is an intriguing result deserving further and deeper investigations. In fact, it seems that $C_a^* = 0.4$ is a threshold for the response to a $2 : 1$ parametric excitation. Recall that $C_a^* = C_a/m^* = m_a/m$ is the ratio between the added mass and the mass of the structure. A number of questions can now be arisen, such as what would explain this threshold, would it be experimentally observable, would it happen for higher modes and higher excitation frequencies?

Fig. 3 presents the steady-state oscillation amplitude at the midpsan (normalized by the diameter) in the plane of parametric excitation parameters h_0/l and $\hat{\Omega}/\hat{\omega}_1$ for two values of drag coefficient, $C_d = 1.2$ and $C_d = 2.5$. As expected, an increase in the value of drag coefficient decreases

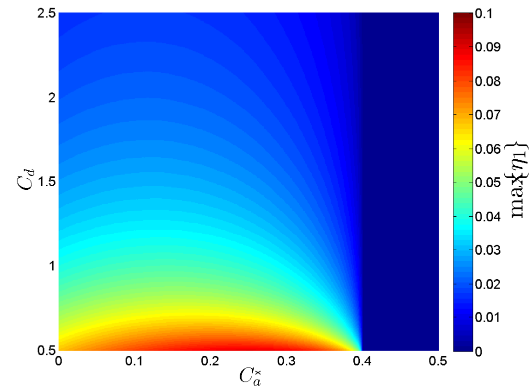


Figure 2 – Post-critical oscillation amplitude as functions of C_d and C_a^*. $\hat{\Omega} : \hat{\omega}_1 = 2 : 1$.

the steady-state oscillation amplitude. Notice also from Figs. 3(a) and 3(b) that close to the principal Mathieu's instability (i.e, $\hat{\Omega}/\hat{\omega}_1 = 2$) there is a marked increase of the steady-state oscillation amplitude. Besides the principal instability region, close to $\hat{\Omega}/\hat{\omega}_1 = 2$, another region of Mathieu's instability appears at $\hat{\Omega}/\hat{\omega}_1 = 1$ and $h_0/l > 0.4\%$, revealing smaller parametric resonance amplitudes.

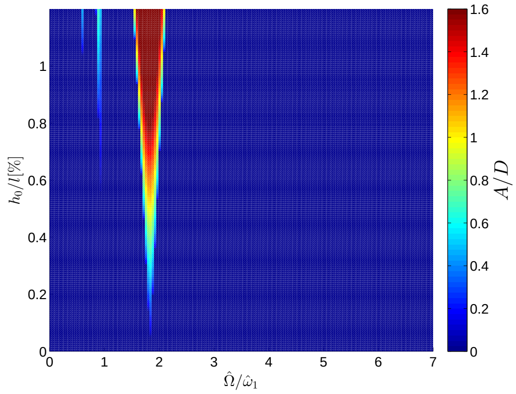
Another comment that should be made concerns the qualitative comparison of the results herein presented with those presented in [6], from multimodal ROM considering the first three natural modes as sinusoidal functions. In this latter reference, other Mathieu's instability regions in the same plane of parameters herein considered are found, probably related to the multimodal character of the ROM. However, similarly to the findings of this paper, the steady-state response at the principal Mathieu's instability showed up much higher than those obtained in the other regions.

4. CONCLUSION

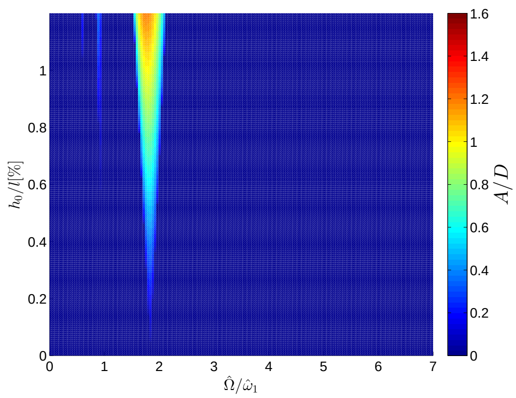
This paper focused on the presentation of a non-linear reduced-order model (ROM) for the problem of a vertical and immersed offshore riser driven by oscillatory vertical excitation at the upper extremity. A "Bessel-like" mode was used as projection function and a single-mode ROM was obtained. The aim was to investigate the influence of two hydrodynamic coefficients, namely the added mass coefficient and the drag coefficient in the steady-state response.

Considering first the post-critical regime (i.e, steady-state oscillations under Mathieu's instability), an interesting result was found. Beyond an added mass threshold value, the steady-state response is close to zero. This is a striking feature that certainly deserves a deeper investigation, since it it might reveal new important aspects in risers and tethers dynamics.

It was also presented post-critical instabilities maps, indicating the steady-state oscillation amplitude as functions of the parameters that govern the parametric excitation (top motion amplitude and frequency) for two different values of



(a) $C_d = 1.2$



(b) $C_d = 2.5$

Figure 3 – Post-critical steady-state response. $C_a = 1$**,** $C_a^* = 0.33$**.**

drag coefficient. As expected, the increase in the mean drag coefficient decreases the steady-state response in the principal Mathieu's instability region. Besides, another Mathieu's instability region was found. However, the values of response in the secondary region are much lower that those obtained in the principal one.

Future work includes a deeper investigation regarding the threshold value of added mass coefficient already pointed out. Additionally, it is planned to carry out further numerical simulations varying the hydrodynamic coefficients and comparing the results with experimental data, by taking into account modal hydrodynamic coefficients, regarding added mass and damping.

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