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MANUFACTURING OPTIMIZATION USING COUPLED LOT SIZING AND STOCK CUTTING PROBLEMS

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Abstract: The goal of this work is to propose the application of a mathematical model, which couples the lot sizing and stock cutting problems, to a real case of a factory, in order to minimize the production costs. Two final products of the manufactory are considered. Costs and demands vary over periods of planning. Numerical results from this study are compared with the case whose production is made considering the demand by period. The optimal solutions obtained from the model are promising, since they provide a considerable profit in relation to the production by demand.

Keywords: Modeling, Optimization, Numerical Simulation, Lot sizing, Stock cutting

1. INTRODUCTION

Considering the worldwide intensification of technology in the 21st century, as well as computational advances, industries have been encouraged to make their production processes more efficient and competitive. Thus, the study of optimization models for the control and production planning becomes an essential tool for industrial advancement.

Usually, industries produce pieces of different sizes and materials in order to construct a final product. In this process, industries worry about the waste of raw materials, since this implies a profit reduction. Then, the question is to solve an optimization problem, which consists of cutting objects, observing these issues.

The paper [1] investigates an integrated production planning and cutting stock problem that is common in small-scale furniture plants, in which the production costs involved in the manufacturing process and the demands for the products are not precisely known. To deal with these uncertainties, robust optimization models that control the conservatism of the solution according to the attitude of the decision maker towards risk were proposed.

The purpose of the paper [2] is to solve the problem of optimizing the production on the basis of the commercial value of the cuts, since the loss of raw materials in wood cutting industries has reached high. This work was completed with the design and the presentation of a software package called cutting optimizer. In [3], the proposed model for multiperiod cutting stock problems allows flexibility in analyzing a solution to be put in practice. The multiperiod cutting problem can be a tool that provides the decision maker a wide view of the problem and it may help him/her on making decisions.

In the context of stock cutting and lot sizing problems, the goal of this work is to propose the application of a mathematical model to a real case of a factory, in order to minimize the global production costs. Costs and demands vary over periods of planning; this consideration represents a more realistic situation. Several works, like [4] and [5] consider these parameters just constants. Numerical results from this study are compared with the case whose production is made considering the demand by period. The model couples the lot sizing and stock cutting problems.

2. THE MATHEMATICAL MODEL

In the process of cutting a board into smaller pieces, for producing items, material loss tends to be increasingly reduced if the cuts are rearranged in a convenient way on the board. Due to this fact, there is an economic pressure to manufacture some products in advance in order to minimize losses. However, this stock can generate costs that can make the production slower [4]. This decision problem is called *Combined Problem*, which couples two optimization problems: the lot sizing and cutting stock. The lot-sizing problem is to plan the amount of items to be produced in various stages in each period over a horizon of finite time, in order to meet demand and to optimize an objective function, as minimizing production and storage costs. The two-dimensional cutting stock problem consists in optimizing the process of cutting boards into smaller pieces in quantities and sizes demanded. Cutting pattern is defined as the arrangement of the pieces within each board, i.e., the way an object (piece) is cut to produce demanded items.

Thus, the goal of the combined problem is to decide the number of final products to be produced in each planning horizon period in order to minimize production, preparation and storage costs (lot sizing) and the number of boards to be cut, as well as cutting patterns to compose final products (cutting stock) [5]. In real situations, most industries approach these two problems separately, which can raise global costs.

One approach to the combined problem disregards preparation costs and relaxes the integrality of the variables representing the number of boards cut in a given pattern, which requires a large demand. This approach can be applied in the furniture industry where wooden boards must be cut to produce items. It is considered that there is only one type of board and parameters like costs and demands are varied. The mathematical model is formulated according to [6] as equations (1) to (5).

$$min \sum_{i=1}^{M} \sum_{t=1}^{T} (c_{it} .x_{it} + h_{it}. e_{it}) + \sum_{j=1}^{N} \sum_{t=1}^{T} cp.y_{jt} +$$
$$+ \sum_{p=1}^{P} \sum_{t=1}^{T} hp_{pt} .ep_{pt} \tag{1}$$

s.t:

$$x_{it} + e_{i,t\text{-}1} - e_{it} = d_{it} \qquad \forall t = 1,...,T, \ \forall i = 1,...,M \tag{2}$$

$$\sum_{j=1}^{N} a_{pj} .y_{jt} + ep_{p,t\text{-}1} - ep_{pt} = \sum_{i=1}^{M} r_{pi} .x_{it} \tag{3}$$

$$\sum_{j=1}^{N} v_j .y_{jt} \le u_t \qquad \forall t = 1,...,T, \ \forall j = 1, ..., N \tag{4}$$

$$x_{it} , f_{it} , y_{it}, e_{pt} \ge 0 \ \ \forall t = 1,...,T \tag{5}$$

Indexes

$t = 1, ..., T$ number of periods of time.
$p = 1, ..., P$ number of different pieces to be cut.
$j = 1, ..., N$ number of different cutting pattern.
$i = 1, ... , M$ number of different final products.

Parameters

c_{it}; production cost of the final product *i* in the period *t*.
hp_{pt}: stocking cost of the piece *p* in the period *t*.
h_{it}: stocking cost of the final product *i* in the period *t*.
d_{it}: demand of the final product *i* in the period *t*.
r_{pi}: number of pieces *p* required to make product *i*.
v_j: time to cut a board using cutting pattern *j*.
a_{pj}: number of piece *p* cut using cutting pattern *j*.
u_t: maximum time of saw capacity
cp: cost of the board to be cut.

Decision Variables

x_{it}: number of final product *i* produced in the period *t*.
ep_{pt}: number of piece *p* stocked in the end of the period *t*.
e_{it}: number of final product *i* stocked in the end of the period *t*.
y_{jt}: number of boards cut using cutting pattern *j* in the period *t*.

The restrictions (2) are related to the stock balance equations in relation to final products (to satisfy items demand). The restrictions (3) are related to stock balance equations in relation to pieces (to satisfy pieces demand). The restrictions (4) are related to saw capacity (maximum time of operation) and, in the end, (5) represents non-negativity conditions. The application of algorithm Simplex [7] with columns generation [8] has been presented, in the literature, as a very good strategy to solve this kind of problem.

3. CASE STUDIES

Numerical data are collected from a small-scale furniture plant, located in the city of Cornélio Procópio, in the state of Paraná, Brazil. The cutting patterns (the way the piece is cut) can be seen in Table 1.

Table 1 – Cutting Patterns.

Pattern	Piece 1	Piece 2	Piece 3	Time
j=1	2	0	0	v_1=1s
j=2	1	88	0	v_2=1.2s
j=3	0	0	35	v_3=1.5s
j=4	0	0	45	v_4=1.4s
j=5	1	8	15	v_5=1.5s

The pieces to be cut are cover (piece 1), foot (piece 2) and seat (piece 3). There are two production scenarios to be considered, which are described as follows.

3.1. The first production planning

The first production planning considers that costs and demands of the final products are constants. The planning period of the factory is 6 months.

The variable x_{1t} represents the product "table", whose production cost is c_{1t} = 255 and the demand is d_{1t} = 2 for t=1,...,6. The variable x_{2t} represents the product "chair", whose production cost is c_{2t} = 80 and the demand is d_{2t} = 3 for t=1,...,6. The other parameters provided from the factory are cp = 120, u_t = 300 hours/period, r_{11} = 1, r_{21} = 5, r_{32} = 2, r_{22} = 6, h_{1t} = 3, h_{2t} = 1, hp_{1t} = 0.2, $hp_{2\,t}$ = 0.3, hp_{3t} = 0.5. Running the model using algorithm Simplex, the optimal solution obtained presents the production total minimum cost of 4508,09, for period t = 6. The solution suggests to anticipate the chair production, x_{21}=18, generating stock, and to postpone the table production, x_{13}=12 in the t = 3. Considering this result, the optimal solution provides profit of 15.2% when it is compared with the production cost that satisfies the demand in each time period.

3.2. The second production planning

The second production planning, considers that costs and demands vary over periods of planning. The new parameters provided from the factory in study are described as follows. The costs of the final products are : $c_{11} = c_{12} = c_{13} = 255$; $c_{14} = c_{15} = c_{16} = 267.75$; $c_{21} = c_{22} = c_{23} = 80$; $c_{24} = c_{25} = c_{26} = 84$. The cost of the board to be cut is $cp = 135.07$. The stocking costs of the final products are $h_{11} = h_{12} = 3$; $h_{13} = 5$; $h_{14} = h_{15} = h_{16} = 5$; $h_{21} = h_{22} = 1$; $h_{23} = h_{24} = h_{25} = h_{26} = 3$ and the stocking costs of the pieces are $hp_{11} = hp_{12} = 0.2$; $hp_{13} = hp_{14} = hp_{15} = hp_{16} = 0.5$; $hp_{21} = hp_{22} = 0.3$; $hp_{23} = hp_{24} = hp_{25} = hp_{26} = 0.6$; $hp_{31} = hp_{32} = 0.5$; $hp_{33} = hp_{34} = hp_{35} = hp_{36} = 0.8$. Demands are $d_{11} = 2$; $d_{12} = 3$; $d_{13} = 2$; $d_{14} = 4$; $d_{15} = 1$; $d_{16} = 3$; $d_{21} = 3$; $d_{22} = 5$; $d_{23} = 4$; $d_{24} = 6$; $d_{25} = 3$; $d_{26} = 4$.

Running the new combined model using algorithm Simplex, considering the variation of the parameters for $t = 6$ periods, the optimal solution obtained presents the production total minimum cost of 5833.09. The solution suggests to anticipate the chair production, $x_{21}=25$, and to postpone the table production, $x_{13} = 15$ in the $t = 3$. Considering this new result, the optimal solution provides profit of 29.7% when it is compared with the production cost that satisfies the demand in each time period.

In practical situations, according to [9], a strategy to round the fractional variables can be applied in order to obtain a viable solution. Since the number of boards, in a numerical solution, must be an integer number, an optimal solution must use at least one board. Thus, if the solution obtained by rounding is not optimal, at the most one more board than the optimal solution will be used.

4. CONCLUSION

This paper presented case studies approaching the Problem Combined, which couples two important optimization problems: cutting stock and lot sizing. Treating them separately can raise global production costs, particularly if a significant portion of the cost of the final product is formed by the material to be cut. Despite the fact that the combination of these problems is still little explored in the literature, the relevance of this combination in different situations elects this problem as an important issue to be researched.

The results of the application of the mathematical model, which considers the study of the Combined Problem, in the presented case studies, are promising and efficient, highlighting the importance of this type of model in the production area.

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