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Discrete-Time Descriptor Kalman Filter in Data Fusion Scenario

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Resumo. This paper deals with recursive estimation of descriptor systems in data fusion scenario. This scenario occurs in cases where the system can operate under different failure conditions. Numerical examples are provided to show the functionality of the estimator.

Keywords. Descriptor Systems, Kalman Filter, Data Fusion

1 Introduction

Descriptor systems (also known as singular systems or implicit systems) has been considered in some important applications related with economical systems [11], image modeling [7], and robotics [12]. Besides, the descriptor formulation contains the usual state-space system as a special case and can describe some dynamical systems for which state-space description does not exist.

State estimation of discrete-time descriptor systems have received great attention in the literature see, e.g., [2] - [4]. Different formulations have been proposed in order to improve performance of the estimators, among which we can highlight filters in data fusion scenario. This filters are useful when the system is subject to modeling errors as can be viewed in [16] and [6].

In this paper is developed discrete-time descriptor Kalman filter to situations in that the system involve a multitude of measurement models. The filter is developed based on deterministic approach of the least square problems in data fusion scenario proposed in [13]. An important issue of this approach is the guarantee of stability of the filter in

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online applications. Simulations was performed to show the effectiveness of the proposed filter.

2 Problem Statement

Consider the discrete-time linear descriptor system

$$\begin{aligned} E_{i+1}x_{i+1} &= F_i x_i + w_i, \quad i = 1, 2, ... && (1) \\ z_{i,k} &= H_{i,k}x_i + v_{i,k}, \; k = 1, 2, ..., L && (2) \end{aligned}$$

where $x_i \in \mathbb{R}^n$ is the state vector; $z_{i,k} \in \mathbb{R}^p$ are the measures output; $w_i \in \mathbb{R}^m$, and $v_{i,k} \in \mathbb{R}^p$ are the process and measurement noises; $E_{i+1} \in \mathbb{R}^{m\times n}$, $F_i \in \mathbb{R}^{m\times n}$, and $H_{i,k} \in \mathbb{R}^{p\times n}$ are the known system matrices, where E_{i+1} is a singular matrix . Assuming $\{x_0, w_i, v_{i,k}\}$, initial condition and the process and measurement noises, are uncorrelated zero-mean random variables with second-order statistics:

$$cov\Big(\begin{bmatrix} x_0^T & w_i^T & v_{i,k}^T \end{bmatrix}^T \Big) = diag(P_0, Q_i\delta_{i,j}, R_{i,k}\delta_{i,j}) \tag{3}$$

where, $\delta_{i,j} = 1$, if $i = j$ and $\delta_{i,j} = 0$, if otherwise.

In this paper, it is proposed a solution for the problem of finding a recursive state estimation algorithm for this descriptor system in data fusion scenario. In the same way, one can say that given a sequence of measures output $\{z_{0,k}, z_{1,k}, ..., z_{i,k}\}$, where $k = 1, ..., L$, the main objective is to develop the best estimates to the state x_i. For this, suppose that, at step i it is obtained an estimate for the state x_i, defined by $\hat{x}_{i|i}$, and there exists a positive-definite weighting matrix $P_{i|i}$ for the state estimation error $x_i - \hat{x}_{i|i}$, along with the new observation at time $i+1$, i.e., z_{i+1}. To update the estimate of x_i from $\hat{x}_{i|i}$ to $\hat{x}_{i+1|i+1}$, it is proposed the solution of following problem:

$$\min_{x_i, x_{i+1}} \left[||x_i - \hat{x}_{i|i}||^2_{P_{i|i}^{-1}} + ||E_{i+1}x_{i+1} - F_i x_i||^2_{Q_i^{-1}} + \sum_{k=1}^{L} ||z_{i+1,k} - H_{i+1,k}x_{i+1}||^2_{R_{i+1,k}^{-1}} \right]. \tag{4}$$

The solution of (4) is given in the next section.

3 Filtering For Discrete-Time Descriptor Systems in Data Fusion Scenario

The filtered estimate to be considered in this section is based on fundamental optimization problem defined in the following Lemma.

Lemma 3.1. *[13] Consider the problem of solving*

$$\hat{x} = arg\min_x \left[||x||^2_Q + \sum_{k=1}^{L} ||A_k x - b_k||^2_{W_k} \right], \tag{5}$$

where A is the data matrix, b_k are the measurement vectors which are assumed to be known, x is the unknown vector, x^TQx is the regularization term and $Q = Q^T > 0$ and $W_k = W_k^T \geq 0$ are given weighting matrices. The solution of (5) is given by

$$\hat{x} = \left[Q + \sum_{k=1}^{L} A_k^T W_k A_k \right]^{-1} \sum_{k=1}^{L} A_k^T W_k b_k. \tag{6}$$

The Kalman filter for descriptor systems in data fusion scenario problem, for the filtered estimates, will be calculated according to the following Theorem.

Theorem 3.1. *Suppose that* $\begin{bmatrix} E_i \\ \overline{H}_i \end{bmatrix}$ *has full-column rank and* $k = 1, ..., L$ *the quantity of measurement equations, for all* $i \geq 0$. *The optimum filtered estimates in data fusion scenario* $\hat{x}_{i|i}$, *resulting from (4), can be obtained from the following recursive algorithm:*

Step 0 *(Initial Condition):*

$$\begin{aligned} P_{0|0} &:= (P_0^{-1} + \overline{H}_0^T \overline{R}_0^{-1} \overline{H}_0)^{-1} \\ \hat{x}_{0|0} &:= P_{0|0} \overline{H}_0^T \overline{R}_0^{-1} \overline{z}_0. \end{aligned} \tag{7}$$

Step 1*: Update* $\{\hat{x}_{i|i}, P_{i|i}\}$ *to* $\{\hat{x}_{i+1|i+1}, P_{i+1|i+1}\}$ *as follows:*

$$P_{i+1|i+1} \;:=\; \left(\begin{bmatrix} E_{i+1} \\ \overline{H}_{i+1} \end{bmatrix}^T \begin{bmatrix} Q_i + F_i P_{i|i} F_i^T & 0 \\ 0 & \overline{R}_{i+1} \end{bmatrix}^{-1} \begin{bmatrix} E_{i+1} \\ \overline{H}_{i+1} \end{bmatrix} \right)^{-1}, \tag{8}$$

$$\hat{x}_{i+1|i+1} \;:=\; P_{i+1|i+1} \begin{bmatrix} E_{i+1} \\ \overline{H}_{i+1} \end{bmatrix}^T \begin{bmatrix} Q_i + F_i P_{i|i} F_i^T & 0 \\ 0 & \overline{R}_{i+1} \end{bmatrix}^{-1} \begin{bmatrix} F_i \hat{x}_{i|i} \\ \overline{z}_{i+1} \end{bmatrix}, \tag{9}$$

where

$$\overline{H}_{i+1} := \begin{bmatrix} H_{i+1,1} \\ H_{i+1,2} \\ \vdots \\ H_{i+1,L} \end{bmatrix}; \quad \overline{R}_{i+1} := \begin{bmatrix} R_{i+1,1} & 0 & \cdots & 0 \\ 0 & R_{i+1,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{i+1,L} \end{bmatrix}; \quad \overline{z}_{i+1} := \begin{bmatrix} z_{i+1,1} \\ z_{i+1,2} \\ \vdots \\ z_{i+1,L} \end{bmatrix}. \tag{10}$$

Proof. The cost functional of the minimization problem (4) is rewritten as

$$\begin{bmatrix} x_i - \hat{x}_{i|i} \\ x_{i+1} \end{bmatrix}^T \begin{bmatrix} P_{i|i}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_i - \hat{x}_{i|i} \\ x_{i+1} \end{bmatrix}$$

$$+ \sum_{k=1}^{L} \left[\left(\begin{bmatrix} -F_i & E_{i+1} \\ 0 & H_{i+1,k} \end{bmatrix} \begin{bmatrix} x_i - \hat{x}_{i|i} \\ x_{i+1} \end{bmatrix} - b_k \right)^T \begin{bmatrix} \frac{1}{L} Q_i^{-1} & 0 \\ 0 & R_{i+1,k}^{-1} \end{bmatrix} \left(\bullet \right) \right] \tag{11}$$

comparing (5) and (11), we obtain the following identifications

$$\begin{aligned} A_k &\leftarrow \begin{bmatrix} -F_i & E_{i+1} \\ 0 & H_{i+1,k} \end{bmatrix}, \quad W_k \leftarrow \begin{bmatrix} \frac{1}{L} Q_i^{-1} & 0 \\ 0 & R_{i+1,k}^{-1} \end{bmatrix}, \\ b_k &\leftarrow \begin{bmatrix} F_i \hat{x}_{i|i} \\ z_{i+1,k} \end{bmatrix}, \qquad Q \leftarrow \begin{bmatrix} P_{i|i}^{-1} & 0 \\ 0 & 0 \end{bmatrix}, \end{aligned} \quad x \leftarrow \begin{bmatrix} x_i - \hat{x}_{i|i} \\ x_{i+1} \end{bmatrix}. \tag{12}$$

According to (12) and (6), the optimum filtered estimate of x_{i+1} in data fusion scenario, $\hat{x}_{i+1|i+1}$, is obtained with the solution of the following expression:

$$\begin{bmatrix} \hat{x}_{i|i+1} - \hat{x}_{i|i} \\ \hat{x}_{i+1|i+1} \end{bmatrix} = \begin{bmatrix} P_{i|i}^{-1} + F_i^T Q_i^{-1} F_i & -F_i^T Q_i^{-1} E_{i+1} \\ -E_{i+1}^T Q_i^{-1} F_i & E_{i+1}^T Q_i^{-1} E_{i+1} + \sum_{k=1}^{L} [H_{i+1,k}^T R_{i+1,k}^{-1} H_{i+1,k}] \end{bmatrix}^{-1} \times \begin{bmatrix} -F_i^T Q_i^{-1} F_i \hat{x}_{i|i} \\ E_{i+1}^T Q_i^{-1} F_i \hat{x}_{i|i} + \sum_{k=1}^{L} [H_{i+1,k}^T R_{i+1,k}^{-1} z_{i+1,k}] \end{bmatrix} \tag{13}$$

pre-multiplying in both sides of the equality (13) by $\begin{bmatrix} 0 \\ I \end{bmatrix}^T$, one can obtain the following equation:

$$\hat{x}_{i+1|i+1} = \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} P_{i|i}^{-1} + F_i^T Q_i^{-1} F_i & -F_i^T Q_i^{-1} E_{i+1} \\ -E_{i+1}^T Q_i^{-1} F_i & E_{i+1}^T Q_i^{-1} E_{i+1} + \sum_{k=1}^{L} [H_{i+1,k}^T R_{i+1,k}^{-1} H_{i+1,k}] \end{bmatrix}^{-1} \times \begin{bmatrix} -F_i^T Q_i^{-1} F_i \hat{x}_{i|i} \\ E_{i+1}^T Q_i^{-1} E_{i+1} + \sum_{k=1}^{L} [H_{i+1,k}^T R_{i+1,k}^{-1} z_{i+1,k}] \end{bmatrix}. \tag{14}$$

After some algebra, the filtered estimate equation $\hat{x}_{i+1|i+1}$ is given by (9) in Step i with the auxiliary variable $P_{i+1|i+1}$, defined as (8). □

Remark 3.1. *Note that, from (8) and (9), there is a similarity between this algorithm and the usual descriptor filtered estimates of [8].*

Remark 3.2. *From (8) and (9), one can verify that for descriptor systems with one measurement equation, i.e., assuming $k = 1$ in (2), this algorithm is the usual descriptor filtered estimates of [8].*

Remark 3.3. *For the usual state-space systems ($E_{i+1} = I$), the descriptor filter in data fusion, developed in this paper, can be compared with the filter given by ([8], Remark 4.1). In this case, the proposed descriptor filter in data fusion (8) and (9) reduces to*

$$P_{i+1|i+1} := ((Q_i + F_i P_{i|i} F_i^T)^{-1} + \overline{H}_{i+1}^T \overline{R}_{i+1}^{-1} \overline{H}_{i+1})^{-1}, \tag{15}$$

$$\hat{x}_{i+1|i+1} := P_{i+1|i+1}(Q_i + F_i P_{i|i} F_i^T)^{-1} F_i \hat{x}_{i|i} + P_{i+1|i+1} \overline{H}_{i+1}^T \overline{R}_{i+1}^{-1} \overline{z}_{i+1}, \tag{16}$$

when k=1, one can obtain the same filter given by ([8], Remark 4.1).

4 Numerical Examples

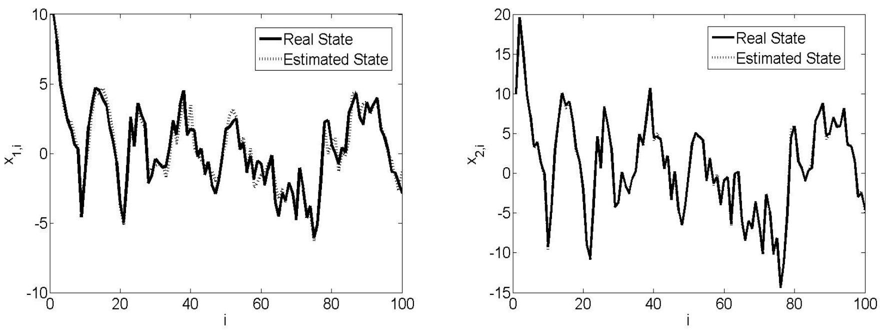
In the numerical examples, two comparative studies are presented to show the effectiveness of the descriptor filter in data fusion scenario. Both studies we consider the parameter matrices defined as:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, G = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, F = \begin{bmatrix} 0.8 & 0 \\ -1 & 0.5 \end{bmatrix}, H_1 = \begin{bmatrix} 3 & 7 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 0.1 & 2 \end{bmatrix}, H_3 = \begin{bmatrix} 0.2 & 3.1 \end{bmatrix}, Q = \begin{bmatrix} 3 & 0 \\ 0 & 0.8 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.8 \end{bmatrix}, R_2 = \begin{bmatrix} 0.8 \end{bmatrix}, R_3 = \begin{bmatrix} 0.8 \end{bmatrix}.$$

First, a comparative study between the true value of the state and the filtered estimate is shown in Figure 1. Notice the descriptor filter in data fusion scenario presented satisfactory estimate. Second, a comparative study between the descriptor filter proposed in [8] and the descriptor filter in data fusion is shown in Figure 2. In this case, the parameter matrix H_1 used in the descriptor filter of [8] is simulating a modeling error. The square root of the mean square error (rms) estimator was simulated. The curves were obtained from $i = 0, ...100$ for each recursive step of 4000 Monte Carlo simulations. Our estimator outperforms the estimator given in [8].



(a) (b)

Figure 1: Real and Estimated States.

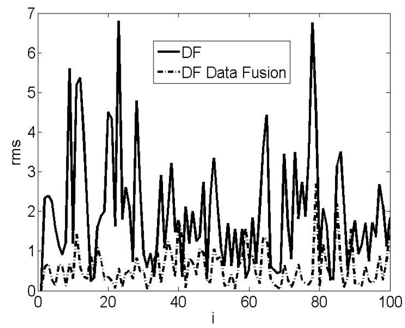


Figure 2: Comparison between Descriptor Filter of [8] and Descriptor Filter in Data Fusion.

5 Conclusion

In this paper was developed the Kalman-type recursive filters for descriptor systems in data fusion scenario. The problems was solved using the data fusion techniques proposed in [13], which considers estimation as a data fitting problem. The numerical example showed the effectiveness of this approach.

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