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Filtering for Discrete-Time Markovian Jump Linear Systems in Data Fusion Scenario

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Resumo. This paper considers the problem of recursive filtering for discrete-time markovian jump linear systems subject to unobserved chain state in data fusion scenario. The estimator presented here, enable that this kind of systems operate with more than one measurement equation. Numerical examples are presented to verify the effectiveness of the proposed algorithm.

Palavras-chave. Markovian Systems, Data Fusion, Kalman Filter

1 Introduction

Markovian jump linear systems (MJLS) have been subject of intensive study in the last years. The MJLS are models to dynamic systems subject to abrupt changes in their strutures. We assume that the abrupt changes can be modeled by a finite state-space Markov chain.

Recursive filters for MJLS have been considered in the literature based on the classical approach established by Rudolf Kalman in [4], see for instance [1], [3], [6], [5], [2], [9]. In particular [1], developed a recursive filter based on Riccati difference equation, that is useful to be used in online applications, thanks to the augmented Markovian model defined to deal with the unknown Markovian chain.

In this note, we extend the filter in [1] for an approach in data fusion scenario. This filters are useful when the system is subject to modeling errors. The estimate is based on optimization problems proposed in [3]. Numerical example are performed to show the effectiveness of the proposed filter.

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2 Problem Statement

Consider the following DMJLS:

$$x_{i+1} = F_{i,\Theta_i}x_i + G_{i,\Theta_i}w_i, \quad i = 0, 1, ... \tag{1}$$
$$y_i^{(k)} = H_{i,\Theta_i}^{(k)}x_i + J_{i,\Theta_i}^{(k)}v_i^{(k)}, \quad k = 1, ..., L \tag{2}$$

where, for each time instant i and jump parameter $\Theta_i \in S$, with $S := 1, .., N$, being F_{i,Θ_i}, G_{i,Θ_i}, $H_{i,\Theta_i}^{(k)}$ and $J_{i,\Theta_i}^{(k)}$ parameter matrices of appropriate dimensions. Suppose that, $\Theta := \{\Theta_i, i = 0, 1, ...|\Theta_i \in S\}$ is a finite state discrete-time Markov chain with $\pi_{i,j} := P\{\Theta_i = j\}$ and state transition probability matrix $P = [p_{jr}] \in \mathbb{R}^{N\times N}$ whose entries are given by $p_{jr} := P\{\Theta_{i+1} = r|\Theta_i = j\}$ for all $j, r \in S$. Besides, as usually, $x_i \in \mathbb{R}^n$ is the random state vector, $y_i^{(k)} \in \mathbb{R}^m$ is the random outputs, $w_i \in \mathbb{R}^p$ is the random state noise, $v_i^{(k)} \in \mathbb{R}^m$ are the random outputs noises, where $\{w_i\}$ and $\{v_i^{(k)}\}$ are independent wide sense stationary sequences of mutually independent random variables with zero mean and covariance matrices U_i and $V_i^{(k)}$, respectively. Assume that x_0 is a random initial state with $\mathbb{E}\{x_0 1_{\{\Theta_0=r\}}\} = 0$ and $\mathbb{E}\{x_0 x_0^T 1_{\{\Theta_0=r\}}\} = \Xi_r$, where $1_. : A \subset X \to \{0, 1\}$, which is the indicator function defined as $1_{A(x)} = 1$, if $x \in A$ and $1_{A(x)} = 0$, if $x \notin A$. And still, $\{\Theta_i\}$ and x_0 are independent of $\{w_i\}$ and $\{v_i^{(k)}\}$; and $G_{i,j}G_{i,j}^T > 0$, for all instant i and $j \in S$.

With the following definition for $i \geq 0$ and $r \in \{1, ..., N\}$,

$$z_{i,r} := x_i 1_{\{\Theta_i=r\}} \in \Re^n \tag{3}$$
$$z_i := \begin{bmatrix} z_{i,1}^T \\ \vdots \\ z_{i,N}^T \end{bmatrix} \in \Re^{Nn}, \tag{4}$$

in [1] it was defined an augmented model of (2) in terms of z_i,

$$z_{i+1} := \mathcal{F}_i z_i + \psi_i, \quad i = 1, 2, ... \tag{5}$$
$$y_i^{(k)} := \mathcal{H}_i^{(k)} z_i + \varphi_i^{(k)}, \quad k = 1, ..., L \tag{6}$$

where

$$\mathcal{F}_i := \begin{bmatrix} p_{11}F_{i,1} & \cdots & p_{N1}F_{i,N} \\ \vdots & \ddots & \vdots \\ p_{1N}F_{i,1} & \cdots & p_{NN}F_{i,N} \end{bmatrix}, \mathcal{H}_i^{(k)} := \begin{bmatrix} H_{i,1}^{(k)} & \cdots & H_{i,N}^{(k)} \end{bmatrix}, \tag{7}$$
$$\varphi_i^{(k)} := J_{i,\Theta_i}^{(k)} v_i^{(k)}, \quad \psi_i := \mathcal{M}_{i+1} z_i + \vartheta_i, \tag{8}$$

$$\mathcal{M}_{i+1} := \begin{bmatrix} \mathcal{M}_{i+1,1} \\ \vdots \\ \mathcal{M}_{i+1,N} \end{bmatrix}, \quad (9)$$

$$\mathcal{M}_{i+1,r} := \begin{bmatrix} (1_{\{\Theta_{i+1}=r\}} - p_{1r})F_{i,1} & \cdots & (1_{\{\Theta_{i+1}=r\}} - p_{Nr})F_{i,N} \end{bmatrix}, \quad (10)$$

$$\vartheta_i := \begin{bmatrix} 1_{\{\theta_{i+1}=1\}} G_{i,\Theta_i} w_i \\ \vdots \\ 1_{\{\theta_{i+1}=N\}} G_{i,\Theta_i} w_i \end{bmatrix}, \quad j,r = 1,...,N;\ k = 1,...,L. \quad (11)$$

$$(12)$$

From hypothesis about u_i and $w_i^{(k)}$, the following properties are valid to the system augmented (5): $\mathbb{E}\{z_0\} = 0$, $\mathbb{E}\{\psi_i\} = 0$, $\mathbb{E}\{\varphi_i^{(k)}\} = 0$, $\mathbb{E}\{z_i \psi_i^T\} = 0$, $\mathbb{E}\{z_i \varphi_i^{(k)T}\} = 0$, $\mathbb{E}\{\psi_i \varphi_i^{(k)T}\} = 0$.

Consider the following definitions for the second-order moment variables, for $i \geq 0$ and $r \in \{1,..,N\}$:

$$Z_{i,r} := \mathbb{E}\{z_{i,r} z_{i,r}^T\} \in \mathbb{R}^{n \times n}, \quad (13)$$

$$Z_i := \mathbb{E}\{z_i z_i^T\} = diag[Z_{i,r}] \in \mathbb{R}^{Nn \times Nn} \quad (14)$$

where $Z_{i,r}$ is given for the following recursive equation

$$Z_{i+1,r} := \sum_{j=1}^{N} p_{jr} F_{i,j} Z_{i,j} F_{i,j}^T + \sum_{j=1}^{N} p_{jr} \pi_{i,j} G_{i,j} U_i G_{i,j}^T, \quad (15)$$

being $Z_{0,r} = \Xi_r$ the initial condition.

Calculating the variances of ψ_i and $\varphi_i^{(k)}$, it is obtained, respectively

$$\Pi_i := \mathbb{E}\{\psi_i \psi_i^T\} = diag\left[\sum_{j=1}^{N} p_{jr} F_{i,j} Z_{i,j} F_{i,j}^T\right] - \mathcal{F}_i Z_i \mathcal{F}^T \quad (16)$$

$$+ \; diag\left[\sum_{j=1}^{N} p_{jr} \pi_{i,j} G_{i,j} U_i G_{i,j}^T\right], \quad (17)$$

$$R_i^{(k)} := \mathbb{E}\{\varphi_i^{(k)} \varphi_i^{(k)T}\} = \sum_{j=1}^{N} \pi_{ij} J_{i,j}^{(k)} V_i^{(k)} J_{i,j}^{(k)T}. \quad (18)$$

The Kalman filter for DMJLS in data fusion scenario problem, for the filtered estimate, will be defined as follows. Let $\hat{z}_{i|i}$ be the filtered estimate of z_i, obtained at step i, then, there exists a weighting matrix $\tilde{Z}_{i|i}$ for the filtering error $z_i - \hat{z}_{i|i}$, along with the new observation at time $i+1$, i.e., $y_{i+1}^{(k)}$, where $k = 1,...,L$. To update the filtered estimate of z_i from $\hat{z}_{i|i}$ to $\hat{z}_{i+1|i+1}$, it is proposed the solution of following problem, based in augmented system (5)

$$\min_{z_i, z_{i+1}} \left[||z_i - \hat{z}_{i|i}||^2_{\tilde{Z}_{i|i}^{-1}} + ||z_{i+1} - \mathcal{F}_i z_i||^2_{\Pi_i^{-1}} + \sum_{k=1}^{L} ||y_{i+1}^{(k)} - \mathcal{H}_{i+1}^{(k)} z_{i+1}||^2_{R_{i+1}^{(k)-1}} \right] \quad (19)$$

3 Filtering For DMJLS in Data Fusion Scenario

To solve this problem, it is necessary enunciate the following lemma, proposed by [3].

Lemma 3.1. *[3] Consider the problem of solving*

$$\hat{x} = arg\min_{x} \left[||x||^2_Q + \sum_{k=1}^{L} ||A_k x - b_k||^2_{W_k} \right], \tag{20}$$

where A is the data matrix, b_k are the measurement vectors which are assumed to be known, x is the unknown vector, $x^T Q x$ is the regularization term and $Q = Q^T > 0$ and $W_k = W_k^T \geq 0$ are given weighting matrices. The solution of (20) is given by

$$\hat{x} = \left[Q + \sum_{k=1}^{L} A_k^T W_k A_k \right]^{-1} \sum_{k=1}^{L} A_k^T W_k b_k. \tag{21}$$

Theorem 3.1. *Let $k = 1, ..., L$ be the quantity of measurement equations, for all $i \geq 0$. The optimum filtered estimates $\hat{z}_{i|i}$, resulting from (19), can be obtained from the following recursive algorithm:*

Step 0 *(Initial Condition):*

$$\begin{aligned} \tilde{Z}_{0|0} &:= (\tilde{Z}_0^{-1} + \overline{\mathcal{H}}_0^T \overline{R}_0^{-1} \overline{\mathcal{H}}_0)^{-1} \\ \hat{z}_{0|0} &:= \tilde{Z}_{0|0} \overline{\mathcal{H}}_0^T \overline{R}_0^{-1} \overline{y}_0. \end{aligned} \tag{22}$$

Step 1*: Update $\{\hat{z}_{i|i}, \tilde{Z}_{i|i}\}$ to $\{\hat{z}_{i+1|i+1}, \tilde{Z}_{i+1|i+1}\}$ as follows:*

$$\tilde{Z}_{i+1|i+1} := ((\Pi_i + \mathcal{F}_i \tilde{Z}_{i|i} \mathcal{F}_i^T)^{-1} + \overline{\mathcal{H}}_{i+1}^T \overline{R}_{i+1}^{-1} \overline{\mathcal{H}}_{i+1})^{-1}, \tag{23}$$

$$\hat{z}_{i+1|i+1} := \tilde{Z}_{i+1|i+1} (\Pi_i + \mathcal{F}_i \tilde{Z}_{i|i} \mathcal{F}_i^T)^{-1} \mathcal{F}_i \hat{z}_{i|i} + \tilde{Z}_{i+1|i+1} \overline{\mathcal{H}}_{i+1}^T \overline{R}_{i+1}^{-1} \overline{y}_{i+1}, \tag{24}$$

where

$$\overline{\mathcal{H}}_{i+1} := \begin{bmatrix} \mathcal{H}_{i+1}^{(1)} \\ \mathcal{H}_{i+1}^{(2)} \\ \vdots \\ \mathcal{H}_{i+1}^{(L)} \end{bmatrix}; \quad \overline{R}_{i+1} := \begin{bmatrix} R_{i+1}^{(1)} & 0 & \cdots & 0 \\ 0 & R_{i+1}^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{i+1}^{(L)} \end{bmatrix}; \quad \overline{y}_{i+1} := \begin{bmatrix} y_{i+1}^{(1)} \\ y_{i+1}^{(2)} \\ \vdots \\ y_{i+1}^{(L)} \end{bmatrix}. \tag{25}$$

Step 2*: The state of the original system (2) is given by*

$$\hat{x}_{i|i} := \sum_{j=1}^{N} \hat{z}_{i,j|i}. \tag{26}$$

Proof. The recursive equation is obtained from application of the Lemma 3.1. This requires prior identification between the parameters of (19) and (21), as follow:

$$\begin{aligned} x &\leftarrow \begin{bmatrix} z_i - \hat{z}_{i|i} \\ z_{i+1} \end{bmatrix}, b_k \leftarrow \begin{bmatrix} \mathcal{F}_i \hat{z}_{i|i} \\ y_{i+1}^{(k)} \end{bmatrix}, Q \leftarrow \begin{bmatrix} \tilde{Z}_{i|i}^{-1} & 0 \\ 0 & 0 \end{bmatrix}, \\ A_k &\leftarrow \begin{bmatrix} -\mathcal{F}_i & I \\ 0 & \mathcal{H}_{i+1}^{(k)} \end{bmatrix}, W_k \leftarrow \begin{bmatrix} \frac{1}{L}\Pi_i^{-1} & 0 \\ 0 & R_{i+1}^{(k)-1} \end{bmatrix}. \end{aligned} \tag{27}$$

With the previous identifications, the cost functional of the minimization problem (19) is rewritten as:

$$\begin{aligned} & \begin{bmatrix} z_i - \hat{z}_{i|i} \\ z_{i+1} \end{bmatrix}^T \begin{bmatrix} \tilde{Z}_{i|i}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_i - \hat{z}_{i|i} \\ z_{i+1} \end{bmatrix} \\ +\sum_{k=1}^{l} & \left[\left(\begin{bmatrix} -\mathcal{F}_i & I \\ 0 & \mathcal{H}_{i+1}^{(k)} \end{bmatrix} \begin{bmatrix} z_i - \hat{z}_{i|i} \\ z_{i+1} \end{bmatrix} - \begin{bmatrix} \mathcal{F}_i \hat{z}_{i|i} \\ y_{i+1}^{(k)} \end{bmatrix} \right)^T \begin{bmatrix} \frac{1}{L}\Pi_i^{-1} & 0 \\ 0 & R_{i+1}^{(k)-1} \end{bmatrix} \left(\bullet \right) \right]. \end{aligned} \tag{28}$$

According to (27) and (21), the optimum filtered estimate of x_{i+1} in data fusion scenario, $\hat{x}_{i+1|i+1}$, is obtained with the solution of the following expression:

$$\begin{aligned} \begin{bmatrix} \hat{z}_{i|i+1} - \hat{z}_{i|i} \\ \hat{z}_{i+1|i+1} \end{bmatrix} = & \begin{bmatrix} \tilde{Z}_{i|i}^{-1} + \sum_{k=1}^{L}[\frac{1}{L}\mathcal{F}_i^T \Pi_i^{-1} \mathcal{F}_i] & \sum_{k=1}^{L}[-\frac{1}{L}\mathcal{F}_i^T \Pi_i^{-1}] \\ \sum_{k=1}^{L}[-\frac{1}{L}\Pi_i^{-1}\mathcal{F}_i] & \sum_{k=1}^{L}[\frac{1}{L}\Pi_i^{-1} + \mathcal{H}_{i+1}^{(k)T} R_{i+1}^{(k)-1} \mathcal{H}_{i+1}^{(k)}] \end{bmatrix}^{-1} \\ \times & \begin{bmatrix} \sum_{k=1}^{L}[-\frac{1}{L}\mathcal{F}_i^T \Pi_i^{-1} \mathcal{F}_i \hat{z}_{i|i}] \\ \sum_{k=1}^{L}[\frac{1}{L}\Pi_i^{-1} \mathcal{F}_i \hat{z}_{i|i} + \mathcal{H}_{i+1}^{(k)T} R_{i+1}^{(k)-1} y_{i+1}^{(k)}] \end{bmatrix} \end{aligned} \tag{29}$$

pre-multiplying in both sides of the equality (29) by $\begin{bmatrix} 0 \\ I \end{bmatrix}^T$, one can obtain the following equation:

$$\begin{aligned} \hat{z}_{i+1|i+1} = \begin{bmatrix} 0 & I \end{bmatrix} & \begin{bmatrix} \tilde{Z}_{i|i}^{-1} + \mathcal{F}_i^T \Pi_i^{-1} \mathcal{F}_i & -\mathcal{F}_i^T \Pi_i^{-1} \\ -\Pi_i^{-1}\mathcal{F}_i & \Pi_i^{-1} + \sum_{k=1}^{L}[\mathcal{H}_{i+1}^{(k)T} R_{i+1}^{(k)-1} \mathcal{H}_{i+1}^{(k)}] \end{bmatrix}^{-1} \\ \times & \begin{bmatrix} -\mathcal{F}_i^T \Pi_i^{-1} \mathcal{F}_i \hat{z}_{i|i} \\ \Pi_i^{-1} \mathcal{F}_i \hat{z}_{i|i} + \sum_{k=1}^{L}[\mathcal{H}_{i+1}^{(k)T} R_{i+1}^{(k)-1} y_{i+1}^{(k)}] \end{bmatrix}. \end{aligned} \tag{30}$$

After some algebra, the filtered estimate equation $\hat{z}_{i+1|i+1}$ is given by (24) in Step i with the auxiliary variable $\tilde{Z}_{i+1|i+1}$, defined as (24). □

4 Numerical Examples

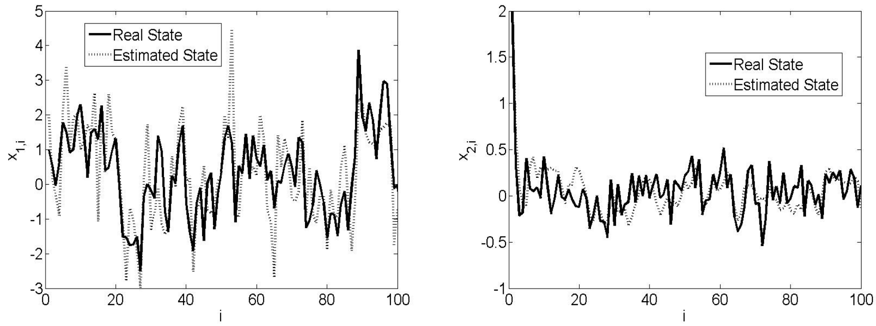
In the numerical examples, two comparative studies are presented to show the effectiveness of the DMJLS filter in data fusion scenario. Both studies we consider the probability and the parameter matrices are defined as

$$P = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix},\ F_1 = \begin{bmatrix} 0.7 & 0 \\ 0.1 & 0.2 \end{bmatrix},\ F_2 = \begin{bmatrix} 0.6 & 0 \\ 0.1 & 0.2 \end{bmatrix},\ G_1 = G_2 = \begin{bmatrix} 0.8731 & 0 \\ 0 & 0.2089 \end{bmatrix},$$

$$H_1^{(1)} = H_2^{(1)} = \begin{bmatrix} 50 & 40 \end{bmatrix},\ H_1^{(2)} = H_2^{(2)} = \begin{bmatrix} 0.00003 & 0.00002 \end{bmatrix},\ H_1^{(3)} = H_2^{(3)} = \begin{bmatrix} 0.00002 & 0.00001 \end{bmatrix},$$

$$J_1^{(1)} = J_2^{(1)} = 80, J_1^{(2)} = J_2^{(2)} = 0.00006, J_1^{(3)} = J_2^{(3)} = 0.00005.$$

First, a comparative study between the real value of the state and the filtered estimate is shown in Figure 1. Notice the DMJLS filter in data fusion scenario presented satisfactory estimate. Second, a comparative study between the DMJLS filter proposed in [1] and the DMJLS filter in data fusion scenario is shown in Figure 2.



(a) (b)

Figure 1: Real and Estimated States.

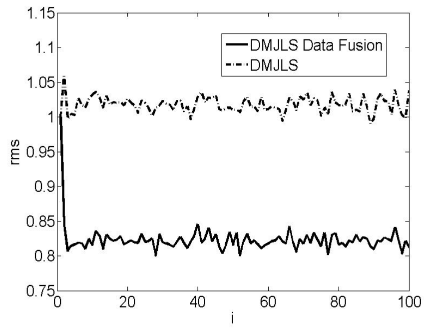


Figure 2: Comparison between Discrete-time MJLS Filter of [1] and Discrete-time MJLS Filter in Data Fusion.

In this case, the parameter matrix $H_i^{(1)}$ used in the DMJLS filter of [1] is simulating a modeling error. The square root of the mean square error (rms) estimator was simulated. The curves were obtained from $i = 0, ...100$ for each recursive step of 4000 Monte Carlo simulations with the values of Θ_i generated randomly. The initial condition x_0 is considered Gaussian with mean $\begin{bmatrix}0.196 & 0.295\end{bmatrix}^T$ and variance $\begin{bmatrix}0.0384 & 0.0578\\ 0.0578 & 0.870\end{bmatrix}$, $\Theta_i \in \{1, 2\}$, v_i and ω_i are independent sequences of noises, $\pi_1(0) = 0.05$ and $\pi_2(0) = 0.95$. Our estimator outperforms the estimator given in [1].

5 Conclusion

In this paper was developed the DMJLS Kalman filter in data fusion scenario. The estimate was deduced based on the assumption that jump parameter is not accessible. Numerical example showed the effectiveness of this approach.

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References

[1] O. L. V. Costa and S. Guerra, Stationary Filter for Linear Minimum Mean Square Error Estimation for Discrete-Time Markovian Jump Linear Systems, IEEE Transactions on Automatic Control, vol. 47, 1351–1356, (2002).

[2] Y. Gao, X. R. Li, and Z. Duan. Estimation fusion for Markovian jump linear system via data transformation. IEEE Transactions on Aerospace and Electronic Systems, vol. 50, 240-253, (2014).

[3] A. P. C. Gonçalves, A. R. Fioravanti and J. C. Geromel, Markov jump linear systems and filtering through network transmitted measurements, Signal Processing, vol. 90, 2842–2850, (2010).

[4] R. E. Kalman, A New Approach to Linear Filtering and Prediction Problems, Transactions of the ASME - Journal of Basic Engineering, vol. 82D, 35–45, (1960).

[5] W. Li and Y. Jia, Adaptive filtering for jump Markov systems with unknown noise covariance, IET Control Theory and Applications, vol. 7, 1765–1772, (2013).

[6] I. Matei and J. S. Baras, A linear distributed filter inspired by the Markovian jump linear system filtering problem, Automatica, vol. 48, 1924–1928, (2012).

[3] A. H. Sayed, T. Y. Al-Naffouri and T. Kailath, Robust Estimation for Uncertain Models in a Data Fusion Scenario, System Identification, vol. 3, 899–904, (2000).

[9] Y. Yin, P. Shiand, F. Liu and K. L. Teo, Filtering for discrete-time nonhomogeneous Markov jump systems with uncertainties, Information Sciences, vol. 259, 118–127, (2014).