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Dynamical properties in the discontinuous standard mapping

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Abstract. The dissipative discontinuous standard mapping described in the action and angle variables is considered. Defined the model, we build the phase space and chaotic attractors are observed. To characterize the chaotic attractors we have used the Lyapunov exponents. The root mean square of the action variable along of chaotic orbits is investigated. We observe that the behaviour of the root mean square diffuses unlimitedly according power law. The approach used is general and can be used in several many other systems.

Key words. Discontinuous standard mapping, dissipation, chaotic attractors, root mean square.

1 Introduction

The investigation of discrete nonlinear dynamical systems has aroused the interest of several authors in the last decades. Such mappings are parameterized by control parameters that play a fundamental role in the evolution of a system, so that the occurrence of chaos and stability can be observed.

A mapping carefully studied by many researchers is the standard mapping introduced by Chirikov [1, 2]. This mapping may be described by the variables action, angle and a control parameter k to illustrate the motion of a pendulum under the action of periodic gravitational pulses. The phase space that illustrates the behaviour of the dynamical system may present a mixed structure composed of a large chaotic sea surrounding periodic

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islands and limited by invariant spanning curves. Choosing $k = 0.9716\ldots$ controls the transition between local to global chaos.

The introduction of dissipation in the system affects the behaviour of the dynamics. There are several ways to introduce dissipation into a system [3–6]. However one of these may be done by introducing damping forces in a system by considering the relative motion of a particle within a fluid with a gas [3, 7]. In this case, the system does not area preserving in the phase space and the mixed structure is destroyed. Thus, it is possible to observe different asymptotic behaviour as the damping parameter is varied. Among them, transient effects [8], attractors fixed points [6, 9], periodic orbits [7] and chaotic attractors [10, 11].

In this paper we consider the dissipative discontinuous standard mapping [12] in the action and angle variables respectively. We build the phase space and observe a large chaotic attractor, that is characterized by Lyapunov exponents. We extend our investigation to analyze analytically and numerically the root mean square along chaotic orbits using a ensemble of different initial conditions.

To express our intentions, we organize this paper as follows: In Section 2 we present the model. In Section 3 we discuss the results. Finally in Section 4 we make the final comments.

2 The model

We consider the dissipative discontinuous standard mapping given by

$$\begin{cases} I_{n+1} = (1-\gamma)I_n + kf(\theta_n)\sin(\theta_n) \ , \\ \theta_{n+1} = \theta_n + I_{n+1} \ , \quad \mod(2\pi) \ , \end{cases}$$

where I and θ are action and angle variables respectively, $\gamma \in [0, 1]$ is the dissipation parameter and the function $f(\theta_n)$ is assumed to be a periodic function with period 2π and zero average $\overline{f(\theta_n)} = 0$. The mapping (1) to assumes the canonical form of the *dissipative standard map*, introduced by Zaslavskii [13]. However, since the function $f(\theta)$ has not been specified, the mapping (1) may also include dissipative *discontinuous* maps [12] or even dissipative polynomial maps [14] if the conditions stated above for $f(\theta)$ are relaxed. The determinant of the Jacobian matrix of the mapping (1) is $(1-\gamma)$. Thus, area preserving only when $\gamma = 0$. A large chaotic attractor is shown in Fig. 1 for the control parameters $k = 1000$ and $\gamma = 10^{-2}$ and different initial conditions.

The chaotic attractors were characterized using Lyapunov exponents. The procedure considers in evolving two neighbouring initial conditions and check whether they diverge exponentially from each other with the time. If the Lyapunov exponent is positive, say $\lambda > 0$, the system has a chaotic component. The Lyapunov exponents are defined as [15]

$$\lambda_j = \lim_{n\to\infty} \frac{1}{n} \ln |\Lambda_j^{(n)}|, \quad j = 1, 2, \tag{1}$$

where $\Lambda_j^{(n)}$ are the eigenvalues of the matrix $M = \Pi_{i=1}^{n} J_i(I, \theta)$ where J_i is the Jacobian matrix of the mapping evaluated along the orbit.

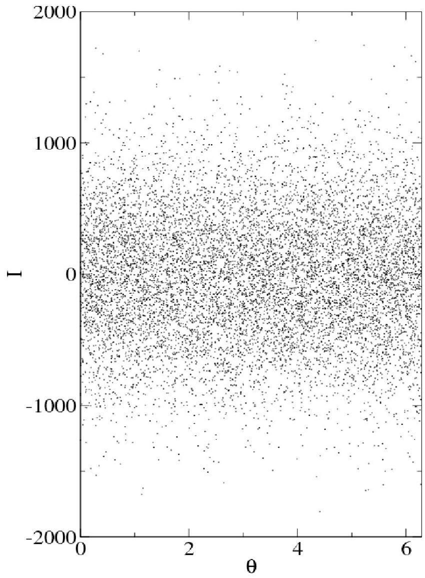

Figura 1: Phase space of the mapping (1) using the control parameters $k = 1000$ and $\gamma = 10^{-2}$.

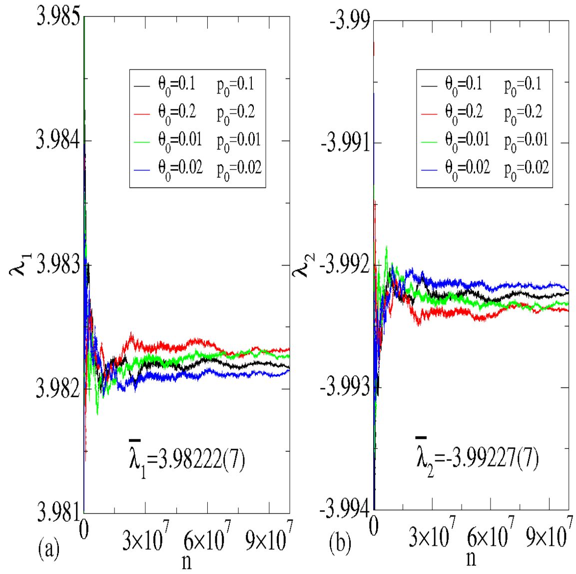

Figura 2: Plots of λ vs. n for the control parameter $k = 100$ and $\gamma = 10^{-2}$ and four different initial conditions. (a) positive and (b) negative Lyapunov exponents.

Figure 2 shows the Lyapunov exponents (a) positive and (b) negative using the control parameters $k = 100$ and $\gamma = 2$ and four different initial conditions (as labeled on the figure). We obtained a convergence for the positive Lyapunov exponent given by $\bar{\lambda}_1 = 3.98222(7)$, while the negative Lyapunov exponent provides $\bar{\lambda}_2 = -3.99227(7)$. The summation of $\bar{\lambda}_1 + \bar{\lambda}_2$ is from the order of -10^{-2}, which is of the same magnitude of

$(1-\gamma)$.

3 Results

Let us now we discuss about root mean square, $I_{RMS} = \sqrt{\overline{I^2}}$, along of chaotic orbits. It is defined as

$$I_{RMS} = \sqrt{\frac{1}{M}\sum_{j=1}^{M}\left[\frac{1}{n}\sum_{j=1}^{n} I_{j,i}^2\right]} \tag{2}$$

where M corresponds to an ensemble of different initial conditions $\theta_i \in (0, 2\pi)$ randomly chosen for a fixed $I_0 \approx 0$. Fig. 3 shows the behaviour of $I_{RMS} \times n$ using $k = 100$ and $\gamma = 0$. We observe that I_{RMS} grows according power law and a fitting provides $\beta = 0.498(8) \simeq 1/2$.

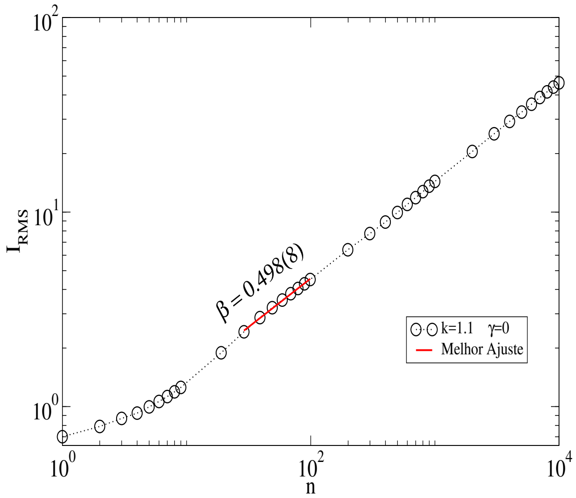

Figura 3: Plot of $I_{RMS} \times n$ of the mapping (1). The control parameters used are $k = 100$ and $\gamma = 0$. A power law fittings to provides $\beta \simeq 1/2$.

Let us now discuss the behaviour of the I_{RMS}. From the first equation of mapping (1) using $\gamma = 0$ and after squaring both sides, we obtain

$$I_{n+1}^2 = I_n^2 + 2KI_n f(\theta_n)\sin(\theta_n) + k^2 f^2(\theta_n)\sin^2(\theta_n) \ .$$

An average over an ensemble of $\theta \in [0, 2\pi]$ and knowing that $\overline{\sin(\theta_n) = 0}$ and $\overline{\sin^2(\theta_n)} = \frac{1}{2}$, we have

$$\overline{I_{n+1}^2} = \overline{I_n^2} + \frac{k^2}{4} \ . \tag{3}$$

Using the approximation $\overline{I^2}_{n+1} - \overline{I^2}_n \cong \frac{d\overline{I^2}^2}{dn}$ and after doing the corresponding interation using the appropriate limits, we end up with

$$\overline{I^2} = I_0^2 + \frac{k^2}{4}n \ . \tag{4}$$

According to the definition of $I_{RMS} = \sqrt{\overline{I^2}}$, we obtain

$$I_{RMS} = \sqrt{I_0^2 + \frac{k^2}{4} n} \ . \tag{5}$$

Let us the discuss the implication of equation (5) for a specific range of n. When $\frac{k^2}{4} n \gg I_0^2$, we obtain

$$I_{RMS} \propto n^{1/2} \tag{6}$$

This result confirm that, in the absence of invariant spanning curves, the action variable diffuses unlimitedly according a power law with $\beta \propto 1/2$.

4 Comments

To summarize our conclusions, we have studied in this work some dynamical properties in dissipative discontinuous standard mapping. The Lyapunov exponents were used to characterize the chaotic attractors. The root mean square was analyse numerically and analytically along chaotic orbits and shown grows according power law provides $\beta \simeq 1/2$.

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