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Robust $\mathfrak{D}$-Stable Control with Gain Minimization of a 2DOF Helicopter

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Abstract. This work presents the design of a robust $\mathfrak{D}$-stable state-feedback controller for a 2 degrees of freedom (DOF) helicopter with uncertain dynamics. The controller is suited for practical implementation through gain minimization and the controller design is carried out with linear matrix inequalities (LMIs). The objective is obtain good performance despite the model uncertainties. A practical implementation validates the effectiveness of the controller in an experimental environment.

Palavras-chave. Robust control, LMI, Gain Minimization, 2DOF Helicopter, Uncertain systems, $\mathfrak{D}$-stability.

1 Introduction

The linear matrix inequality formulation has been consistently used throughout years to achieve an extensive number of control objectives such as eigenvalues restriction [6], controller norm minimization [1] and disturbance rejection [7]. As stated in [2], recent developments in the optimization algorithms made the process of solving these problems extremely efficent [5], with convergence in polynomial time [4].

In this context, the main focus of this work is the design of a state feedback robust controller for a 2DOF didactic helicopter with uncertain yaw and pitch dynamics. Moreover, aiming at adequate performance characteristics, a $\mathfrak{D}$-stability approach was used to allow the eigenvalues to be placed in a desirable region. Finally, to guarantee a state-feedback controller suitable for practical implementation, a minimization of the norm of the controller was carried out. The solution of the ensuing semidefinite programming problem was performed in Matlab® 2014a environment through the default package of the robust control toolbox, LMI Lab.

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Throughout this work, the following notation was used: $He\{.\}$ is the hermitian operator, $He\{Z\} = Z^T + Z$ with $Z \in \mathbb{R}^{q \times q}$ and the symbol (*) denotes a generic symmetric block in a symmetric matrix.

2 Robust $\mathfrak{D}$-Stable Control with Gain Minimization

Consider a linear, time invariant system with polytopic uncertanties described as:

$$\begin{aligned} \dot{x}(t) &= A(\gamma)x(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ the control signals vector and $A(\gamma)$, B and C are properly dimensioned matrices which characterize the dynamic system, being the first one uncertain and defined as:

$$A(\gamma) = \sum_{i=1}^{v} \gamma_i A_i, \tag{2}$$

with

$$\gamma_i \geq 0,\ \sum_{i=1}^{v} \gamma_i = 1,\ \gamma = [\gamma_1, \gamma_2, ..., \gamma_v]^T, \tag{3}$$

where $v = 2^s$ and s is the number of uncertain parameters in $A(\gamma)$.

An usual control law is given by

$$u(t) = -Kx(t), \tag{4}$$

where K is a state-feedback gain. Applying the law in (4) to the system in (1) the following closed loop system is obtained:

$$\begin{aligned} \dot{x}(t) &= (A(\gamma) - BK)x(t), \\ y(t) &= Cx(t). \end{aligned} \tag{5}$$

To ensure a desirable transient response to the system, one could restrict the closed loop eigenvalues to a adequate region in the complex plane. Let $\mathcal{D}_r$, $\mathcal{D}_\theta$ and $\mathcal{D}_\alpha$ denote subsets of the complex plane, defined as:

$$\mathcal{D}_r = \{(x, y) \in \mathbb{R}^2 \mid |x + jy| < r\}, \tag{6}$$

$$\mathcal{D}_\theta = \{(x, y) \in \mathbb{R}^2 \mid x \tan\theta < -|y|\}, \tag{7}$$

$$\mathcal{D}_\alpha = \{(x, y) \in \mathbb{R}^2 \mid x < -\alpha < 0\} \tag{8}$$

then the imposition of the three regions concurrently places the eigenvalues in a target region given by

$$\mathcal{D} = \mathcal{D}_r \cap \mathcal{D}_\theta \cap \mathcal{D}_\alpha. \tag{9}$$

The necessary conditions to solve the problem of finding a state-feedback gain K that places the eigenvalues of (5) in the target area (9) are given by the next theorem.

Theorem 2.1. *Consider the system* (5). *If there exist matrices* $X = X^T > 0$ *and* M*, with apropriate dimensions, such that*

$$He\{A_iX - BM\} + 2\alpha X < 0, \tag{10}$$

$$\begin{bmatrix} -rX & A_iX - BM \\ * & -rX \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} He\{A_iX - BM\}\sin\theta & \Gamma \\ * & He\{A_iX - BM\}\sin\theta \end{bmatrix} < 0, \tag{12}$$

where $\Gamma = \cos\theta(A_iX - XA_i^T - BM + M^TB^T)$ *and* $i \in \mathbb{N}^*$*, then a state-feedback gain that places the eigenvalues in the target area is given by* $K = MX^{-1}$ *[3].*

Additionaly, one could impose a restriction in the magnitude of the state-feedback gain to avoid high control signal amplitudes. A solution for this problem is presented in [1] with the following theorem.

Theorem 2.2. *Given a constant* $\mu_0 > 0$*, then the requirement of the restriction of the state-feedback gain* K *is obtained by the minimization of a term* $\beta > 0$*, such that* $KK^T < \beta I \mu_0^2$*. The optimal value of* β *can be obtained with the resolution of the following optimization problem:*

$$\begin{aligned} & min\ \beta \\ & s.t \begin{bmatrix} \beta I & M \\ * & I \end{bmatrix} > 0, \end{aligned} \tag{13}$$

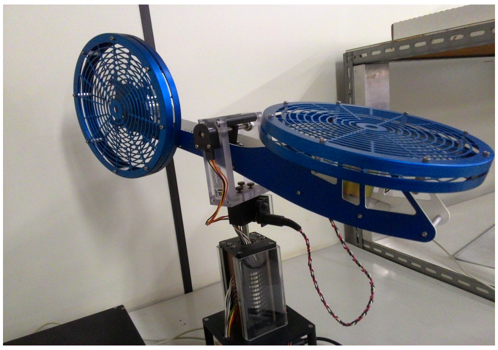
$$X > \mu_0 I. \tag{14}$$

If (14) *is feasible, then the minimum state-feedback gain is given by* $K = MX^{-1}$.

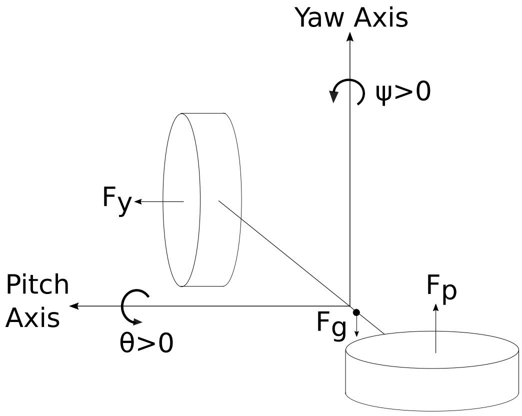
3 2-DOF Helicopter

The plant and its respective schematic diagram can be seen in Fig. 1. It is a didactic system that aims to reproduce the dynamic of a conventional helicopter with a front horizontal rotor and a back vertical rotor. The plant has two degrees of freedom in terms of movement: the rotations in the pitch and yaw axes, defined in Fig. 1. The actuadors of the system are two direct current motors, coupled to the center and tail of the helicopter and responsable for pitch and yaw movements through pitch (F_p) and yaw forces (F_y), respectively. The gravity force (F_g) in the center of mass is parallel to the pitch force.

The equations of motion of the 2DOF helicopter were derived in [8]. The state and control vectors are defined as $x(t) = [\theta\ \psi\ \dot{\theta}\ \dot{\psi}]^T$ and $u(t) = [V_p\ V_y]^T$, where θ is the pitch angle, ψ the yaw angle, $\dot{\theta}$ and $\dot{\psi}$ the respective angular velocities and V_p and V_y the voltages from the front and back motors. The system was linearized around the equilibrium point $\bar{x} = [0\ 0\ 0\ 0]^T$ and $\bar{u} = [12.535V - 3.1827V]^T$. The resulting state-space realization is



(a)



(b)

Figure 1: a) 2-DOF QUANSER® Helicopter b) Schematic Diagram

equivalent to (1), with:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -9.260 & 0 \\ 0 & 0 & 0 & -3.487 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2.361 & 0.079 \\ 0.240 & 0.790 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{15}$$

4 Results and Discussion

In this section the controller design and practical implementation will be displayed and discussed.

4.1 Controller design

Initially, integrators were added to the system to obtain zero error in steady state for a step reference. Due to the model simplification assumptions made in obtaining the linear model (15), model-plant mismatch might be an issue. To adress such mismatch during the design, robust control techniques may be employed. This mismatch can be formulated as polytopic uncertanties in the state-space matrix A, more specifically, by deviations in the yaw and pitch velocity dynamics. In order to achieve the desired degree of robustness, the following polytope vertices are proposed:

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -9.260 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.487 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 9.260 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.487 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -9.260 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.487 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 9.260 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.487 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{16}$$

In this context, the control technique presented in (5) with a state-feedback gain K given by Theorems 2.1 and 2.2 is utilized. The $\mathfrak{D}$-stability parameters are tuned as $r = 32$, $\theta = 35^\circ$ and $\alpha = 0.5$ to ensure appropriate response velocity and overshoot characteristics. Moreover, the minimization parameter is tuned as $\mu_0 = 0.02$ to guarantee a state-feedback gain with adequate magnitude for practical implementations. The following gain and restriction parameter β were obtained:

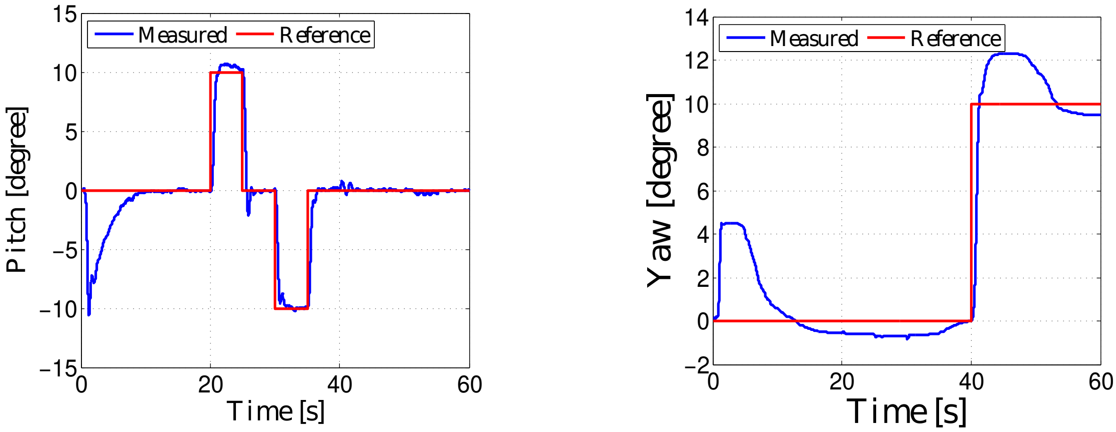
$$K = \begin{bmatrix} 20.1361 & -1.5329 & 10.2203 & -0.9802 & 8.5657 & -0.5892 \\ -4.7979 & 30.2227 & -2.3946 & 19.5421 & -2.0657 & 11.7122 \end{bmatrix}, \ \beta = 106.6110, \tag{17}$$

validating the effectiveness of the procedure due to the appropriate magnitude and correct placement of the eigenvalues.

4.2 Practical implementation

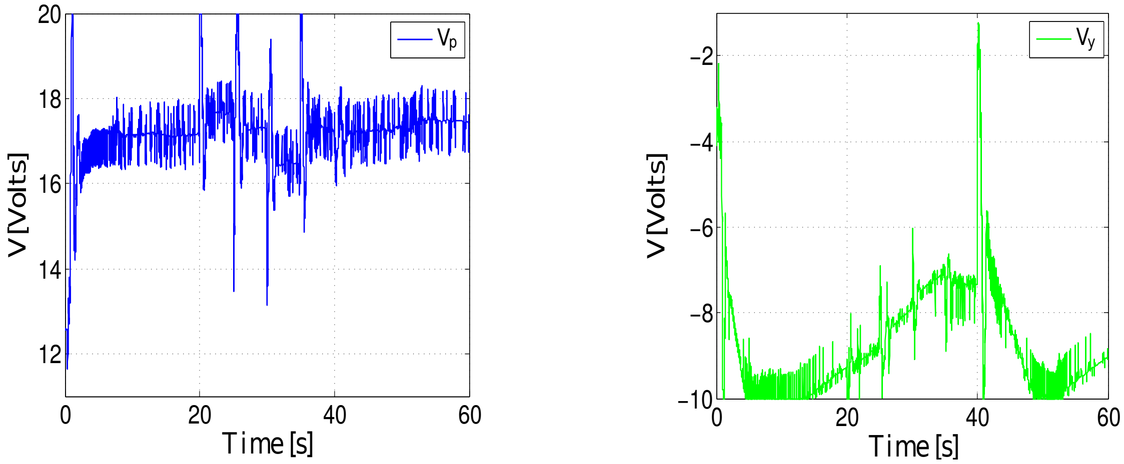
In order to verify the effectiveness of the controller (17), experiments were carried out with the plant. To evaluate the performance of the system in terms of tracking, references were imposed to both pitch and yaw angles. Since the helicopter was linearized around perfect horizontal alignment, the manuever consists primarily in pitch deviations of $\pm 10^\circ$ around that angle (0°) and keeping the yaw regulated around 0°. Then a yaw movement of 10° is requested while the pitch is regulated around 0°. The results can be observed in Figs. 2(a) and 2(b).

After an accomodation period of 10 seconds, the tracking capabilities of the system could be analyzed. It is clear from Fig. 2(a) that satisfactory pitch tracking was obtained, both positive and negative pitch values were reached sucessfully with adequate overshoot values. One can observe from Fig. 2(b) that the more complicated yaw dynamic is well handled by the robust controller, providing acceptable values of error and overshoot. It is worth noting a meaningful improvement in terms of decoupling of pitch and yaw manuevers compared to previous results as [8]. The voltage deviations can be observed next.



(a) (b)

Figure 2: Step responses: (a) Pitch and (b) Yaw.



(a) (b)

Figure 3: Control signals: a) Pitch motor voltage b) Yaw motor voltage.

For a safe operation it is desirable to keep the excursion of the control signal within the following margins: $0 \leq V_p \leq 20$ [V] and $-10 \leq V_y \leq 10$ [V]. From Fig. 3 it can be stated that the control signal that remains most time saturated is V_y. The initial saturation between 5s and 15s can be attributed to the beginning of the maneuver far from the equilibrium point. The shorter saturation around 50s is responsible for the prolonged time that the yaw angle remains above the reference.

5 Conclusions

In this paper the incidence of polytopic uncertanties in the model of the 2DOF helicopter were considered and treated by a $\mathfrak{D}$-stable robust controller suited for practical implementation through gain minimization. The experiments showed good results in terms of reference tracking, with adequate transient response. The pitch and yaw dynamics were sufficiently decoupled allowing the manuevers to occur separately with no observable interference.

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