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An Earth-to-Earth Mission with a Lunar Flyby

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Abstract. The present work describes an Earth-to-Earth mission with a lunar flyby: initially, the spacecraft is at a low Earth orbit and it is desired to perform an orbital transfer assisted by a lunar flyby to another orbit around the Earth with a different inclination. In order to solve the transfer problem, two models are considered: the first one is based on a tridimensional patched-conic approximation, and, the second one is based on the spatial circular restricted three-body problem (SCR3BP).

Keywords. Lunar flyby, spatial patched-conic approximation, spatial three-body problem, Earth-to-Earth trajectory

1 Introduction

Several authors study space trajectories considering the planar motion of the spacecraft; however, if a spacecraft departs from a low Earth orbit (LEO) towards the Moon and with a considerable inclination related to the orbital motion of the Moon, the transfer problems based on the planar models is not suited for the determination of the trajectory. Besides that, the fuel consumption can be highly altered in this kind of transfer problem with changing plane [4]. For instance, the velocity increment applied to a satellite to produce a changing of 60° in the inclination of its orbit around the Earth is equivalent to its circular velocity in the context of the two-body problem [1]. In order to decrease the fuel consumption during the change of inclination between two orbits around the Earth, tridimensional models with the gravitational influence of the Moon can be utilized.

The present work concentrates in analyzing this kind of maneuver where, initially, the spacecraft is at a low Earth orbit and it is desired to perform an orbital transfer assisted by a lunar flyby to another orbit around the Earth with a different inclination. In order to solve the transfer problem, two models are considered: the first one is based on a tridimensional patched-conic approximation [2], and, the second one is based on the spatial circular restricted three-body problem (SCR3BP) [3]. This kind of transfer is

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useful for several applications, which include missions to transfer a spacecraft from a low inclination orbit into a geosynchronous orbit, or a sun-synchronous orbit, or even orbits with the inclination value of the Molniya orbit.

2 Objectives

The present work concentrates in analyzing the fuel consumption of Earth-to-Earth missions where, initially, the spacecraft is inserted at a circular low Earth orbit (LEO) with a given altitude h_i, inclination I_i, and longitude of the ascending node Ω_i. After an application of a first impulsive velocity increment, Δv_1, the spacecraft is inserted into a transfer trajectory which performs a flyby maneuver with the Moon that changes the plane of motion of the spacecraft. When the spacecraft returns close to Earth, a second impulsive velocity increment, Δv_2, is applied to decelerate and circularize the movement of the spacecraft in a prescribed circular low or high Earth orbit with a given altitude h_f, inclination I_f, and longitude of the ascending node Ω_f. The total velocity increment represents the fuel consumption.

In order to solve this problem, a new 3D patched-conic approximation associated with a two-point boundary value problem is proposed. This new 3D patched-conic approximation is an extension of the 3D lunar patched-conic approximation [2]. The same transfer problem is formulated considering the spatial restricted three-body problem (SR3BP). The results of the patched-conic approximation is compared with the results of the SR3BP.

3 Formulation

In this section, the spatial patched-conic approximation and the spatial circular restricted three-body problem (SCR3BP) for an Earth-to-Earth mission are briefly formulated by means of two two-point boundary value problems (TPBVP). These two models (patched-conic approximation and SCR3BP) are extensions of the formulations of the Earth-Moon missions presented by [2].

3.1 The Transfer Problem

For an Earth-to-Earth mission the spacecraft is, initially, inserted at a circular low Earth orbit (LEO) O_i with a given altitude h_i, inclination I_i, and longitude of the ascending node Ω_i. The subscript i indicates that the quantities are related to the initial parking orbit O_i. With an application of an accelerating velocity increment Δv_1, the spacecraft is inserted into a transfer trajectory which performs a flyby maneuver with the Moon. When the spacecraft returns close to Earth, a second velocity increment but decelerating, Δv_2, is applied in order to circularize its motion around the Earth at the final orbit O_f with a given altitude h_f, inclination I_f, and longitude of the ascending node Ω_f. The subscript f indicates that the quantities are related to the final orbit O_f around the Earth. The total velocity increment, given by

$$\Delta v_{Total} = \Delta v_1 + \Delta v_2 \tag{1}$$

represents the fuel consumption of the spacecraft [1].

3.2 The Patched-Conic Approximation

For the patched-conic approximation, the transfer trajectory has three distinct phases: a first elliptical geocentric trajectory that describes the departure of the spacecraft from the initial orbit at the Earth; an hyperbolic selenocentric trajectory that describes the lunar flyby; and a second elliptical geocentric trajectory that describes the arrival of the spacecraft at the final orbit around the Earth. The characterization of these phases is done utilizing the concept of the sphere of influence (SOI); in this way, the selenocentric phase is described inside the SOI of the Moon. The bi-impulsive model is considered with the velocities increment applied at the initial and final orbits O_i and O_f, respectively. The impulses are not, necessarily, tangential to these terminal orbits as they are responsible by changing in the orbital plane of motion of the spacecraft. However, these impulses are perpendicular to the respective position vector of the spacecraft. The Moon's orbit around the Earth is considered to be circular.

After the first velocity increment, the spacecraft is inserted into an elliptic geocentric orbit O_g, whose Keplerian elements are: a_g (semi-major axis), e_g (eccentricity), I_g (inclination), Ω_g (longitude of the ascending node), and ω_g (pericenter argument). The subscripts g indicates the quantities are related to the geocentric orbit O_g. In the geocentric phase, only the gravitational field of the Earth is considered. When the space vehicle reaches the SOI of the Moon, the selenocentric phase begins, where only the gravitational field of the Moon is considered. In this phase, the trajectory of the vehicle is characterized by a hyperbolic selenocentric orbit O_s, whose Keplerian elements are: a_s, e_s, I_s, Ω_s, and ω_s. The subscripts s indicates the quantities related to the selenocentric orbit O_s. When the spacecraft reaches the limit of the SOI of the Moon, a second geocentric trajectory begins, whose Keplerian elements are: $a_{g,2}$, $e_{g,2}$, $I_{g,2}$, $\Omega_{g,2}$, and $\omega_{g,2}$. The subscripts $g, 2$ indicates the quantities are related to the geocentric orbit $O_{g,2}$. The inertial reference frame B_{XYZ} adopted has its origin centered at Earth, the x-axis points towards the Moon at the initial time t_0 (instant of application of the first velocity increment), the y-axis is orthogonal to the x-axis and positive to the direction of the orbital motion of the Moon around Earth, thus the xy-plane matches with the orbital plane of the Moon, and the z-axis is orthogonal to both x-axis and y-axis according to the right hand system. The patched-conic approximation is solved when the velocity vector $\mathbf{v_0}$ of the space vehicle relative to Earth and the phase angle $\theta_{EP}(0)$ of the spacecraft relative to Earth, both defined immediately after the first velocity increment, are determined. Since the orbit O_i is circular, the phase angle $\theta_{EP}(0)$ is the true latitude; that is, $\theta_{EP}(0)$ is the angle measured from the line of nodes to the initial position vector $\mathbf{r_0}$, in the sense of movement, beginning at the ascending node. The determination of the velocity vector $\mathbf{v_0}$ depends on two unknown: the magnitude v_0 and the angle α_E defining the mutual inclination between the orbits O_i and O_g. This angle defines the direction of the velocity vector $\mathbf{v_0}$. The velocity increment Δv_1 is assumed to occur at the pericenter of the geocentric orbit O_g, therefore the true anomaly $f_g(0)$ and the flight path angle $\varphi_g(0)$ at the initial time are equal to $0°$. In this way, the pericenter distance $r_g(0)$ is equal to the radial distance r_0.

The mathematical formulation as seen in [2] describes the motion of the spacecraft from the departure of the orbit O_i until the periselenium of the selenocentric trajectory. In order to extend this formulation, the motion of the spacecraft must by continued until it reaches the limit of the SOI of the Moon performing a complete flyby maneuver. At this point, the selenocentric hyperbolic trajectory ends, and, the an second geocentric elliptical trajectory begins. When the spacecraft reaches the perigee of this second geocentric trajectory, the second velocity increment is applied to circularize the motion of the spacecraft in the orbit O_f around the Earth.

The two-point boundary value problem to solve this transfer is enunciated as the same way as in [2]. Therefore, the set of variables $(v_0, \theta_{EP}(0))$ is solved, with a given value of α_E, to satisfy two constraints. The first constraint satisfies the specified altitude h_f of the orbit O_f; and, the second constraint satisfies the arrival geometry at the orbit O_f. This TPBVP is solved by means of Newton-Raphson algorithm.

3.3 The Spatial Circular Restricted Three-Body Problem

The Earth-to-Earth transfer problem based on the spatial circular restricted three-body problem (SCR3BP) is formulated in the same way as seen in [2] with the same hypotheses. For this model Earth and Moon move in circular orbits around the center of mass (barycenter) of the Earth-Moon system.

The inertial reference frame B_{XYZ} to describe the motion of the spacecraft for this model has its origin at the center of mass B of the Earth-Moon system; the x-axis points towards to the Moon position at the initial time $t_0 = 0$ (time immediately after the application of the velocity increment), the y-axis is perpendicular to the x-axis and positive to the direction of the orbital motion of the Moon around Earth, thus the xy-plane matches with the orbital plane of the Moon, and the z-axis is orthogonal to both x-axis and y-axis according to the right hand system

In the B_{XYZ} reference frame, the motion of the spacecraft (P) is described by the following differential equations:

$$\ddot{x}_P = -\frac{\mu_E}{r_{EP}^3}(x_P - x_E) - \frac{\mu_M}{r_{MP}^3}(x_P - x_M) \tag{2}$$

$$\ddot{y}_P = -\frac{\mu_E}{r_{EP}^3}(y_P - y_E) - \frac{\mu_M}{r_{MP}^3}(y_P - y_M) \tag{3}$$

$$\ddot{z}_P = -\frac{\mu_E}{r_{EP}^3}(z_P - y_E) - \frac{\mu_M}{r_{MP}^3}(z_P - y_M) \tag{4}$$

where x_P, y_P, e z_P are the position vector components of the spacecraft; x_E, y_E, e z_E are the position vector components of the Earth; x_M, y_M, e z_M are the position vector components of the Moon; and r_{EP}, and r_{MP} are, respectively, the spacecraft distance from Earth and from Moon. μ_E and μ_M are the gravitational parameters of the Earth and the Moon,respectively.

For this model, the TPBVP has the formulation as described by [3], since the lunar flyby maneuver occurs naturally from the dynamics of the three-body problem. But, it should be noted that the final constraints and the arrival geometry are now related to the

Earth. Therefore, the set of variables $(v_0, \theta_{EP}(0), T)$ is solved, with a given value of α_E, to satisfy three constraints. The first constraint satisfies the specified altitude h_f of the orbit O_f; the second constraint satisfies the arrival geometry at the orbit O_f; and, the third constraint satisfies the orthogonality between the position and the velocity vectors of the spacecraft related to Earth at the final time. This TPBVP is solved by means of Newton-Raphson algorithm.

4 Results

In this section is discussed an Earth-to-Earth mission which consists in transferring a spacecraft from a circular LEO to a circular LEO with same radius considering a large modification in the inclination of the orbital plane and the longitude of the ascending node. Table 1 shows the values of the quantities that specifies the transfer problem. The TPBVP for both model are solved by means of Newton-Raphson algorithm. These both TPBVP utilize the variable α_E as a parameter, i.e., the value of this variable is given together with the specification of the terminal orbits.

Tabela 1: Parameters specifications.

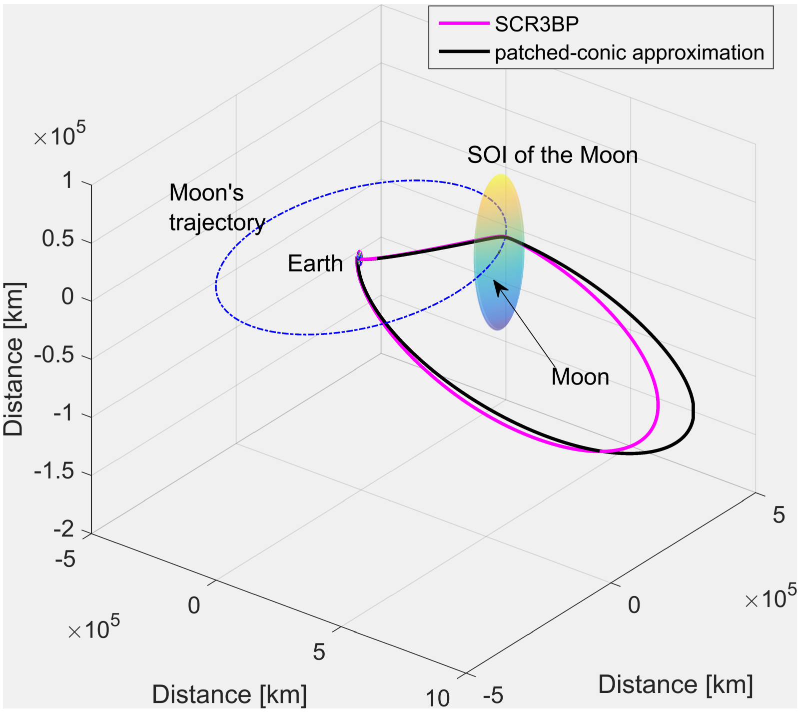
R_E (Earth radius)	6378.2 km
R_M (Moon radius)	1738 km
μ_E	398600 km^3/s^2
μ_M	4902.83 km^3/s^2
D (mean Earth-Moon distance)	384400 km
R_S (radius of SOI of the Moon)	66300 km

Table 2 shows that Δv_{Total} of the patched-conic approximation and the Δv_{Total} of the transfer based on the SCR3BP are close to Δv_{Total} of the bi-parabolic transfer, which is a theoretical transfers in which the changing of inclination of the motion is performed at an infinite distance.

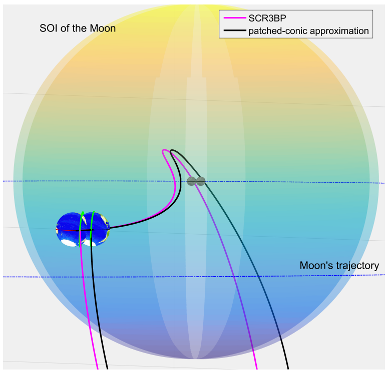
The comparison between the lunar flyby transfer with the single-impulse maneuver reveals that the lunar flyby saves fuel consumption: for the transfer of Table 2 the decrease of fuel consumption is 4.248 km/s for the model based on the SCR3CP. Figure 1 depicted the trajectories showed by Table 2 considering the patched-conic approximation and the SCR3BP in order to highlight the good agreement between these models.

Tabela 2: Results of orbital transfer considering several models. Terminal orbits: $I_i = 2°$, $\Omega_i = 0.5°$, $h_i = 167\ km$, $I_f = 86°$, $\Omega_f = 212°$ e $h_f = 167\ km$.

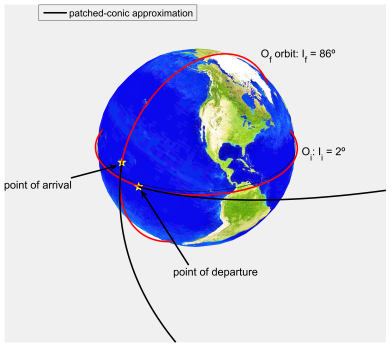
Model	Δv_1 [km/s]	Δv_2 [km/s]	Δv_{Total} [km/s]	Time of flight [days]	h_{sP} [km]	α_E [degrees]
patched-conic	3.247386	3.252111	6.499497	35.802	17130.445	3.00
SCR3BP	3.251273	3.313738	6.565011	30.429	16646.913	3.31
bi-parabolic	3.232448	3.232448	6.464895	infinity	–	0
single-impulse	–	–	10.813121	0	–	87.7



(a) Inertial reference frame centered at the barycenter.



(b) Visualization of lunar flyby.



(c) Departure and arrival points.

Figura 1: Transfer between non coplanar orbits with same altitudes. Terminal orbit specification: $I_i = 2°$, $\Omega_i = 0.5°$, $h_i = 167\ km$, $I_f = 86°$, $\Omega_f = 212°$ and $h_f = 167\ km$.

5 Conclusions

This work studies Earth-to-Earth missions with an intermediate lunar flyby maneuver. Only two-impulsive trajectories are considered. The first velocity increment is accelerative and it inserts the spacecraft into a transfer trajectory, which performs a flyby maneuver with the Moon. As the changing plane of the maneuver is assisted by the gravitational field of the Moon during this flyby maneuver, an amount of fuel consumption can be saved. The second velocity increment is applied to decelerate and circularize the motion of the spacecraft at the final circular orbit around the Earth. In order to solve this problem, a 3D patched-conic approximation associated with a two-point boundary value problem is proposed. The same transfer problem is formulated considering the spatial restricted three-body problem (SR3BP). The results of the patched-conic approximation is compared with the results of the SR3BP showing a good agreement between the models. The changing plane assisted by a lunar flyby can be very favorable for saving fuel. Despite the increase of the time of flight, the saving of fuel is considerable. Indeed, the total velocity increment of this kind of maneuver is close to the velocity increment provided by the bi-parabolic transfer. The saving of fuel consumption of Earth-to-Earth mission with a lunar flyby can exceed the equivalent to a velocity increment equal to 4 km/s when this maneuver is compared to a single-impulse maneuver.

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